DUALISM IN QUANTUM THEORY

Do particles sometimes behave as waves? Can waves be treated as particles? There is still disagreement in the interpretation of the quantum theory.

MAX BORN and WALTER BIEM

IN RECENT YEARS Alfred Landé has tried to give a new foundation for quantum theory, starting with statistical nonquantal principles that have not been taken from classical physics. In his books[1] and articles[2] he attacks the basic concept of quantum theory that is generally taken for granted by most physicists nowadays. He calls this concept "dualistic" and maintains that he can replace it by another more unified one.

We think Landé has not realized the historical origin of the dualistic interpretation and does not correctly describe its physical meaning. Moreover his fight against "dualism" in modern quantum theory seems to be a tilt against windmills.

Landé's attempt at a new foundation for quantum theory was reviewed in detail by Abner Shimony in PHYSICS TODAY[3] so that we need not enter into his concept of quantum theory as a whole.

Landé's approach

As Landé says, dualism is the habit of modern physicists to use two apparently contradictory "theories" side by side, the particle theory and the wave theory, to explain the phenomena of the microcosm. Landé maintains that matter can and must be treated exclusively with a particle theory, and light on the contrary should only be treated with a wave theory; this is what he calls "unity in quantum physics." We think that Landé's interpretation represents a more unsatisfactory dualism that ignores important physical discoveries just for prejudice. Moreover the dualism of waves and particles, according to Landé, appears to be a late invention of the theoreticians (especially Niels Bohr and Werner Heisenberg) who tried to interpret quantum mechanics; this view is historically incorrect.

The founder of the dualistic view was Albert Einstein, that is to say young Einstein, who did not speculate as he did in later years, but analyzed physical experience with remarkable penetration and drew irrefutable inferences. His first work on this subject[4]

Max Born studied at Breslau, Heidelberg, Zürich, Göttingen and Cambridge before moving to Frankfurt in 1919 as professor of theoretical physics. Two years later he took up a similar post at Göttingen. In 1933 he left Germany for Cambridge, Bangalore and Edinburgh. His Nobel Prize was awarded in 1954.

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appeared 60 years ago in the same volume of *Annalen der Physik* as his great paper on electrodynamics of moving bodies (which includes the foundation for the special theory of relativity) and his fundamental work on Brownian motion, in which the existence of atoms and molecules is proved for the first time by empirical facts.

Einstein's derivation

Einstein considered the radiation balance in a cavity and used Planck's radiation law, known since 1900, which gives the average density of energy u as a function of radiation frequency ν and temperature T. The radiation energy $E(\nu)$, at a frequency ν, in a small part V of a cavity is not always exactly uV but is subject to fluctuations. If the radiation consists of waves, as was assumed at that time, the mean square of fluctuation of the energy is proportional to the square of the mean energy density, according to Hendrik Lorentz. That is, $\langle \Delta E^2 \rangle = u^2 V^2$. This quantity can also be calculated from Planck's equation, yielding two terms

$$\langle \Delta E^2 \rangle = u^2 V^2 + \epsilon_0 \cdot uV; \ \epsilon_0 = h\nu \quad (1)$$

