PHYSICS OF THE COLD FUSION PHENOMENON

Hideo Kozima
Cold Fusion Research Laboratory
Oya 597-16, Aoi, Shizuoka, 421-1202, Japan

Abstract

The cold fusion phenomenon (CFP) is investigated quantum mechanically using a recent knowledge of nuclear and solid-state physics: neutron halo of exotic nuclei and non-localized proton/deuteron wavefunctions. Characteristics of the CFP including various nuclear events are essentially explained by the resulting inter-nuclear $n-n$ interaction mediated by protons/deuterons in transition-metal hydrides and deuterides (T-MH/D). It is shown also that events in the CFP have characteristics of complexity in accordance with the complicated structures of T-MH/D where occurs the CFP, thus resolving the controversial lack of the quantitative reproducibility.

1. Introduction

Since the discovery of the cold fusion phenomenon (CFP) by Fleischmann et al. [1] in 1989, hundreds of experimental data sets have been accumulated in both transition-metal hydrides and deuterides. The data sets range widely over various events from the emissions of neutron, charged particles and gamma ray to nuclear transmutations generating almost all elements in the periodic table. Excess energy accompanies each event. We have to construct a science of these facts in the CFP, as H. Poincaré said a hundred years ago: *Science is build up of facts, as a house is built of stones; but an accumulation of facts is no more a science than a heap of stones is a house* [2].

To have a unified point of view for these diverse and complicated experimental data sets [3 – 6], we have developed a phenomenological model (TNCF model) at first and then the neutron drop model for the CFP. These works were published in papers and also in two books [3, 4] and a review paper [5]. Some quantum mechanical investigations on the bases of premises assumed in these models were also performed in recent works [3 – 5].

After publication of the recent book [4], we have further developed the
idea of complexity in several phases of the CFP. Almost all CF systems satisfy the sufficient conditions for the occurrence of complexity which is common in any nonintegrable system with Poincaré resonances (both in classical and quantum) according to I. Prigogine [7]. That is, they are open, non-equilibrium (far-from-equilibrium), many body systems with non-linear interactions between active components (agents). In addition to this general nature of CF systems, we have many experimental data sets in CFP that show characteristics of complexity.

Most outstanding examples of them are the two laws we have found, i.e. (1) the inverse-power law of \( N(P_{\text{ex}}) \) vs. \( P_{\text{ex}} \) relation, where \( N(P_{\text{ex}}) \) is the number of events producing excess power \( P_{\text{ex}} \) ([4] §2.12) and (2) the stability effect in the yield of product elements by the nuclear transmutation ([4] §2.11). In addition to these two laws, we will point out here a third law, (3) the bifurcation of the CFP in occurrence of excess energy and nuclear transmutation reactions [8]. Several features of the CFP related with these laws, especially the third law, are qualitatively depicted in this paper.

The discovery of the cold fusion phenomenon (CFP) was made in an electrolytic system with Pd cathodes by Fleischmann et al. [1] in 1989. At almost the same time, Jones et al. [9] reported independently detection of 2.45 MeV neutrons in similar electrolytic systems (with Pd and Ti cathodes) to that used by Fleischmann et al.

They analyzed their experimental data sets assuming that immense (more than \( 10^{25} \) times) amplification of the probability of fusion reactions of two deuterons occurs in transition-metal deuterides (Fleischmann’s or H-P’s hypothesis).

Due to the experimental and theoretical barriers inherent in the system where occurs the CFP to be accepted straightforwardly, this phenomenon has been excluded from established research fields of science and has survived in obscurity for 18 years.

Experimental barriers against CFP to be accepted by scientists are lack of quantitative reproducibility, difficulty in excess energy measurement, and also in neutron detection. The most important barrier is the lack of quantitative reproducibility which is apparently expected only for simple systems. We have known many examples of this lack of quantitative reproducibility (i.e. existence of qualitative reproducibility). One of these examples is the decay of radioactive nuclide, e.g. \(^{224}\text{Ra}\) which decays by
alpha emission with a half-life of 3.66 days. In this case, we can determine only statistically a number $N(t)$ of nuclei at a time $t$ if we know $N(0)$.

Therefore, we should not expect the quantitative reproducibility for events in CFP where nuclear processes surely participate in.

