Tunneling of classical waves

geometrical optics approximation: light travels as a ray (which is a subset of wave optics), in essence: all apertures are large, true wave properties do not matter (much), pretty similar to classical mechanics as a subset of quantum mechanics

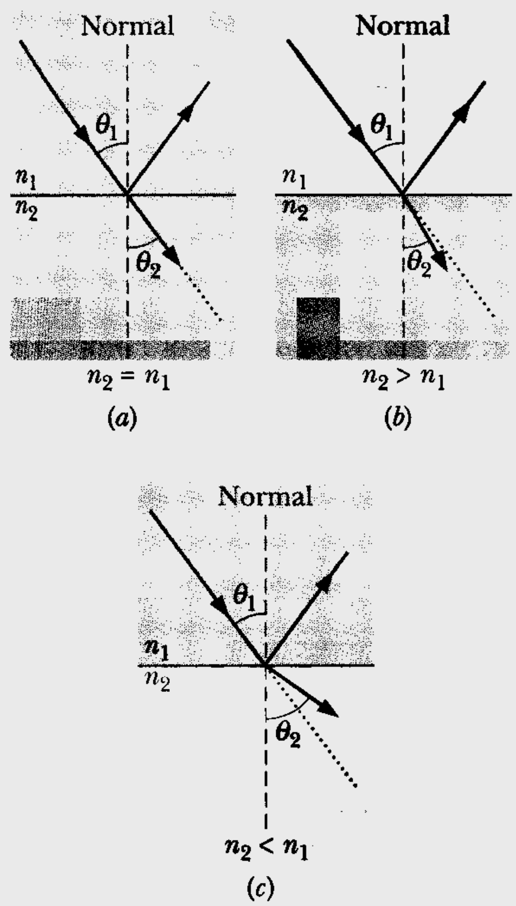

Fig. 34-18 Light refracting from a medium with an index of refraction n_1 and into a medium with an index of refraction n_2. (*a*) The beam does not bend when $n_2 = n_1$; the refracted light then travels in the *undeflected direction* (the dotted line), which is the same as the direction of the incident beam. The beam bends (*b*) toward the normal when $n_2 > n_1$ and (*c*) away from the normal when $n_2 < n_1$.

reflection: angle of incidence equals angle of reflection (Schnell's law)

refraction: travel of light through a surface where there is a change of refractive index usually changes its direction, is said to be bent,

law of refraction:

coming from 1 going to 2

$$\mathbf{n_1 \sin \theta_1 = n_2 \sin \theta_2}$$

for $\theta_2 = 90\,°$, there is no refracted beam any more, this sets a critical angle condition for θ_1

this critical angle $\theta_c = \arcsin \dfrac{n_2}{n_1}$

coming from a denser medium, i.e. $n_1 > n_2$

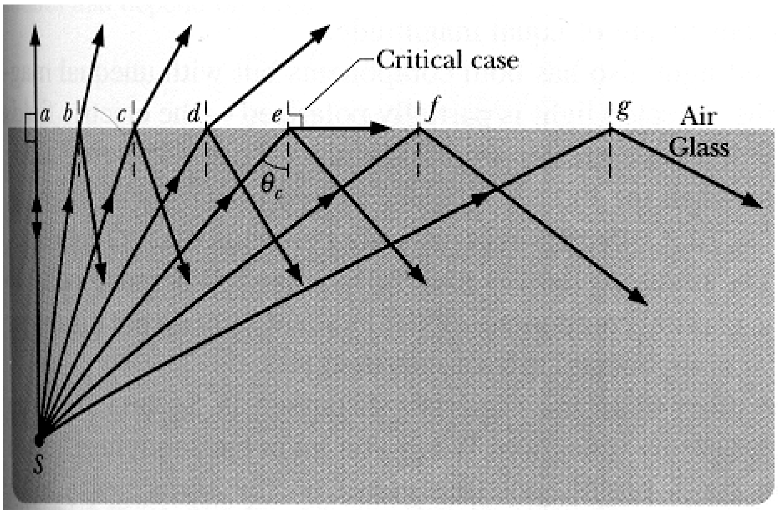

Fig. 34-24 Total internal reflection of light from a point source S in glass occurs for all angles of incidence greater than the critical angle θ_c. At the critical angle, the refracted ray points along the air–glass interface.

with a sufficiently high refractive index n_1) we do not have a refracted beam anymore, because a sine of an angle cannot exceed unity, n_2 cannot exceed n_1 in this equation, ray stays in the medium, e.g. glass, and gets totally internally reflected, in

this means: in ray optics: there is an infinitely high barrier at the interface

now in wave treatment, we have tunneling, frustrated total internal reflection, because

wave equation of sound, light, water, $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$,

which works for all classical waves, that have either a photon associated with it or a pseudo-particle (such as a phonon), we also have wave functions whose amplitude is not zero in a barrier

index of refraction: ***absolute = 1*** is ratio of speed of light in vacuum to speed of light in vacuum

relative: ration of speed of light in vacuum to speed of light in medium, for visible light greater than 1 (for X-rays slightly smaller than 1) light is slower in more dense medium, example: optical "flint" glass: $c_n \approx 1.8\ 10^{8}\ ms^{-1}$, $n \approx 1.65$