$(h = \text{Planck's constant})$

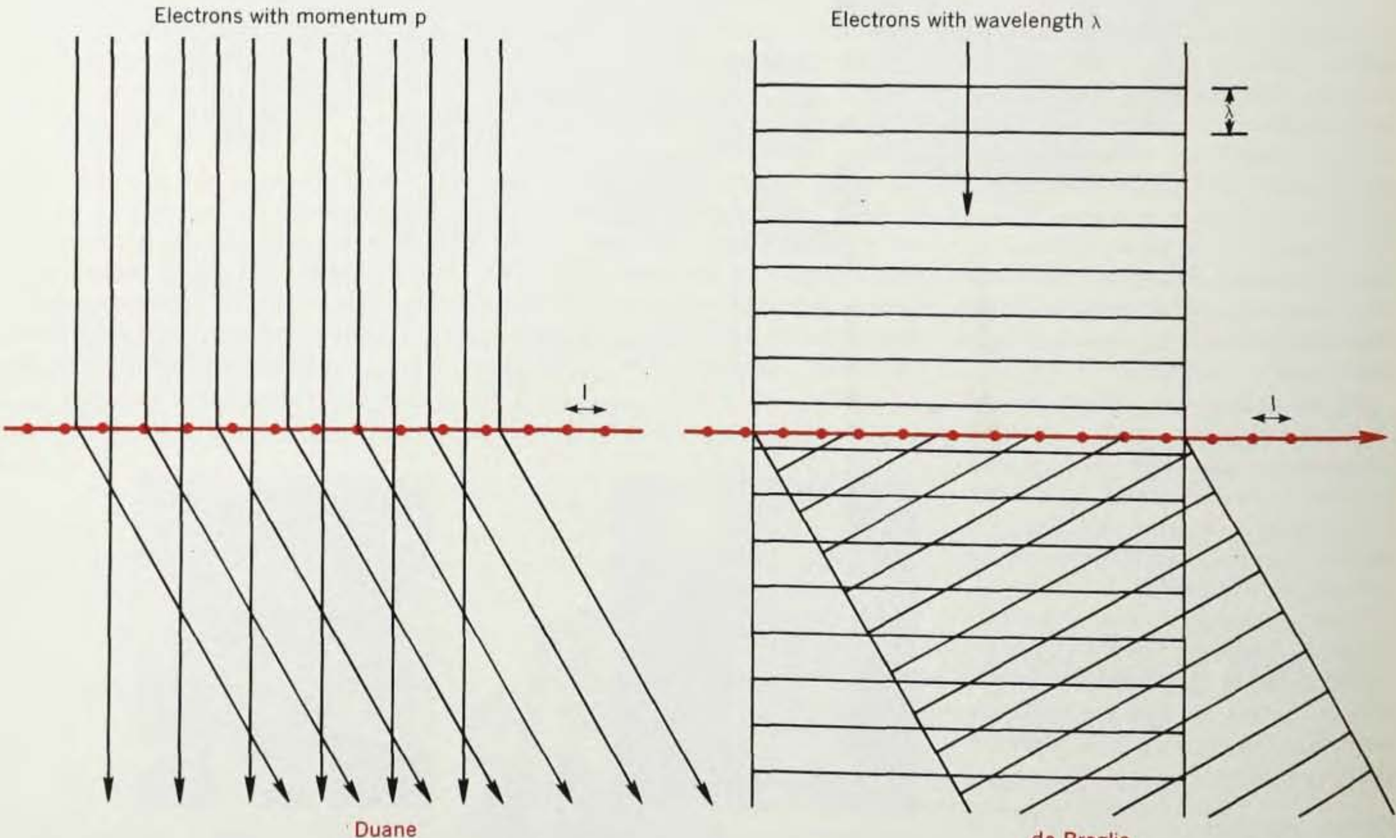
The new linear term has a simple meaning. If we consider the energy to consist of quanta of magnitude $\epsilon_0 = h\nu$, so that $E = \epsilon_0 n$ when n is the number of quanta, the mean square of fluctuation of the quantum gas becomes exactly equal to the second term $\epsilon_0 \cdot uV$ according to elementary equations of statistical mechanics. Equation 1 can therefore be written as a formula for the mean square of fluctuation, $\langle \Delta n^2 \rangle$, of the number of quanta. Calling the average number of quanta $\langle n \rangle$, we have $Vu = \epsilon_0 \langle n \rangle$ from equation 1

$$\langle \Delta n^2 \rangle = \langle n \rangle^2 + \langle n \rangle \quad (2)$$

