EINSTEIN'S PHYSICS

A modern understanding

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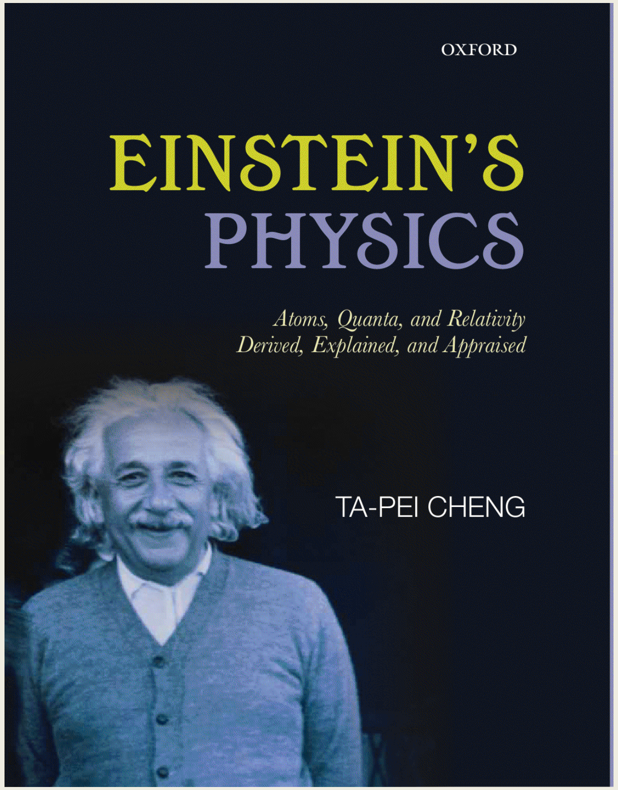
talk based on …

Oxford Univ Press (2013)

Einstein's Physics

Atoms, Quanta, and Relativity --- Derived, Explained, and Appraised



Albert Einstein
1879 – 1955

The book
explains his physics
in equations

1. Atomic Nature of Matter
2. Quantum Theory
3. Special Relativity
4. General Relativity
5. Walking in Einstein's Steps

Today's talk
provides w/o math details
some highlight
historical context
& influence

1. The quantum postulate: Planck *vs* Einstein
2. Einstein & quantum mechanics
3. Relativity: Lorentz *vs* Einstein
4. The geometric theory of gravitation
5. Modern gauge theory and unification

Einstein, like Planc… …esis thru the study of BBR

Blackbody … …ion = rad in thermal

To find the $u(T)$ and … $u(T) = \int_0^\infty \rho(T,\nu)\,d\nu$

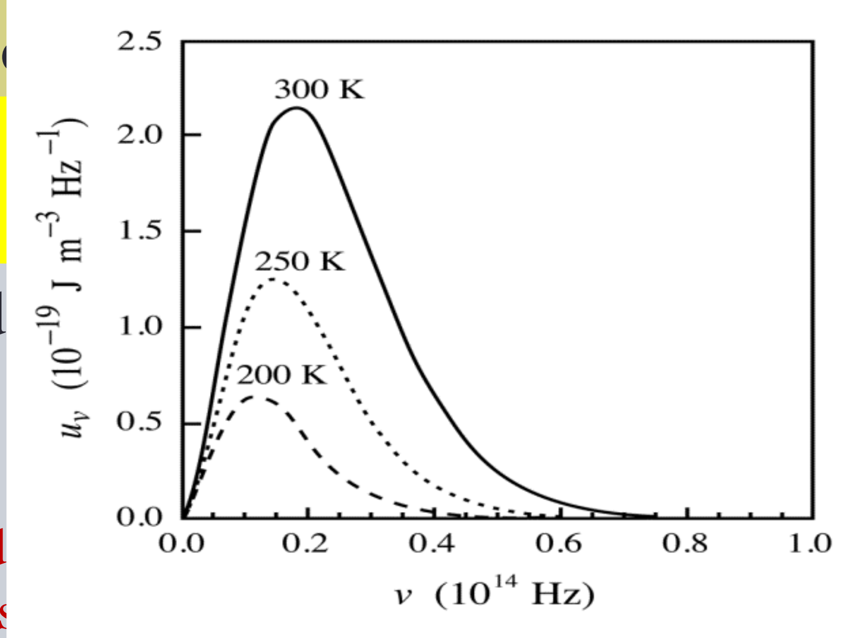
$$u(T) = aT^4$$

…Boltzmann (1884) law:

