Quantum Mechanics
To Angus and Gavin
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Preface to Fourth Edition

When I told a friend that I was working on a new edition, he asked me what had changed in quantum physics during the last ten years. In one sense very little: quantum mechanics is a very well established theory and the basic ideas and concepts are little changed from what they were ten, twenty or more years ago. However, new applications have been developed and some of these have revealed aspects of the subject that were previously unknown or largely ignored. Much of this development has been in the field of information processing, where quantum effects have come to the fore. In particular, quantum techniques appear to have great potential in the field of cryptography, both in the coding and possible de-coding of messages, and I have included a chapter aimed at introducing this topic.

I have also added a short chapter on relativistic quantum mechanics and introductory quantum field theory. This is a little more advanced than many of the other topics treated, but I hope it will be accessible to the interested reader. It aims to open the door to the understanding of a number of points that were previously stated without justification.

Once again, I have largely re-written the last chapter on the conceptual foundations of the subject. The twenty years since the publication of the first edition do not seem to have brought scientists and philosophers significantly closer to a consensus on these problems. However, many issues have been considerably clarified and the strengths and weaknesses of some of the explanations are more apparent. My own understanding continues to grow, not least because of what I have learned from formal and informal discussions at the annual UK Conferences on the Foundations of Physics.

Other changes include a more detailed treatment of tunnelling in chapter 2, a more gentle transition from the Born postulate to quantum measurement theory in chapter 4, the introduction of Dirac notation in chapter 6 and a discussion of the Bose–Einstein condensate in chapter 10.

I am grateful to a number of people who have helped me with this edition. Glenn Cox shared his expertise on relativistic quantum mechanics when he read a draft of chapter 11; Harvey Brown corrected my understanding of the de Broglie–Bohm hidden variable theory discussed in the first part of chapter 13; Demetris Charalambous read a late draft of the whole book and suggested several
improvements and corrections. Of course, I bear full responsibility for the final version and any remaining errors.

Modern technology means that the publishers are able to support the book at the web site http://bookmarkphysics.iop.org/bookpge.htm/book=1107p. This is where you will find references to the wider literature, colour illustrations, links to other relevant web sites, etc. If any mistakes are identified, corrections will also be listed there. Readers are also invited to contribute suggestions on what would be useful content. The most convenient form of communication is by e-mail to 0750308397@bookmarkphysics.iop.org.

Finally I should like to pay tribute to Ann for encouraging me to return to writing after some time. Her support has been invaluable.

Alastair I. M. Rae

2002
In preparing this edition, I have again gone right through the text identifying points where I thought the clarity could be improved. As a result, numerous minor changes have been made. More major alterations include a discussion of the impressive modern experiments that demonstrate neutron diffraction by macroscopic sized slits in chapter 1, a revised treatment of Clebsch–Gordan coefficients in chapter 6 and a fuller discussion of spontaneous emission in chapter 8. I have also largely rewritten the last chapter on the conceptual problems of quantum mechanics in the light of recent developments in the field as well as of improvements in my understanding of the issues involved and changes in my own viewpoint. This chapter also includes an introduction to the de Broglie–Bohm hidden variable theory and I am grateful to Chris Dewdney for a critical reading of this section.

Alastair I. M. Rae
1992
Preface to Second Edition

I have not introduced any major changes to the structure or content of the book, but I have concentrated on clarifying and extending the discussion at a number of points. Thus the discussion of the application of the uncertainty principle to the Heisenberg microscope has been revised in chapter 1 and is referred to again in chapter 4 as one of the examples of the application of the generalized uncertainty principle; I have rewritten much of the section on spin–orbit coupling and the Zeeman effect and I have tried to improve the introduction to degenerate perturbation theory which many students seem to find difficult. The last chapter has been brought up to date in the light of recent experimental and theoretical work on the conceptual basis of the subject and, in response to a number of requests from students, I have provided hints to the solution of the problems at the ends of the chapters.

I should like to thank everyone who drew my attention to errors or suggested improvements, I believe nearly every one of these suggestions has been incorporated in one way or another into this new edition.

Alastair I. M. Rae
1985
Preface to First Edition

Over the years the emphasis of undergraduate physics courses has moved away from the study of classical macroscopic phenomena towards the discussion of the microscopic properties of atomic and subatomic systems. As a result, students now have to study quantum mechanics at an earlier stage in their course without the benefit of a detailed knowledge of much of classical physics and, in particular, with little or no acquaintance with the formal aspects of classical mechanics. This book has been written with the needs of such students in mind. It is based on a course of about thirty lectures given to physics students at the University of Birmingham towards the beginning of their second year—although, perhaps inevitably, the coverage of the book is a little greater than I was able to achieve in the lecture course. I have tried to develop the subject in a reasonably rigorous way, covering the topics needed for further study in atomic, nuclear, and solid state physics, but relying only on the physical and mathematical concepts usually taught in the first year of an undergraduate course. On the other hand, by the end of their first undergraduate year most students have heard about the basic ideas of atomic physics, including the experimental evidence pointing to the need for a quantum theory, so I have confined my treatment of these topics to a brief introductory chapter.

While discussing these aspects of quantum mechanics required for further study, I have laid considerable emphasis on the understanding of the basic ideas and concepts behind the subject, culminating in the last chapter which contains an introduction to quantum measurement theory. Recent research, particularly the theoretical and experimental work inspired by Bell’s theorem, has greatly clarified many of the conceptual problems in this area. However, most of the existing literature is at a research level and concentrates more on a rigorous presentation of results to other workers in the field than on making them accessible to a wider audience. I have found that many physics undergraduates are particularly interested in this aspect of the subject and there is therefore a need for a treatment suitable for this level. The last chapter of this book is an attempt to meet this need.

I should like to acknowledge the help I have received from my friends and colleagues while writing this book. I am particularly grateful to Robert Whitworth, who read an early draft of the complete book, and to Goronwy Jones and George Morrison, who read parts of it. They all offered many valuable and
penetrating criticisms, most of which have been incorporated in this final version. I should also like to thank Ann Aylott who typed the manuscript and was always patient and helpful throughout many changes and revisions, as well as Martin Dove who assisted with the proofreading. Naturally, none of this help in any way lessens my responsibility for whatever errors and omissions remain.

Alastair I. M. Rae
1980