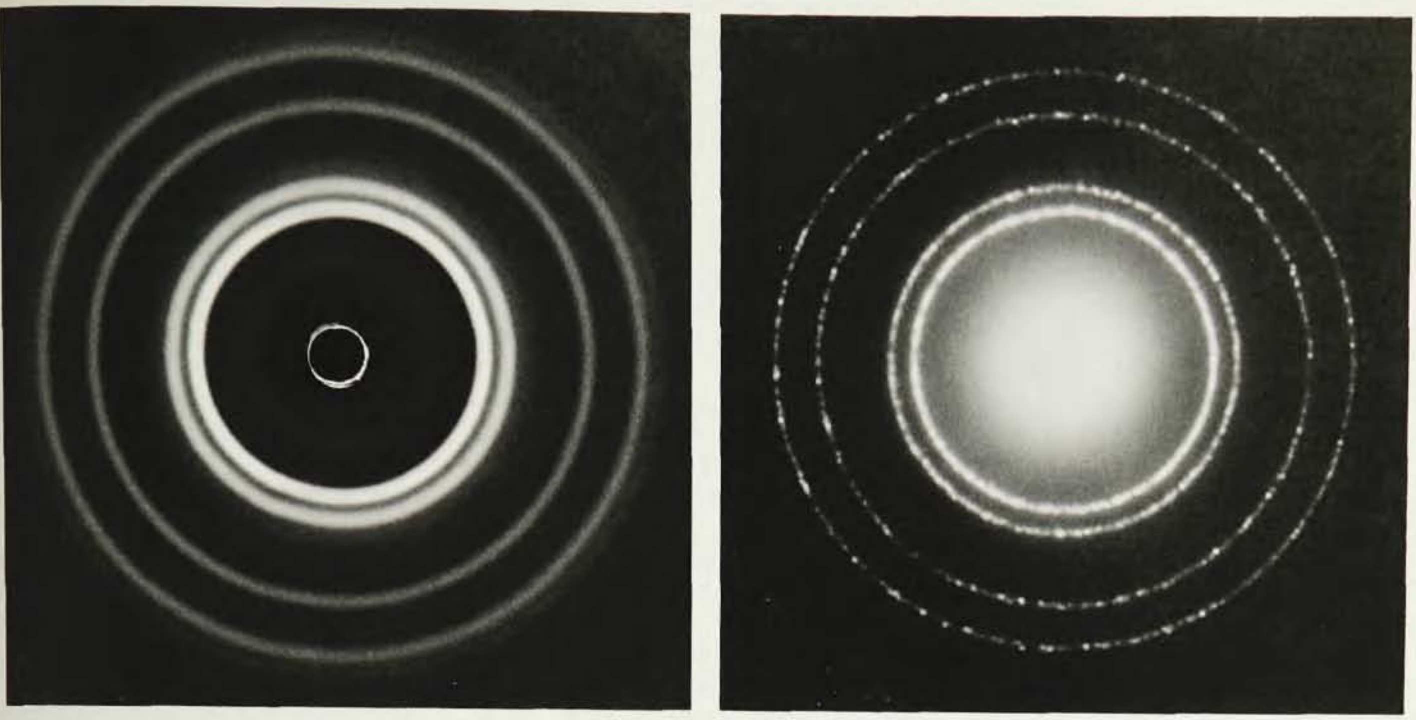
This result (and not the photoelectric effect as has often been maintained) was the starting point for Einstein's statement that the structure of light is not completely described by wave theory, but that there are light quanta (or *photons* as we now call them).

The most astonishing thing is that we need both concepts of light, the corpuscular and the undulatory, to understand the fluctuation equation 1, which as we point out once again, is absolutely incontestable by the derivation from Planck's radiation law.

Thus Landé's thesis that we need only a wave theory for light, whereas we must exclusively use a particle theory for "material" particles, was already refuted by Einstein's first work on this subject. Light can be described, as Einstein's consideration shows, neither with waves alone nor with particles alone. It is not both waves and particles but has something to do with both. We can not understand this fact on the level of classical physics. Classical wave theory is found to be only a model for light, and an imperfect one. At first Einstein and his contemporaries (including



ELECTRON DIFFRACTION by a linear chain of regularly spaced atoms. On the left is Duane's model of March 1923, in which the periodic structure can change the component of momentum p parallel to the chain by amounts $\Delta p = h/L = hf$ ($f = 1/L$ and $L = l, 2l, 3l, \ldots$). On the right is de Broglie's model of Sept. 1923, with electron waves of wavelength λ ($p = h/\lambda$). Change of momentum then proceeds according to Planck's quantum rule. de Broglie founded his rule on Einstein's theory of relativity, unlike Duane whose rule is without relativistic foundation. —FIG. 1



X-RAY AND ELECTRON DIFFRACTION in aluminum, showing the similarity in the diffraction patterns produced. X-ray photograph at left was made with rays of wavelength 7.1 nanometers; electron diffraction photograph at right is for 600-eV electrons (equivalent to about 5 nm). Electron pattern has been enlarged × 1.6. Photos by Film Studio, Educational Development Center. —FIG. 2

one of us) did not see that this was the case, and even later many of them did not understand it.