The property of penetration into a forbidden region is a general property of classical waves. Consider the case of total internal reflection* of light waves, as shown in Figure 5.20*a*. How does the light wave "know" that there is air on the other side of the boundary? As long as the light wave is entirely inside the glass, it cannot "know" what is beyond the glass. To find out, it must penetrate into the forbidden air a short distance, perhaps a few wavelengths, before it realizes that entry into the air is forbidden and that it must return to the glass. It is, of course, never observed in the air; the laws of reflection and refraction forbid its presence there. If, however, a second piece of glass is placed within the penetration distance, the beam can reappear in the second piece of glass, as shown in Figure 5.20*b*. This phenomenon is called *frustrated total internal reflection.* We can imagine that the beam sends out "feelers" into the forbidden air gap, finds the other glass, realizes that it is not forbidden from entering the glass, and proceeds on its way. Just as with the barrier of Figure 5.15, the probability to penetrate the air gap decreases as the thickness of the gap increases.

The same behavior is shown by other classical waves, such as water waves as in Figure 5.21.

water waves are slower close to the shore where the ocean is shallower

by analogy, we can have the equivalent of total internal reflection for a combination of sudden increase in water dept, where the speed of the wave would be higher, with just the right combination of change in "refractive index" and incident angle, we get total internal reflection as evidenced by the interference pattern so the area of the deep water is the barrier over which the water can not penetrate (if we do the experiment just right), there is,

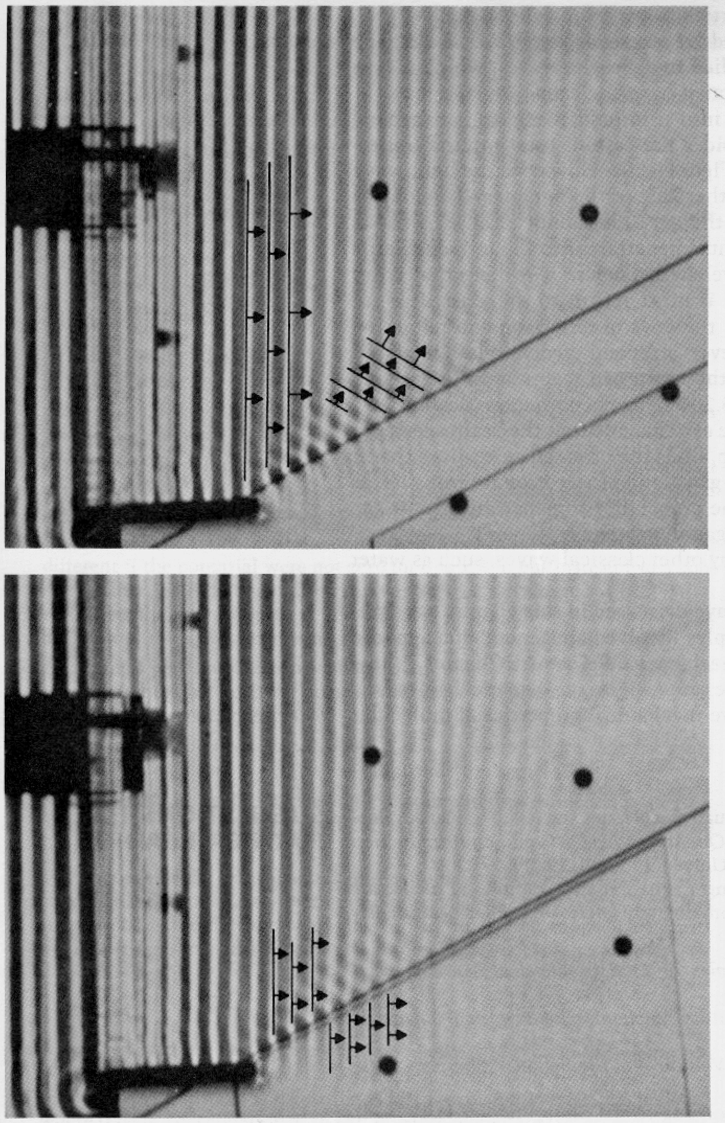

FIGURE 5.21 Frustrated total internal reflection for water waves. At the boundary the depth increases suddenly and the waves are totally reflected. When the gap is made narrow, the waves can penetrate and appear on the other side. (Courtesy of Education Development Center, Inc., Newton, MA.)

however, again tunneling, if the barrier is “thin” enough, we can have a measurable effect, pick up the water wave again, just as we did with frustrated total internal reflection of light

conclusion: ***tunneling is a real wave phenomena, all wave’s can do it, in principle***