Theoretical barriers against CFP are mainly related with the Fleischmann's hypothesis. Based on the firm knowledge established in nuclear physics by the end of 20th century, possibility of any nuclear reaction observable in room-temperature solids without any acceleration mechanism has been denied. It says that nuclear reactions between or among charged nuclei occur effectively only in a high-energy region with mutual nuclear energies of millions electron volt (MeV). In this common sense denying the CFP, however, there is a presupposition that only charged nuclei participate in CFP excluding possible participation of neutrons.

In the eighteen years after the discovery of a part of the CFP by Fleischmann et al.[1] and Jones et al. [9], there have appeared experimental data sets revealing the complicated nature of the CFP [3 – 6]. Events of the CFP have been obtained not only in deuterium systems (systems containing deuterium, D) but also in protium systems (containing hydrogen, H). It became clear that the CFP does not occur when there are no ambient neutrons and that the CFP is enhanced by an artificial thermal neutron irradiation [3 – 5]. Furthermore, there have been observed various new nuclides supposed to be generated in both deuterium and protium systems. These features have clearly shown that the Fleischmann’s hypothesis is not fundamental for the CFP.

The variety of the products of the CFP demands us to have a definite point of view to investigate this phenomenon. It is possible to consider the protium and deuterium systems as independent fields with different fundamental mechanisms for their CF events. Or it is possible to explore a new nuclear physics on the CFP not explored in the orthodox nuclear physics we have at hand. A possible reconciliation of the CFP and the nuclear physics may be attained using the neutron as an agent relying on the unexplored nature of neutrons in solids and on nuclear properties at around the neutron evaporation level that is under development at present [4, 5].

It is noticed also that there are correlations between the CFP and atomic behavior of H and D in the transition-metal hydrides and deuterides (T-MH /D): the CFP occurs only in $fcc$ (and $hcp$) T-MH/D where hydrogen isotopes (H
and/or D) have not localized at a specific interstice. The CFP does not occur in \textit{bcc} T-MH/D where the H and/or D atom localizes at an interstice [4, 5].

Furthermore, the host atoms in T-MD/H where the CFP occurs have characteristic nuclides that seem to have the neutron evaporation level with wavefunctions extended out from ordinary nuclear surfaces [4, 5].

Therefore, physics of the CFP should be investigated in conjunction with nuclear physics of the host nuclides and solid state physics of hydrogen isotopes in T-MH/D.

Turning our viewpoint from microscopic to macroscopic, we notice the systems where the CFP occurs have very complicated structures. They are multi-component, inhomogeneous, non-equilibrium, open systems with components interacting non-linearly each other under influence of mass flow of H/D, of electric field, of irradiation of ambient thermal and epi-thermal neutrons, and sometimes of irradiation of charged particles (in discharge systems).

It is well known that there occurs a phenomenon called complexity in such an open, non-equilibrium system with components interacting non-linearly. Therefore, we have to expect to observe complexity in the systems where we have observed the CFP.

In the CFP, some features characteristic to many-particle systems have been found in the analyses of the experimental data sets [4, 5]. More direct features of the CFP corresponding to complexity was obtained by Dash et al. as reported in the paper presented at ICCF13 [8].

In Chapter 2, we give a brief survey of nuclear properties supposed to be related with CFP and of wavefunctions of H/D in \textit{fcc hcp} T-MH/D. In Chapter 3, we investigate complexity in the CFP. In Conclusion, we summarize physics of the CFP as briefly presented in this paper.

2. Nuclear and Solid-State Properties of Transition-Metal Deuterides and Hydrides (T-MH/D)

In this Chapter, we give an explanation of neutron physics in T-MH/D based on nuclear properties of exotic nuclei developed in these twenty years and solid-state physics of hydrogen isotopes in them.

2.1 Wavefunctions of Neutrons in Evaporation Levels of Exotic Nuclei

Properties of nuclides in their ground and high-energy excited states have
been explored extensively in the last century applying neutrons, gamma rays and high energy particles accelerated by various machines to make charged nuclei approach each other close enough in the range of the short-range nuclear force overcoming Coulomb repulsion between them.

However, there has remained a field kept untouched by these researches until recently; the nuclei in their low excited state are alien to those in the ground states and higher excited states. Especially, investigation of the states of nuclei at their so-called neutron evaporation level had started only twenty years ago. Now, there are several review articles on this problem telling following facts [10, 11]; there exist many nuclei with the so-called neutron halo extending far out of the nuclear core that is the nucleus figured out in orthodox nuclear physics. It is recognized now the necessity to treat nuclei tenderly to know their behavior in states at or near the neutron evaporation level.