Bose statistics

Similar considerations show that the situation is the same for matter as for light. The statistical laws for the motion of molecules have to be altered with regard to the quantization rules. Classical Boltzmann statistics are replaced by Bose statistics, named after Satyandra Nath Bose. Einstein recognized at once the fundamental importance of this change. The fluctuations of energy in a Bose gas are once again represented by two terms; one of them corresponds to the behavior of particles according to classical mechanics, the other to the behavior of classical waves. We find the same duality as with light.

Later, Bose statistics were discovered to rule only a certain class of particles (for example, photons, mesons and atoms of the normal helium isotope He^4). The other class of elementary particles is governed by different statistics, discovered by Enrico Fermi and Paul Dirac. Electrons, neutrons, protons and the rare helium isotope He^3 belong to this class. For systems consisting of particles of this kind we have for the mean square fluctuation

$$\langle \Delta\, n^2 \rangle = -\langle n \rangle^2 + \langle n \rangle \qquad (3)$$

The negative sign of the quadratic term prevents us from interpreting the equation the same way as we did equation 2. We shall return to Fermi statistics later on.

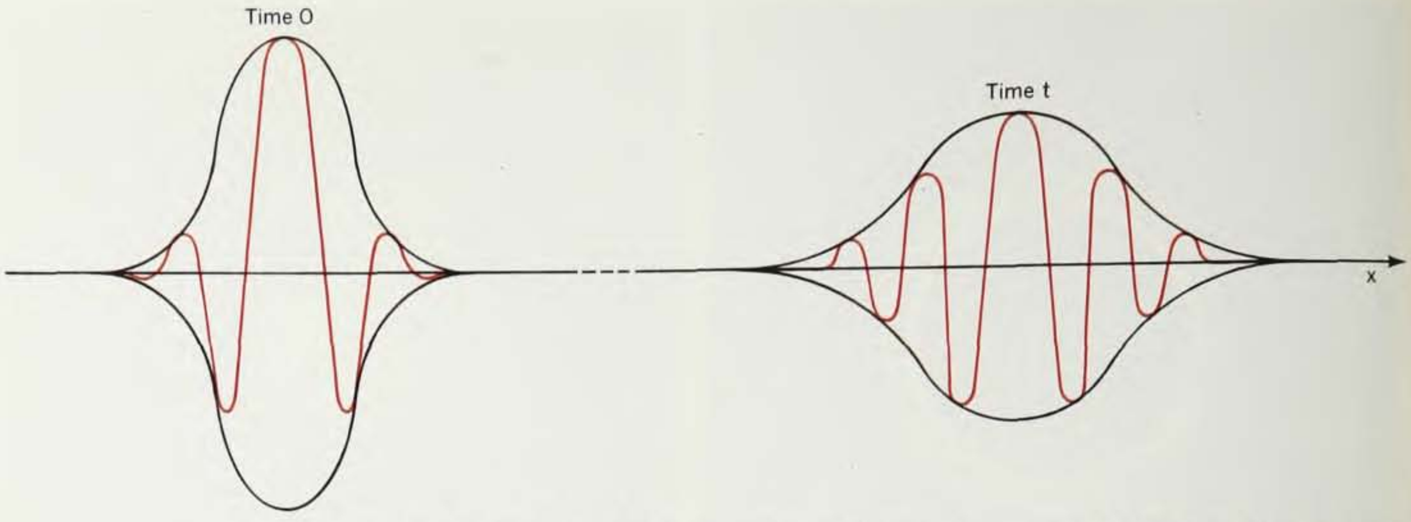
Bose statistics were discovered almost at the same time as Louis de Broglie's wave mechanics, which is based on the idea that, in the theory of relativity, energy E is not a scalar but the fourth component of a four-vector, whose other three components represent the momentum p. Therefore, as well as using Planck's quantum rule for the change in energy of a system periodic *in time* with frequency ν, $\Delta E = h\nu$, de Broglie postulates the change of momentum $\Delta p = hk$, where k is the wave number of the frequency *in space* ($k = 1/\lambda$; λ = wave length).

Duane's formulation

Landé lays much stress on the fact that William Duane formulated the momentum rule before de Broglie, without relativistic foundation, in order to explain x-ray diffracton in crystals by means of the corpuscular theory of light (see figure 1). Landé uses Duane's formulation only for particles, not for x rays, which are waves classically and must be treated as waves if one follows Landé.