…EM (radiation pressure = $u/3$)

electromagnetic rad… …E^2, B^2 ~ oscillator energy kx^2

os… …o of is an *adiabatic invariant*



What is the f function? $\rho(T,\nu) = \nu^3 f(\nu/T)$ **Wien's displacement law** (1893)

Wien's distribution (1896): $\rho(T,\nu) = \alpha\nu^3 e^{-\beta\nu/T}$ fitted data well…. until IR

Wien = high ν of Planck

Planck's distribution (1900): $\rho(T,\nu) = \dfrac{\alpha\nu^3}{e^{\beta\nu/T} - 1}$

Excellent fit of all the data

key: Wien 2 →1 var

Blackbody Radiation Planck (1900): $\rho(T,\nu)=\dfrac{\alpha\nu^3}{e^{\beta\nu/T}-1}$

What's the physics behind this result?

Derived a relation $U=(c^3/8\pi\nu^2)\rho$ *and* $dS=dU/T \rightarrow$ entropy S

Boltzmann's principle $S=k\ln W$

Planck : What microstates ***W*** that can lead to this ***S*** ?

was "compelled" to make the hypothesis of **energy quantization**

$$\varepsilon = nh\nu \qquad n = 0,1,2,...$$

Einstein's 1905 proposal of light quanta

was ***not*** *a direct follow-up* of Planck's

Einstein's 1905 proposal of light quanta $\varepsilon = h\nu$

Einstein used Planck's calculation $U = \left(c^3 / 8\pi\nu^2\right)\rho$ and

invoked the equipartition theorem of stat mech $U = \frac{1}{2}kT$

to derive the **Rayleigh-Jeans law:** $\rho = 8\pi c^{-3} kT\nu^2$

noted its *solid theoretical foundation*

showing BBR = clear challenge to classical physics

poor accounting of the data, notably the problem of *ultraviolet catastrophe* $u = \int_0^\infty \rho \, d\nu = \infty$

Rayleigh-Jeans = the low frequency limit of the successful Planck's distribution

new physics ➡ The high frequency limit (**Wien's distribution**) $\rho = \alpha\nu^3 e^{-\beta\nu/T}$

Einstein undertook a statistical study of $(\text{BBR})_{\text{wien}}$:

instead of W, calculate ΔS due to volume change **$(\text{BBR})_{\text{wien}}$ ~ ideal gas**

→ $(\text{BBR})_{\text{wien}}$= ***a gas of light quanta*** with energy of $\varepsilon = nh\nu$

Einstein arrived at energy quantization **independently**---- cited Planck only in 2 places

A year later……Einstein gave a new derivation of Planck's distribution

Einstein's discoveries in quantum theory

(1900) **Planck:** $\varepsilon = h\nu$ is only a formal relation

(1905) **Einstein:** the quantum idea must represent *new physics*

proposed **photoelectric effect** as test

beyond BBR: **Q theory of specific heat** (1907)

(1909) **Light quanta = particles**

***h* as conversion factor particle ↔ wave**

***Wave-Particle Duality*: a deep riddle**

Einstein stated for the 1st time:

quanta carried by point-like particles

"*point of view of Newtonian emission theory*"

Photon carries energy + momentum $p = h / \lambda$

(1913) Bohr's **quantum jumps** describe *absorption and emission of photons*

(1916–17) Einstein construct a microscopic theory of radiation–matter interaction:

