Albert Einstein
1879 – 1955

The book explains his physics in equations

Today’s talk provides w/o math details some highlight historical context & influence

1. Atomic Nature of Matter
2. Quantum Theory
3. Special Relativity
4. General Relativity
5. Walking in Einstein’s Steps

1. The quantum postulate: Planck vs Einstein
2. Einstein & quantum mechanics
3. Relativity: Lorentz vs Einstein
4. The geometric theory of gravitation
5. Modern gauge theory and unification
Blackbody Radiation

To find the $u(T)$ and $\rho(T,\nu)$ functions:

$$u(T) = aT^4$$

electromagnetic radiation = a collection of oscillators

What is the $f$ function?

$$\rho(T,\nu) = \nu^3 f(\nu/T)$$

Wien’s displacement law (1893)

$$\rho(T,\nu) = \alpha \nu^3 e^{-\beta \nu / T}$$

fitted data well… until IR

Wien = high $\nu$ of Planck

Excellent fit of all the data

key: Wien $2 \to 1$ var

Boltzmann (1884) law:

$EM$ (radiation pressure = $u/3$)

$$\nu^2, B^2 \sim oscillator\ energy \ kx^2$$

The ratio of is an adiabatic invariant

Planck’s distribution (1900):

$$\rho(T,\nu) = \frac{\alpha \nu^3}{e^{\beta \nu / T} - 1}$$

$\nu$, $\nu$, $\nu$
Blackbody Radiation \textbf{Planck (1900)}:
\[ \rho(T,\nu) = \frac{\alpha \nu^3}{e^{\beta \nu/T} - 1} \]

What’s the physics behind this result?

Derived a relation
\[ U = \left( \frac{c^3}{8\pi \nu^2} \right) \rho \quad \text{and} \quad dS = \frac{dU}{T} \rightarrow \text{entropy } S \]

\textit{Boltzmann’s principle } \[ S = k \ln W \]

Planck: \textbf{What microstates } \[ W \text{ that can lead to this } S \text{ ?} \]

was “compelled” to make the hypothesis of \textbf{energy quantization}

\[ \epsilon = n\hbar \nu \quad n = 0, 1, 2, \ldots \]

\textbf{Einstein’s 1905 proposal of light quanta}

was \textbf{not a direct follow-up} of Planck’s
Einstein’s 1905 proposal of light quanta \( \varepsilon = h\nu \)

Einstein used Planck’s calculation \( U = \left( \frac{c^3}{8\pi \nu^2} \right) \rho \) and invoked the equipartition theorem of stat mech \( U = \frac{1}{2} kT \) to derive the Rayleigh-Jeans law: \( \rho = 8\pi c^{-3} kT \nu^2 \)

noted its solid theoretical foundation

Rayleigh-Jeans = the low frequency limit of the successful Planck’s distribution

new physics \( \rightarrow \) The high frequency limit (Wien’s distribution) \( \rho = \alpha \nu^3 e^{-\beta \nu/T} \)

Einstein undertook a statistical study of (BBR)\text{\tiny wien}:

instead of \( W \), calculate \( \Delta S \) due to volume change

\( (\text{BBR})\text{\tiny wien} \sim \text{ideal gas} \)

\( \rightarrow (\text{BBR})\text{\tiny wien} = \text{a gas of light quanta} \) with energy of \( \varepsilon = nh\nu \)

Einstein arrived at energy quantization independently---- cited Planck only in 2 places

A year later……Einstein gave a new derivation of Planck’s distribution
Einstein’s discoveries in quantum theory

(1900) Planck: $\varepsilon = h \nu$ is only a formal relation

(1905) Einstein: the quantum idea must represent new physics

proposed photoelectric effect as test beyond BBR: Q theory of specific heat (1907)

(1909) Light quanta = particles

Einstein stated for the 1st time:

quanta carried by point-like particles

“point of view of Newtonian emission theory”

Photon carries energy + momentum $p = h/\lambda$

(1913) Bohr’s quantum jumps describe absorption and emission of photons

(1916–17) Einstein construct a microscopic theory of radiation–matter interaction: (A and B coeff); The central novelty and lasting feature is the introduction of probability in quantum dynamics