He tries to explain the interference of electrons when passing a crystal lattice by a corpuscular theory; that is, he insists that the crystal interferences of light and electrons are caused by quite different processes. We think this is hardly plausible. Anyone who has seen Laue and Debye–Scherrer photographs taken with electrons and with x rays for the same material (figure 2) knows that they cannot be distinguished from one another without difficulty. We can not see how mechanisms that are physically totally different from each other should produce identical phenomena. Above all there is no reason why a unified interpretation that already exists, the quantum theory as it is generally understood, should be wrong and why it should be replaced by a new interpretation that explains apparently related phenomena in two different ways, and quite dogmatically lays stress on these differences.

Duane's "quantum rule" has hardly been noticed, and this is understandable for the rule is obscure without de Broglie's idea of the correspondence



MOTION OF A WAVE PACKET in quantum mechanics. The same packet is shown at two instants of time; originally concentrated, the wave packet is later smeared out in space. —FIG. 3

of particle and wave and without his proof that the group velocity of a wave packet coincides with the velocity of the corresponding particle. In the same sense, Planck's quantum rule $\Delta E = h\nu$ was obscure at that time. The problem for physicists was to give a meaning to all these rules. It was necessary to see that such a meaning was impossible in the domain of traditional concepts. The first steps to reach the desired aim were matrix mechanics, inspired by Heisenberg and executed by him together with Born and Pascual Jordan, and Dirac's theory of noncommuting quantities, which was also inspired by Heisenberg's ideas but was independently developed. Both theories were quite abstract, and the words "particle," "motion," "momentum," etc, were only used symbolically. Then Erwin Schrödinger published his wave mechanics, which was based on de Broglie's ideas.

Was Schrödinger an opponent?

Landé quotes Schrödinger, as well as Einstein, as an opponent to the dualistic theory. It appears to us that Landé also misunderstood Schrödinger. Schrödinger believed that matter is a wave phenomenon. He denied the existence of particles and "quantum jumps" that occur in Bohr's theory of the electron clouds of atoms, and maintained that the facts explained by this theory could only be described by means of waves and wave packets. He adhered to this opinion during his lifetime. Landé, however, writes about "Einstein's and Schrödinger's realistic point of view" as if both of them had defended the same position, opposite to dualism.

In fact dualism is a discovery, not an invention, of Einstein, as we explained above; it was proved by the equations on fluctuations in gases and in radiation and there are no substantial arguments against it. Einstein himself never tried to deny dualism. It was quite clear for him that one could not avoid dualism with the usual concept of particles. He tried to master the whole question in quite another way, namely with his unified field theory, which combined the fields of gravitation and electromagnetism into a formal unity. The substrata of this theory (or, strictly, theories, for Einstein laid down several theories during his lifetime) were fields of a highly abstract kind, nonsymmetrical tensors, ruled by complicated differential equations. In 1920 he wrote about these ideas:[5] "During my spare time I brood over the quantum problem from the point of view of relativity . . . But I do not succeed in giving a concrete shape to my favorite idea—the comprehension of the quantum structure from an overdetermination by differential equations."

Singularities from a continuum

Einstein rejected quantum mechanics for reasons totally different from Landé's. He never denied the dualism of particle and wave, which he himself had discovered, but pursued the idea that particles as singularities could be derived from a continuum theory in which there is an overdetermination of the variables. He did not succeed in carrying through this project.

Landé's remarks give the impression that Schrödinger defended Landé's thesis that matter consists only of particles, light only of waves; the very contrary, however, was true. Schrödinger wished to regard both matter and light as waves, and maintained that he had found the way back to the good old classical theory.

Schrödinger's opinion is not valid, as has often been shown. First, a concentrated wave packet that, according to his concept, is to represent a particle does not hold together but disperses as shown in figure 3. Second, his wave function ψ is distributed in three-dimensional space only for *one* particle. For two particles one needs a function in six dimensional space $\psi(x_1, y_1, z_1; x_2, y_2, z_2)$, and for n particles a function in a $3n$-dimensional space. These functions are not reducible to functions in three-dimensional space. Thus the desired approach to the classical wave theory has become an illusion.