(*A and B coeff*); The central novelty and lasting feature is the introduction of **probability** in quantum dynamics

General acceptance of the photon idea

Only after Compton scattering (1924)

(1924–25) Bose-Einstein statistics & condensation

Einstein & Quantum Mechanics

Modern quantum mechanics :

states = vectors in Hilbert space (superposition)

observables = operators (commutation relations)

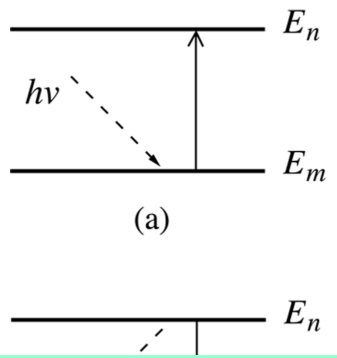
His discoveries in quantum theory:

Wave/particle nature of light, quantum jumps etc. can all be *elegantly* accounted for in the framework of **quantum field theory**

Classical radiation field = collection of oscillators

Quantum radiation field = collection of ***quantum*** **oscillators**

$$E_n = (n+\tfrac{1}{2})h\nu$$



- A firm mathematical foundation for Einstein's photon idea
- *Quantum jumps* naturally accounted for by **ladder operators**

$$[\hat{a}_-, \hat{a}_+] = h\nu, \quad \hat{a}_\pm |n\rangle \sim |n \pm 1\rangle \;\Rightarrow\; \frac{particle}{behavior}$$

QFT description broadens the picture of interactions

not only can alter motion, but also allows for emission and absorption of radiation

$\longrightarrow$ creation and annihilation of particles

Beautiful resolution of *wave-particle duality*

in radiation energy fluctuation $\quad wave \sim \hat{a} e^{ikx}$

Alas, Einstein never accepted this neat resolution as he never accepted the new framework of QM

Einstein & QM: The debate with Bohr

Orthodox interpretation of QM (*Niels Bohr* & co): the attributes of a physical object (position, momentum, spin, etc.) can be assigned *only* when they have been measured.

Local realist viewpoint of reality (*Einstein*,…): a physical object has definite attributes whether they have been measured or not. …. QM is an incomplete theory

Einstein, Podolsky & Rosen (1935) : A thought experiment highlighting the "**spooky action-at-a-distance**" feature: the measurement of one part of an entangled quantum state would instantaneously produce the value of another part, no matter how far the two parts have been separated. the discussion and debate of "EPR paradox" have illuminated some of the fundamental issues related to the meaning of QM

Bell's theorem (1964) : these seemingly philosophical questions could lead to observable results. The experimental vindication of the orthodox interpretation has sharpened our appreciation of the nonlocal features of quantum mechanics.

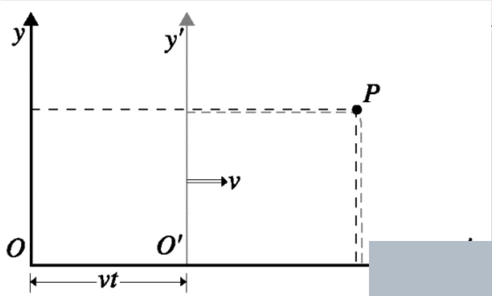
Einstein's criticism allowed a deeper understanding of the meaning of QM.

Nevertheless, the **counter-intuitive picture of objective reality** as offered by QM still troubles many, leaving one to wonder whether quantum mechanics is ultimately a complete theory.

Special Relativity

Maxwell's equations: EM wave – c Contradict relativity?

2 inertial frames $x' = x - vt$ & $d/dt' = d/dt$ → velocity addition rule $u' = u - v$



erpretation: Max eqns valid only in the rest frame of **ether**

Key Q: How should EM be described for sources and observers *moving with respect to the ether-frame*?

