

## Annalen der Physik IV, Folge 4, 553-563 (1901)

TRANSLATED by Peter Möck, parts that require a deeper understanding of statistical mechanics and thermodynamics are omitted, literature references are omitted, slight modifications are made in order to be more easily comprehend by my “Introduction to Modern Physics” students

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### *9. On the distribution law of energy in the normal spectrum*

by Max Planck

(presented in different form at the German Physical Society Meeting on October 19 and December 14, 1900, proceedings 2, p. 202 and p. 237, 1900)

#### **Introduction**

The new measurements of spectra by O. Lummer and E. Pringsheim <sup>1</sup> and even more conspicuously those by H. Rubens and F. Kurlbaum <sup>2</sup>, which simultaneously confirm a result obtained earlier by H. Beckmann <sup>3</sup> have shown that the energy distribution law in the normal spectrum, which was first derived by Wien from molecular kinetic considerations and later by myself from the theory of electromagnetic radiation, is not generally valid.

On any account, the theory behind the energy distribution law needs an improvement and I will try in the following to make an attempt in this direction on the basis of my own theory of electromagnetic radiation. In order to do so, it will be necessary to identify the very link in the chain of reasoning which lead to Wien's energy distribution law that is amendable to change; to remove that link from the chain and replace it with a suitable substitute.

In my last paper <sup>4</sup>, I have shown that the physical foundation of the electromagnetic theory of radiation including the hypothesis of the “natural radiation” withstand severe falsifications. As the calculations do not, as far as I can see, contain errors, the notion stands that the distribution law of energy in the normal spectrum will be entirely determined when the entropy  $S$  of an irradiated, monochromatically oscillating resonators is known as a function of his oscillation energy  $U$ . From the relation  $dS/dU = 1/\vartheta$  one can obtain the dependence of the energy  $U$  on the temperature  $\vartheta$ . Since, on the other hand, the energy  $U$  is connected to the radiation density of the corresponding oscillation number by a simple relation <sup>5</sup>, one obtains by this route also the dependence of the radiation density on the temperature. The normal energy distribution is then the one at which the radiation densities of all the different oscillation numbers possess the same temperature.

The whole problem, thus, is reduced to finding  $S$  as a function of  $U$  and this will constitute the main part of this investigation. In my first paper on this topic I introduced  $S$  directly per definition without further justification as a simple relation of  $U$  and showed that this relation suffices all requirements from thermodynamics. At that time, I believed that this is actually the only relation of its kind and that Wien's law, which follows from

it, must with necessity be generally valid. On closer inspection later on <sup>6</sup>, I realized that there must be other relations that are equally valid and that there must, therefore, be another conditions to calculate S unambiguously. Such a condition, I thought at the time to have found in the plausible appearing notion that the increase in the total entropy  $S_N = N S$  resulting from an infinitesimal small change of a system of N identical resonators that are nearly in thermodynamical equilibrium only depends on the total energy  $U_N = N U$  and its change, but not from the energy U of the individual resonators. This notion, with necessity, lead again to Wien's energy distribution law. Since Wien's law is not bore out by observations, one has to arrive at the conclusion that this notion is incorrect and must therefore be removed from the theory <sup>7</sup>.

What remains to be done is finding another condition which allows for the calculation of S. ... In the following a new relation for the entropy will be derived which results in a novel radiation formula, which does not disagree with any currently know observation.

## I. Calculation of the entropy of a resonator as a function of its energy

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$$\S 2. S_N = k \log W + \text{const.} \quad (3)$$

\S 3. It is now important to find the probability W that the N resonators as a whole possess the vibrational Energy  $U_N$ . For that it is important to consider  $U_N$  not as a smooth, arbitrarily dividable entity. On the contrary,  $U_N$  must be considered as a discrete entity that is composed of an integer number of finite identical parts. Calling such an energy increment  $\epsilon$ , we obtain

$$U_N = P \epsilon \quad (4)$$

whereby P is an integer which is generally large and the value of  $\epsilon$  will be derived later.

.....

\S 3. ...

Hence, after equation (2) the entropy S of a resonator as a function of its energy U is:

$$S = k \left\{ \left(1 + \frac{U}{e}\right) \log \left(1 + \frac{U}{e}\right) - \frac{U}{e} \log \frac{U}{e} \right\} \quad (6)$$

## II. Introduction of Wien's Displacement Law

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\S 10. Applying Wien's law in its last formulation to the expression of the entropy S (6), we see that the energy element  $\epsilon$  must be proportional to the frequency  $\nu$ , i.e.

$$\varepsilon = h \nu$$

and thus:

$$S = k \left\{ \left(1 + \frac{U}{h\nu}\right) \log \left(1 + \frac{U}{h\nu}\right) - \frac{U}{h\nu} \log \frac{U}{h\nu} \right\}$$

whereby h and k are universal constants.

Substituting in (9) one obtains:

$$\frac{1}{n} = \frac{k}{h\nu} \log \left(1 + \frac{h\nu}{U}\right),$$

$$U = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1} \quad (11)$$

and from (8) the energy distribution law we searched for follows

$$u = \frac{8\pi h \nu^3}{c^3} \cdot \frac{1}{e^{\frac{h\nu}{kT}} - 1} \quad (12)$$

and substituting the frequency (?) with the wavelength (?)

$$E = \frac{8\pi ch}{\lambda^5} \cdot \frac{1}{e^{\frac{ch}{\lambda T}} - 1} \quad (13)$$

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### III. Numerical Values

§ 11. The values of the two constants of nature h and k can be calculated rather accurately on the basis of the existing measurements.

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$$h = 6.55 \cdot 10^{-27} \text{ erg sec}$$

$$k = 1.346 \cdot 10^{-16} \text{ erg/grad Boltzmann's constant}$$

(Received January 7<sup>th</sup>, 1901)

## Comment by translator

today's best values are

$$h = 6.6260755(40) \cdot 10^{-34} \text{ Js}$$

$$k = 1.380658(12) \cdot 10^{-23} \text{ J/K}$$

i.e. Plank's calculations were too low by only  $\sim 1.15\%$  and  $2.51\%$ , respectively.