Space and momentum pictures

The discussion on dualism or nondualism appears to be superfluous. Since Einstein's discovery of the fluctuation equation it has become more and more obvious that nature can be described not by particles or waves alone, but by a more sophisticated mathematical theory. This is the quantum theory, which supersedes both models and only in certain limits represents one or the other. Quantum theory has become known to us as a complete whole since the end of the 1920's. We need not turn from a particle picture to a wave picture arbitrarily, and we need not be without real comprehension when using it. It is, on the contrary, possible to represent the states of a system in different ways, and these representations are connected by unique transformations.

Among these representations there is a space representation from which one can easily derive the probability of finding the particles at a certain point in space. There is a momentum representation as well from which one can easily read off the probability of the particles having certain momenta (that is, velocities). As the momentum, according to de Broglie, defines the wavelength one has found the wave properties of the system in this representation. These two representations correspond respectively to the "particle picture" and the "wave picture;" they date from the period when quantum theory as a whole had not yet been developed. But there is an unlimited number of other representations of the states of a system, for example the energy representation, from which one can easily calculate the energies of the system. Moreover, all properties of the system can be calculated in any of these representations (for example, the distribution in space of the particles can also be calculated in the momentum distribution). Thus with quantum theory one treats all systems in the same way, whether they consist classically of particles or are described classically by fields (waves).

Bosons and fermions

Much more fundamental than the distinction between particles and waves is the classification of particles between those that follow Bose statistics (bosons) and those that are ruled by Fermi statistics (fermions). Whereas bosons can be compared to classical particles and waves, as we have shown above by means of their fluctuations, this comparison is not altogether true for fermions, as we have already seen when looking at the fluctuation equation 3.

As to Landé's claim to have given a new derivation of quantum mechanics based on classical ideas though not as such taken from classical physics, it must of course be examined thoroughly; Shimony wrote of such an examination in his review.[3] We will add only the following remarks. Landé starts from statistical postulates when trying to give a new foundation to quantum theory. These postulates are unknown in classical physics, from which Landé takes all his other concepts. It appears to be not very surprising that one can derive theories similar to quantum mechanics from statistical postulates. Such a derivation can be interesting in itself and does not need to be accompanied by attacks on supposed enemies. The strangest aspect of Landé's treatment is his dogmatic use of classical and macroscopic concepts of particles and waves in atomic dimensions and his rejection of obvious explanations of simple experimental results on account of this dogma or prejudice.

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DIALOG ON DUALISM

Replies by ALFRED LANDE to points raised in the preceding article and further comments by MAX BORN and WALTER BIEM.

LANDE: Concerning "the historical origin of the dualistic interpretation" which I "have not realized:" I know of course of Einstein's light quanta in opposition to light waves. But I also know that there is a unitary quantum theory of radiation that has relegated the "photon" to the role of a quantum number attached to the periodic components of the continuous Maxwell field; thereby it has become unnecessary to attribute various *ad hoc* invented quantities—spin, interdependence of electric and magnetic properties of the photon—in order to save a particle picture dual to the wave picture of light. Light waves are real, matter waves are imaginary, in more than one sense.

Duality began to be taken seriously only after the experiment of electron diffraction *seemed* to allow no other explanation than the assumption that particles of matter pass through a wave interlude near a crystal or screen with slits: "An electron spreads out from its original size millions of times to cover both slits; thereafter it interferes with itself." This oddity, together with an associated "new conceptual situation" accounting for the unphysical transmutation magic, could have been avoided if quantum theorists around 1927 had been aware of the quantum rule for linear momentum (Duane, 1923). This rule explains the electronic diffraction patterns in a natural way as due to the quantized momentum activity of the diffractor, including coherence effects as shown in my books and articles. It has been quite a revelation to many younger physicists trained in the dualistic doctrine. To belittle the quantum rule for the momentum p beside those for E and p_φ as is done by Born, is as unphysical as if one would belittle the mechanical conservation law for p beside those for E and p_φ in classical theory. One here really must ask: "Why do quantum theorists ignore the quantum theory?" I would be delighted to be shown a single place in the literature on interpretation where Duane's unitary explanation of diffraction, applied to matter particles, is quoted. Born was one of the few who knew of Duane's March 1923 paper.

BORN AND BIEM: Every physicist must accept Duane's rule,[6] which de-