"The electrodynamics of a moving body"

The 1895 *dynamics* theory of ether/matter by **Lorentz**

Michelson-Morley null result @ O(v^2/c^2) → ***length contraction***

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1} \qquad x' = \gamma\,(x - vt) \qquad [\text{+ a math construct 'local time'}] \quad t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

Lorentz transformation Maxwell 'covariant' to all orders (1904)

Still in the framework of ether …Applicable only for EM

A very different approach by Einstein…

Special Relativity

With a *keen sense of aesthetics* in physics
Einstein was ***troubled by***

- **Dichotomy** of matter ~ particles, radiation ~ waves
  → **Light quanta**
- **EM singles out** 1 particular frame: the ether frame

Relativity is a symmetry in physics
Physics unchanged under some transformation
Relativity = same physics in all coordinate frames

How to reconcile (Galilean) relativity $u' = u - v$ with the constancy of c?

Resolution: simultaneity is relative

Time is not absolute, but frame dependent $t' \neq t$

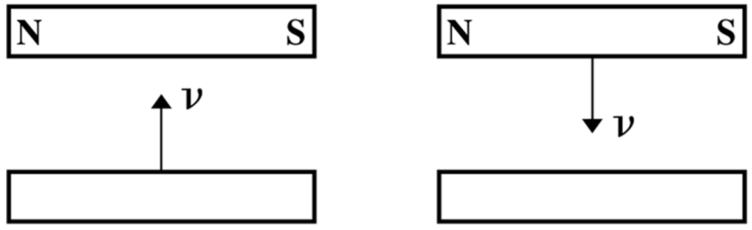
From this 1905 realization to full theory in 5 weeks 10yrs

Relation among *inertial frames*
correctly given by **Lorentz transformation**,
with Galilean transformation as low v/c approximation

All nontrivial results follow from this new conception of time

The new kinematics applicable to all physics

Magnet-conductor thought experiment



Case I Case II

Case I: moving charge in ***B*** *(ether frame)*
Lorentz force (per unit charge)

$$\frac{f}{e} = v \times B$$

Case II: changing ***B*** induces an ***E*** via Faraday's law, resulting exactly the **same force, yet such diff descriptions**

Seeking a more symmetric picture
valid in all frames

- *Dispense with **ether***
- *Invoke the **principle of relativity***

Special Relativity

Geometric formulation

Even simpler perspective

Hermann Minkowski (1907)

*Essence of SR: __**time** is on an equal footing as **space**__.*

To bring out this symmetry, unite them in a single math structure, **spacetime**

Emphasizes the **invariance** of the theory: $c \rightarrow s$

$$s^2 = -c^2t^2 + x^2 + y^2 + z^2 = [x][g][x]$$

$$= \begin{pmatrix} ct & x & y & z \end{pmatrix} \begin{pmatrix} -1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \end{pmatrix} = length^2$$

***c* as the __conversion factor__** **space $\leftrightarrow$ time**

Lorentz-transformation = rotation $\hat{R}$ in spacetime

metric $[g] \rightarrow$ all SR features

SR: The arena of physics is the 4D spacetime

Einstein was initially not impressed,

.. until he tried to formulate

General relativity (non-inertial frames)

= Field theory of gravitation

Gravity = structure of spacetime

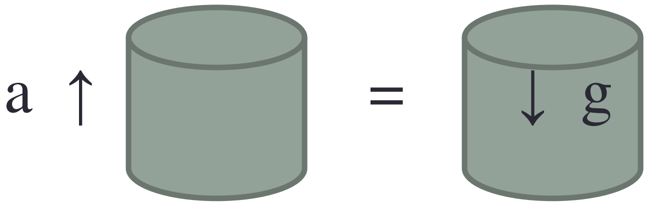
SR = flat spacetime

GR = curved spacetime

Why does GR principle automatically bring gravity into consideration?
How is gravity related to spacetime?