On the other hand, there appeared a new field of investigation related with the states of nuclei at their lower excited states; the cold fusion phenomenon (CFP). At first, this field was considered as a part of the orthodox nuclear physics when a part of its events was discovered and proclaimed in 1989 [1, 9]. The content of the phenomenon unexpectedly increased into huge amount with profound variety in these 18 years after its discovery. Now, we know that the CFP occurs both in protium and deuterium systems and includes such events as nuclear transmutations producing almost all nuclides from pre-existed nuclei Ti, Ni, Pd, n, p, d, Li, K, S, O, and so forth in CF systems, emissions of neutrons, photons, and charged particles (p, alpha, and so forth) accompanying excess energy.
Fig. 1. Plot of the $^{11}\text{Be}$ density [10]. The upper right corner shows the simplified picture of a halo nucleus as a two-body system with an inert core and a halo neutron. Dotted line, core density as a function of radius; solid line, the density obtained in a Hartree–Fock calculation with the neutron in a 2$s_{1/2}$ orbital and a single-neutron separation energy adjusted (to agree better with experiment) to 0.51 MeV. Note the very far-extended, dilute tail that is the characterizing feature of a halo [10].

To show some phases of the recent development in nuclear physics, we cite here a figure (Fig. 1) showing extension of neutron density outside the nuclear core and a table (Table 1) showing confirmed nuclides in the halo states [10]. The investigation of the halo states of nuclei is in an infantile state and only a few data has been obtained for small mass-number nuclides as shown in Table 1.

### Table 1. Halo states. For each state the excitation and separation energies and the angular momentum of the halo particle(s) are listed.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$E_x$ (MeV)</th>
<th>$S - E^*$ (keV)</th>
<th>Configuration</th>
<th>$l$</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{11}\text{Be}$</td>
<td>g.s.</td>
<td>504</td>
<td>$n + ^{10}\text{Be}$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$^{11}\text{Be}$</td>
<td>0.32</td>
<td>184</td>
<td>$n + ^{10}\text{Be}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$^{17}\text{F}$</td>
<td>0.50</td>
<td>105</td>
<td>$p + ^{16}\text{O}$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$^{6}\text{He}$</td>
<td>g.s.</td>
<td>973</td>
<td>$n + n + ^{4}\text{He}$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>$^{11}\text{Li}$</td>
<td>g.s.</td>
<td>310</td>
<td>$n + n + ^{9}\text{Li}$</td>
<td>?</td>
<td></td>
</tr>
<tr>
<td>$^{14}\text{Be}$</td>
<td>g.s.</td>
<td>1340</td>
<td>$n + n + ^{12}\text{Be}$</td>
<td>?</td>
<td></td>
</tr>
</tbody>
</table>

*From Audi and Wapstra, 1993.*

Table 1. (Table 1 of Reference [10].)

To know the present extent of our knowledge in this field, we summarize the nuclear properties of the neutron halo closely related to the CFP from our point of view [10, 11]:
1) “Thresholds,” the transition points between bound discrete states and a continuous spectrum, give rise to many fascinating phenomena. Among the newest of them are the so-called halo states that occur in some nuclei near the limit of particle stability.
2) Only very loosely bound neutrons in an $s$ state relative to the core provide an ideal halo.
3) Arbitrary large values of the mean-square radius could occur in nuclei, at
least in principle.

4) If a neutron halo is defined as a divergent root-mean-square (r.m.s.) radius of the halo for the neutron separation energy $S_n$ approaching zero, then a two-body (i.e. single-neutron) halo is possible only if the orbital angular momentum $l = 0, 1$.

5) The conditions for a three-body (two neutron) halo are more restrictive and the radius is at most logarithmically divergent.

6) The known cases of even-$n$ halos have the Borromean property, namely that both the two-body sub-systems $(A + n)$ and $(n + n)$ are unbound.

7) Mass calculations suggest that halo states will occur symmetrically at the neutron drip line for most elements.

8) Our knowledge is restricted to five cases in total (He, Li, Be). (Cf. Table 1)

9) Halo states have unusually large nuclear and electric reaction cross sections.

10) The large size of the halo implies a narrow momentum distribution.

11) The halo neutron is in a mixed state containing its ground state and continuum states.

12) A halo neutron has a probability distribution that extends far beyond the core.