General acceptance of the photon idea

Only after Compton scattering (1924)

(1924–25) Bose-Einstein statistics & condensation
Einstein & Quantum Mechanics

His discoveries in quantum theory:
Wave/particle nature of light, quantum jumps etc. can all be elegantly accounted for in the framework of quantum field theory

• A firm mathematical foundation for Einstein’s photon idea
• Quantum jumps naturally accounted for by ladder operators

\[
[\hat{a}_-, \hat{a}_+] = \hbar \nu, \quad \hat{a}_\pm |n\rangle \sim |n \pm 1\rangle \Rightarrow \text{particle behavior}
\]

QFT description broadens the picture of interactions
not only can alter motion, but also allows for emission and absorption of radiation
→ creation and annihilation of particles

Beautiful resolution of wave-particle duality in radiation energy fluctuation
wave \sim \hat{a} e^{ikx}

Modern quantum mechanics:
states = vectors in Hilbert space (superposition)
observables = operators (commutation relations)

Classical radiation field = collection of oscillators
Quantum radiation field = collection of quantum oscillators

\[ E_n = (n + \frac{1}{2}) \hbar \nu \]

Alas, Einstein never accepted this neat resolution as he never accepted the new framework of QM
### Einstein & QM: The debate with Bohr

<table>
<thead>
<tr>
<th>Orthodox interpretation of QM (Niels Bohr &amp; co):</th>
<th>the attributes of a physical object (position, momentum, spin, etc.) can be assigned only when they have been measured.</th>
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<tbody>
<tr>
<td>Local realist viewpoint of reality (Einstein,…):</td>
<td>a physical object has definite attributes whether they have been measured or not.</td>
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<td>.... QM is an incomplete theory</td>
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**Einstein, Podolsky & Rosen (1935):** A thought experiment highlighting the “spooky action-at-a-distance” feature: the measurement of one part of an entangled quantum state would instantaneously produce the value of another part, no matter how far the two parts have been separated. The discussion and debate of “EPR paradox” have illuminated some of the fundamental issues related to the meaning of QM.

**Bell’s theorem (1964):** these seemingly philosophical questions could lead to observable results. The experimental vindication of the orthodox interpretation has sharpened our appreciation of the nonlocal features of quantum mechanics.

**Einstein’s criticism allowed a deeper understanding of the meaning of QM.** Nevertheless, the **counter-intuitive picture of objective reality** as offered by QM still troubles many, leaving one to wonder whether quantum mechanics is ultimately a complete theory.
**Special Relativity**

Maxwell’s equations:  EM wave $- c$  Contradict relativity?

2 inertial frames  $x' = x - vt$  &  $d/dt' = d/dt$  $\rightarrow$ velocity addition rule  $u' = u - v$

Interpretation:  Max eqns valid only in the rest-frame of ether

**Key Q:** How should EM be described for sources and observers moving with respect to the ether-frame?

“The electrodynamics of a moving body”

The 1895 dynamics theory of ether/matter by Lorentz

Michelson-Morley null result  $\rightarrow$ length contraction

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1} \quad x' = \gamma \left(x - vt\right) \quad t' = \gamma \left(t - \frac{v}{c^2} x\right)$$

Lorentz transformation  Maxwell ‘covariant’ to all orders (1904)

Still in the framework of ether …Applicable only for EM

A very different approach by Einstein…
Special Relativity

With a *keen sense of aesthetics* in physics

Einstein was *troubled by*

- **Dichotomy** of matter ~ particles, radiation ~ waves → **Light quanta**

- **EM singles out 1 particular frame:** the ether frame

**Relativity is a symmetry in physics**

Physics unchanged under some transformation

Relativity = same physics in all coordinate frames

How to reconcile (Galilean) relativity $u' = u - v$ with the constancy of $c$?