The Equivalence Principle (1907)
played a key role in the formulation of
general theory of relativity

starting from Galileo Remarkable ***empirical observation***
All objects fall with the same acceleration



accelerated frame = inertial frame w/ gravity

EP as the handle of going from SR to GR

SR $\to$ GR, flat $\to$ curved spacetime

"Gravity disappears in a free fall frame"

Einstein: "*My happiest thought*"

EP: Motion in grav field totally independent of properties of the test body,
attributable directly to underlying spacetime?

Einstein proposed in 1912:
gravitational field = warped spacetime

Relativity as a coordinate symmetry

Equations written in terms of 4-tensors are automatically relativistic

Special relativity

flat spacetime

$\hat{R}$ = spacetime-independent transformation

Global symmetry

General relativity

curved spacetime with *moving* basis vectors

general coord transf = spacetime dependent

$$\hat{R} = \hat{R}(x)$$

Local symmetry

Must replace ordinary derivative by *covariant differentiation*

$$[d] \to [D = d + \Gamma]$$

$\Gamma = \partial[g]$ compensate the moving bases

$SR \to GR$ with $d \to D$

gravity is brought in

local symmetry → dynamics

General Relativity

gravitational field = warped spacetime

metric tensor $[g_{\mu\nu}]$ =
rela. grav. potential

Field theory of gravity

Source particle → Curved spacetime → Test particle

Source particle → Field → Test particle

Einstein field eqn (Field eqn)

Geodesic Eqn (Eqn of motion)

1915

$$G_{\mu\nu} = g_N T_{\mu\nu}$$

g_N Newton's constant

$T_{\mu\nu}$ energy momentum tensor

$G_{\mu\nu}$ curvature tensor = nonlinear 2nd derivatives of $[g_{\mu\nu}]$

The **Einstein equation**
10 coupled PDEs
solution = $[g_{\mu\nu}]$

Metric being gravi pot,
Curvature = tidal forces

g_N as the conversion factor geometry ↔ mass/energy

General Relativity

$$G_{\mu\nu} = g_N T_{\mu\nu}$$

The **Einstein field equation**

In the limit of test particles moving with $v \ll c$
in **a static** and **weak grav field**
Einstein → Newton (the $1/r^{2}$ law explained!)
GR can depict new realms of gravity:
time-dep & strong

time-dep: **GR → gravitational wave**
Hulse-Taylor binary pulse system

Strong: **Black Holes** *full power & glory of GR*
Role of space and time is reversed
Light-cones tip over instead of → ($t = \infty$), → ($r = 0$)
Even light cannot escape

Einstein & unified field theory

the last 30 years of his life , strong conviction:

GR + ED $\longrightarrow$ solving the *quantum* mystery?

Was not directly fruitful, but his insight *Symmetry & Geometry* fundamentally influenced effort by others:

Gauge theories and *KK unification*, etc.

But both possible only with modern QM

The symmetry principle

Before Einstein, symmetries were generally regarded as mathematical curiosities of great value to crystallographers, but hardly worthy to be included among the fundamental laws of physics. We now understand that a symmetry principle is not only an organizational device, but also **a method to discover new dynamics**.

Gauge Theory

Relativity

Transformation $\hat{R}$

in coordinate space

$_{\text{global}}SR \rightarrow {}_{\text{local}}GR$

$$[d] \rightarrow [D = d + \Gamma]$$

$$\Gamma = \partial[g]$$

compensate moving bases

Gauge symmetry

Local transformation $\hat{R}(x)$

in the *internal charge* space

"changing particle label"

Gauge principle:

Change from *global* to ***local*** transformation on QM states

$$[d] \rightarrow [D = d + A]$$

brings in the compensating field A , the ***gauge field***

$\hat{R} = e^{i\theta(x)} \rightarrow$ a single $A(x)$

SR + gauge principles $\longrightarrow$ Maxwell

Electrodynamics as a gauge interaction

Gauge principle can be used to extend consideration to other interactions

Particle physics

Special relativity, photons, & Bose-Einstein statistics = key elements
But Einstein did not work directly on any particle physics theory
Yet, **the influence of his ideas had been of paramount importance**
to the successful creation of **the Standard Model of particle physics**