13) $^{11}$Be, an example of the neutron halo size: The wave functions for the case of a single-neutron halo correspond to calculated (with Woods – Saxon potential) r.m.s. radii of the $l = 0, 1$ states of 5.99 and 5.66 fm, respectively. For comparison, the r.m.s. radius of a normal $p$-shell nucleus is 2.5 fm.

Thus, we do not know yet exactly what kind of wavefunctions the halo neutrons have in the medium mass-number nuclides such as Ti, Ni and Pd. However our knowledge about them is scarce yet, there surely exist neutrons with extended wavefunctions in lattice nuclei in CF materials.

We have utilized this property of neutron wavefunctions expected from experimental data sets in the estimation for the possible interaction of two neutrons in different lattice nuclei mediated by interstitial protons or deuterons which we called the super-nuclear interaction [4, 5]. The super-nuclear interaction makes possible formation of the neutron bands at the zero-energy level in which high density neutron states (CF-matter) are formed at surface/boundary regions. This state of neutrons has been the essential factor to make possible the CFP to occur.

Therefore, it is necessary to have not only the extended neutron
wavefunction but also the proton/deuteron wavefunction (cf. § 2.2 below) having contact interaction with the neutron to realize the CF-matter and finally the CFP.

2.2 Wavefunctions of Protons/Deuterons in Transition-Metal Hydrides and Deuterides (T-MH/D)

It is a surprising fact that almost all cold fusion phenomenon (CFP) has been observed in fcc (and hcp) transition-metal hydrides and deuterides exclusively in metals Ti, Ni, and Pd. It is also true that there are selective combinations among the metal and hydrogen isotopes (and alkaline elements and other elements in electrolyte) such as Pd-D-Li, Ni-H-K and others.

![Density profiles of H and D on T-sites in bcc metals (Nb, V, Ta) in the x-direction shown in the inset. Profiles calculated for a) a H-atom and b) a D-atom in Nb. Profiles observed by Reidinger et al. for c) VD$_{0.79}$ at 70 degC, d) NbH$_{0.82}$ at 119 degC and NbD$_{0.73}$ at 140 degC and e) TaH$_{0.20}$ at 150 degC. (Fig. 5 of H. Sugimoto and Y. Fukai, J. Phys. Soc. Japan 51 (8), pp. 2554 – 2561 (1982) )](image)

On the other hand, the transition-metal hydrides and deuterides (T-MH/D) have been investigated for more than 100 years for their peculiar diffusion and occlusion properties of hydrogen isotopes.

Therefore, it is natural to investigate the physical properties of these transition-metal hydrides and deuterides in connection with the occurrence of the CFP. We have had speculated this problem a little in the recent papers and compiled them in a recent report [5] and a book [4].

It is interesting to know that the physical behavior of hydrogen isotopes
in transition metals is divided into two classes; one in \textit{bcc} metals (V, Nb, Ta etc.) and the other in \textit{fcc} (and \textit{hcp}) metals (Ti, Ni, Pd etc.). In the first class (\textit{bcc} metals) a nucleus of hydrogen isotopes are localized in an interstitial site (interstice) and there are a few works to determine the profile of their wavefunctions as illustrated in Figs. 2 and 3. Accordingly, vibration and diffusion properties of these localized hydrogen isotopes have characteristics in accordance with the localization of the nuclei [5 §2.2, 4 §3.6].

![Fig. 3. Localization of a hydrogen atom at an interstice in a \textit{bcc} transition metal Ta (after Y. Fukai, \textit{Kotai-Butsuri} (in Japanese) 16, 253 (1981)). Left: potential contour lines around a self-trapped H-atom in Ta. Right: Contour lines of H-atom wavefunction self-trapped in Ta. From top to bottom: (a) Ground state, (b) – (d) excited states with excitation energies of $\Delta E = 109$, 158 and 168 meV, respectively. In excited states, wavefunctions extend to the nearest lattice points and interstices.](image)

On the other hand, in the second class (\textit{fcc} and \textit{hcp} metals), a nucleus of hydrogen isotopes is not localized at an interstitial site but extend to several interstices [5 §2.2, 4 §3.6]. And, therefore, vibration and diffusion characteristics of hydrogen isotopes are not discussed in parallel to the first class (\textit{bcc} metals). In this class, there are no work on the wavefunctions of a proton and a deuteron in these metals suggesting difficulty to determine wavefunctions of non-localized particles (proton and deuteron).