Resolution: simultaneity is relative

Time is not absolute, but frame dependent $t' \neq t$

From this 1905 realization to full theory in 5 weeks 10yrs

Relation among **inertial frames**

correctly given by **Lorentz transformation**.

with Galilean transformation as low $v/c$ approximation

All nontrivial results follow from this new conception of time

The new kinematics applicable to all physics

**Magnet-conductor thought experiment**

![Diagram of magnet-conductor thought experiment]

Case I: moving charge in $B$ *(ether frame)*

**Lorentz force** (per unit charge)

$$\frac{f}{e} = v \times B$$

Case II: changing $B$ induces an $E$ via Faraday’s law, resulting exactly the same force, *yet such diff descriptions*

Seeking a more symmetric picture valid in all frames

- **Dispense with ether**

- **Invoke the principle of relativity**
Even simpler perspective

**Hermann Minkowski** (1907)

*Essence of SR:* time is on an equal footing as space.

To bring out this symmetry, unite them in a single math structure, spacetime.

Emphasizes the **invariance** of the theory: \( c \rightarrow s \)

\[ s^2 = -c^2t^2 + x^2 + y^2 + z^2 = [x][g][x] \]

\[ = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \begin{pmatrix} -1 & 1 & 1 & 1 \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \text{length}^2 \]

\( c \) as the **conversion factor** space ↔ time

**SR:** The arena of physics is the 4D spacetime

Einstein was initially not impressed,

.. until he tried to formulate

**General relativity** (non-inertial frames)

= Field theory of gravitation

Gravity = structure of spacetime

SR = flat spacetime

GR = curved spacetime

Lorentz-transformation = rotation \( \hat{R} \) in spacetime

**metric** \([g] \rightarrow \) all SR features
The Equivalence Principle (1907) played a key role in the formulation of general theory of relativity.

Why does GR principle automatically bring gravity into consideration? How is gravity related to spacetime?

starting from Galileo Remarkable empirical observation All objects fall with the same acceleration

accelerated frame = inertial frame w/ gravity

EP as the handle of going from SR to GR

SR → GR, flat → curved spacetime

“Gravity disappears in a free fall frame”

EP: Motion in grav field totally independent of properties of the test body, attributable directly to underlying spacetime?

Einstein proposed in 1912: gravitational field = warped spacetime

Einstein: “My happiest thought”
Relativity as a coordinate symmetry

Special relativity
flat spacetime
\( \hat{R} = \) spacetime-independent transformation

Global symmetry

General relativity
curved spacetime with moving basis vectors
general coord transf = spacetime dependent
\( \hat{R} = \hat{R} (x) \)

Local symmetry

Equations written in terms of 4-tensors are automatically relativistic

Must replace ordinary derivative by covariant differentiation

\[ [d] \rightarrow [D = d + \Gamma] \]
\[ \Gamma = \partial [g] \]
compensate the moving bases

\( SR \rightarrow GR \) with \( d \rightarrow D \)

gravity is brought in

local symmetry \( \rightarrow \) dynamics
**General Relativity**

Gravitational field = warped spacetime

*Field theory of gravity*

Source particle $\rightarrow$ Curved spacetime $\rightarrow$ Test particle

Einstein field eqn

Geodesic Eqn

The **Einstein equation**

10 coupled PDEs

Solution $= [g_{\mu\nu}]$

$g_N$ as the conversion factor  geometry $\leftrightarrow$ mass/energy

1915

$g_N$ Newton’s constant

$T_{\mu\nu}$ energy momentum tensor

$G_{\mu\nu}$ curvature tensor = nonlinear 2nd derivatives of $[g_{\mu\nu}]$

Metric being gravi pot, Curvature = tidal forces

Rela. grav. potential

$[g_{\mu\nu}]$ metric tensor

$T_{\mu\nu}$ energy momentum tensor

$G_{\mu\nu}$ curvature tensor

$g_N$ Newton’s constant
In the limit of test particles moving with $v \ll c$
in a static and weak grav field
Einstein $\rightarrow$ Newton (the $1/r^2$ law explained!)
GR can depict new realms of gravity:
*time-dep* & *strong*

**time-dep:** GR $\rightarrow$ gravitational wave
Hulse-Taylor binary pulse system

**Strong:** *Black Holes* full power & glory of GR

*Role of space and time is reversed*
Light-cones tip over instead of $\rightarrow (t = \infty)$, $\rightarrow (r = 0)$
Even light cannot escape

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**General Relativity**

$G_{\mu\nu} = g_N T_{\mu\nu}$

The Einstein field equation
Einstein & unified field theory

the last 30 years of his life, strong conviction:

GR + ED $\rightarrow$ solving the quantum mystery?