Strong, weak & electromagnetic interactions
are all *gauge interactions*
Symmetry principle allowed us to discover the basic eqns of SM
QCD, electroweak field eqns = generalization of Maxwell's eqns

- Non-commutative transformation:
  Multiple gauge (Yang-Mill) fields
- Spontaneous symmetry breaking:
  The symmetry is hidden

- Quantization & renormalization
  ■ truly **relevant degrees of freedom** for strongly interacting particles are **hidden** (*quark confinement*)

Q: What is the charge space?
What's the origin of gauge symmetry?

*Gauge transf = coord transf in extra D *Internal charge space = extra D

Foreshadowed modern unification theories
GR + SM require
compactified multi-dim extra D space

Einstein's influence lives on!

Kaluza-Klein theory
unification of GR+Maxwell

1919 Theodor **Kaluza** : GR in 5D
extra dimension w/ a particular geometry $[g]_{kk}$

$$GR_5^{kk} = GR_4 + ED_4$$

The Kaluza-Klein miracle!

In physics , a miracle requires an explanation

1926 Oskar **Klein** explained in modern QM

Compactified extra D → a tower of KK states
the decoupling of heavy particles simplifies the metric to $[g]_{kk}$

Last slide : summarizing

the central nature of Einstein's physics

Besides his legacy on geometry & symmetry principle,

his fundamental contribution =

Ability to connect disparate phenomena

They are conversion factors connecting disparate phenomena

h: Wave & Particle (QT)

c: Space & Time (SR)

g_N: Energy & Geometry (GR)

A pithy description *via* **the fundamental constants**

***h* -- *c* -- g_N**

form an unit system of *mass/length/time*

(The **Planck unit system**)

All due to **Einstein**'s essential contribution

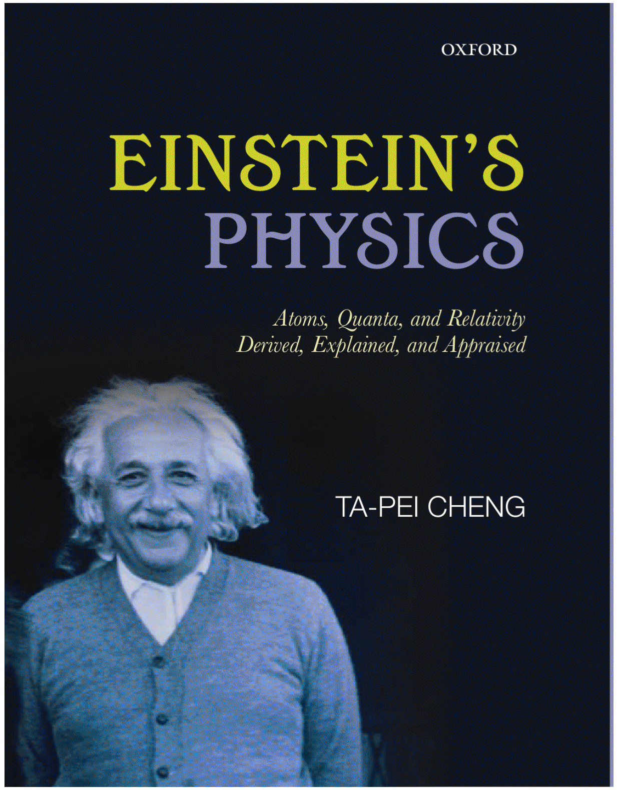
h = *Planck's constant*

g_N = *Newton's constant*

c = *Einstein's constant ?!*

$$t_P = \sqrt{\frac{h g_N}{c^5}} = 5.4\times10^{-44}\, s$$

These PowerPoint slides are posted @
www.umsl.edu/~chengt/einstein.html



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