In relation to the occurrence of the super-nuclear interaction proposed by us [4 §3.7.2, 5 §3], we have to estimate the wavefunction $\Phi(R; b)$ of interstitial proton (or deuteron) at around a nearest neighbor lattice nucleus. The super-nuclear interaction determines the width of the neutron band and
then the neutron density at a boundary/surface region. The latter is related to the CF matter that interacts especially with adsorbed or absorbed nuclei and also with lattice nuclei at the boundary/surface region.

We have formulated this scenario in the frame of the second-order perturbation theory [4, 5]. On the same line described in these publications, we discuss here a possibility to determine the proton/deuteron wavefunction in terms of the data obtained in the CFP research.

Let us write down the proton/deuteron wavefunction \( \Phi(R, b_j) \) at an interstice \( b \) (the suffix \( j \) specifies the interstice) as a linear combination of localized wavefunctions (Wannier functions) \( W(R, a_i) \) at nearest-neighbor lattice points \( a \) (the suffix \( i \) specifies the lattice point) around the interstice \( b \) as follows (neglecting the spin part):

\[
\Phi(R, b_j) = \sum_i A_{ji} W(R, a_i). \tag{1}
\]

The coefficients \( A_{ji} \) give a density of a proton/deuteron at one of the adjacent lattice points \( a \) of the interstice \( b \).

If we know \( \Phi(R, b_j) \), the Wannier function \( W(R, a_i) \) at the site \( a \) is calculated inversely from \( \Phi(R, b_j) \) by an inverse transformation of the equation (1):

\[
W(R, a_i) = \sum_j B_{ij} \Phi(R, b_j). \tag{2}
\]

The coefficients \( B_{ij} \) is calculated by \( A_{ji} \) and vice versa. In the simplest case where the Wannier function is expressed as a 1s-type function that has a finite value at the origin, we can determine the wavefunction of the proton/deuteron if we know the value \( A_{ji} \) by some means.

From the estimation of the width of the neutron valence band ([4 § 3.7, 5 § 3.2]) responsible to the experimental data of the CFP, we can determine the matrix element of the proton/deuteron – neutron interaction if we know the latter wavefunction. The neutron wavefunction in a nucleus at its halo state has been investigated intensively for light nuclei in nuclear physics. We will be able to know concrete information about neutron wavefunctions in medium mass-number nuclei like Ni and Pd in near future. Then, we might be able to calculate the proton/deuteron wavefunctions from experimental data obtained in the CFP. The resulting wavefunction will give information about the nature of the protons/deuterons related to their diffusion and vibration properties in fcc and hcp transition metals.

The interrelation between the solid-state physics of the T-MH/D and the CFP is not explored at all yet. However, as we have pointed out recently [4, 5],
there are apparent correlations between them. Intentional researches of this interrelation will be fruitful for both fields which have been investigated independently hitherto as if they are isolated fields.

2.3 Possible States of Neutrons in T-MH/D and the CFP

When optimum conditions for wavefunctions of neutrons in lattice nuclei (§2.1) and of hydrogen isotopes (§2.2) were satisfied in the transition-metal hydrides/deuterides (T-MH/D), we can expect formation of neutron bands around zero and then the CF matter in surface/boundary regions of the sample [4, 5].

2.3.1 Neutron – Proton Interaction in Solids

The overlapping of the proton wavefunction $\psi_p(R - b_j)$ on an interstice at $b_j$ with a nucleon (neutron) wavefunction $\psi_n(x, a_i)$ on an adjacent lattice nucleus at $a_i$ results in a proton-neutron interaction through the nuclear force. The nuclear interaction is expressed by a potential whose form is taken, for example, as the square-well type (neglecting spin-dependence and the imaginary part):

$$V_s(r-R) = \begin{cases} V_s^0, & (|r-R| < b) \\ 0, & (|r-R| > b) \end{cases}$$  \hspace{1cm} (3)

where $V_s^0 \approx 3.5$ MeV and $b \approx 2.2 \times 10^{-13}$ cm. The choice of this potential out of several possible types does not make a large difference to the result for the low energy phenomena we are considering in this paper.

This interaction pulls two neutron states in different lattice nuclei into coupling as shown below. We will call this coupling the super-nuclear interaction.

2.3.2 Super-Nuclear Interaction between Neutrons in Different Lattice Nuclei and Neutron Valence Band

If there is an interaction between a neutron in a lattice nucleus at $a_i$ and a proton/deuteron at an interstice $b_j$ next to the lattice point $a_i$, there appears an interaction between two neutrons in two lattice nuclei at $a_i$ and $a_i'$ interacting with the same proton/deuteron at $b_j$ when $a_i'$ is also a neighbor of $b_j$. The coupled neutrons in lattice nuclei reveal a dispersed energy spectrum called the energy band. We investigate this possibility below.