Was not directly fruitful, but his insight Symmetry & Geometry
fundamentally influenced effort by others:

Gauge theories and KK unification, etc.
But both possible only with modern QM

The symmetry principle

Before Einstein, symmetries were generally regarded as mathematical curiosities of great value to crystallographers, but hardly worthy to be included among the fundamental laws of physics. We now understand that a symmetry principle is not only an organizational device, but also a method to discover new dynamics.
Gauge Theory

**Relativity**
Transformation $\hat{R}$ in coordinate space

**Gauge symmetry**
*Local* transformation $\hat{R}(x)$ in the *internal charge* space
“changing particle label”

**Gauge principle:**
Change from *global* to *local* transformation on QM states

$$ [d] \rightarrow [D = d + \Gamma] $$

$$ \Gamma = \partial[g] $$

brings in the compensating field $A$, the *gauge field*

$$ \hat{R} = e^{i\theta(x)} \rightarrow \text{a single } A(x) $$

**Electrodynamics as a gauge interaction**

SR + gauge principles $\rightarrow$ Maxwell

Gauge principle can be used to extend consideration to other interactions
**Particle physics**

Special relativity, photons, & Bose-Einstein statistics = key elements

But Einstein did not work directly on any particle physics theory

Yet, the influence of his ideas had been of paramount importance
to the successful creation of the **Standard Model of particle physics**

**Strong, weak & electromagnetic interactions**

are all *gauge interactions*

Symmetry principle allowed us to discover the basic eqns of SM

QCD, electroweak field eqns = generalization of Maxwell’s eqns

- Non-commutative transformation: Multiple gauge (Yang-Mill) fields
- Spontaneous symmetry breaking: The symmetry is hidden

- Quantization & renormalization
  - truly relevant degrees of freedom for strongly interacting particles are hidden *(quark confinement)*
### Q: What is the charge space? What’s the origin of gauge symmetry?

*Gauge transf = coord transf in extra D *Internal charge space = extra D

### Kaluza-Klein theory

**unification of GR+Maxwell**

1919 Theodor **Kaluza** : GR in 5D extra dimension w/ a particular geometry $[g]_{kk}$

$GR_{5 \, kk} = GR_4 + ED_4$

*The Kaluza-Klein miracle!*

In physics, a miracle requires an explanation

1926 Oskar **Klein** explained in modern QM

Compactified extra D → a tower of KK states

the decoupling of heavy particles simplifies the metric to $[g]_{kk}$

### Foreshadowed modern unification theories

GR + SM require compactified multi-dim extra D space

**Einstein’s influence lives on!**
A pithy description

via the fundamental constants

\[ h \quad \text{--} \quad c \quad \text{--} \quad g_N \]

form an unit system of

mass/length/time

(The Planck unit system)

All due to Einstein’s essential contribution

They are conversion factors connecting disparate phenomena

- \( h \): Wave & Particle (QT)
- \( c \): Space & Time (SR)
- \( g_N \): Energy & Geometry (GR)

Besides his legacy on geometry & symmetry principle, his fundamental contribution = Ability to connect disparate phenomena

Natural units, not human construct

Last slide: summarizing the central nature of Einstein’s physics

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\[ t_P = \sqrt{\frac{g_N}{c^5}} = \sim 4 \times 10^{-44} \text{ s} \]
These PowerPoint slides are posted @ www.umsl.edu/~chengt/einstein.html

Copies of the book are left in PSU Physics Dept Office for your perusal
Sign-up & Check-out

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