(a) Total Energy of fcc Transition-Metal Hydrides and Deuterides

In the second-order perturbation approximation taking the square well potential (3) for the nuclear interaction, the total energy $E_{k,\{pa\}}$ of this system composed of occluded protons (deuterons) and a neutron in an excited level of
lattice nuclei expressed as a Bloch state is written down as follows \cite{5}:

\[ E_{k,\{p\alpha\}} = E_{n,\{p\alpha\}} + \sum_{k',i,i',j} \exp[-i(k_i - k_{i'}a_i)] v_{np}(ii') \]  \hspace{1cm} (4)

\[ E_{n,\{p\alpha\}} = E_{n,\{p\}} + \sum_i \epsilon_{pj} \]  \hspace{1cm} (5)

\[ v_{np}(ii') = \sum_{\{p'\neq\{p\}} P[dE/\rho_h(E) \langle np;ij| V| n'p';ij\rangle \langle n'p';i'j| V| np;i'j\rangle]/(E + \epsilon_{pj}), \]  \hspace{1cm} (6)

\[ V(r) = V_{s}(r), \]  \hspace{1cm} (7)

\[ <np;ij| V| n'p';ij> = \int d\mathbf{R}_j \psi_{n\{p\}}^*(\mathbf{r} - a_i) \phi_{p}(\mathbf{R}_j - b_j) V_{s}(\mathbf{r} - R_j) \psi_{n'}(\mathbf{r} - a_{i'}) \phi_{p'}(\mathbf{R}_j - b_{i'}), \]  \hspace{1cm} (8)

where summations over \( i \) and \( i' \) in (4) are only over the nearest neighbor lattice points \( a_i \) and \( a_{i'} \) of an interstice \( b_j \), \( \rho_h(E) \) is a density of states for neutron energy levels, and

\[ \epsilon_{p'p} \equiv \epsilon_{p'} - \epsilon_{p}, \quad E \equiv E_{n}\{p\} - E_{n\{p\}}. \]

Further, the summation over \( p' \) in (6) reduces to \((n_p+1)(n_p+2)\), the degeneracy of the energy \( \epsilon_{np} \), times summation over \( n_p \). \( E_{n\{p\}} \) is an energy of a neutron in an excited state \( \psi_{n\{p\}}(\mathbf{r} - a_i) \) in a lattice nucleus at \( a_i \) when occluded protons are in states \( p_h \), and \( \epsilon_{pj} \) in (5) is an energy of a proton in a state \( \phi_{pj}(\mathbf{R}_j - b_j) \) at an interstice \( b_j \). We ignore the \( p \)-dependence of \( E_{n\{p\}} \) hereafter in this work.

Thus, the matrix elements (8) calculated for neutron states around the evaporation level and proton (deuteron) states of Bloch functions are at most \(10^{20}\) times larger than the value calculated for a lower excited state of a neutron in nucleus and a localized state of a proton (deuteron). If this matrix element is effective, there appears an interaction between two neutrons in different lattice nuclei mediated by occluded hydrogen isotopes at interstices. This interaction is a long-range one reaching from a lattice point to another having its origin in the internucleon strong interaction; we may call it the **super-nuclear interaction** between neutrons in different lattice nuclei, or the inter-nuclear n-n interaction mediated by protons/deuterons in T-MH/D. The super-nuclear interaction is illustratively depicted in Fig. 4.
Fig. 4 Super-nuclear interaction between two neutrons $a$ and $b$ in lattice nuclei $i$ and $i'$ mediated by a proton (or a deuteron) $k$ at an adjacent interstitial site $j$.

Finally, the effective potential $v_{np}(ii')$ (6) is estimated as a function of the principal value $I$ of the integration which appeared in that equation using the result of an order of magnitude calculation
\[<np;ij|V|n'p';ij> = 3.2 \times 10^{-14}, \quad (eV)\]
and assuming the insensitiveness of the matrix elements to the energy:
\[v_{np}(ii') = 1 \times 10^{-27} I, \quad (eV)\]
\[I \equiv P\int dE \rho_n(E)/E = (\rho_n(E)/\delta\varepsilon) \Delta\varepsilon.\]

We can, then, estimate the approximate value of the integral (principal value) $I$, taking following values $\rho_n(E) \approx 10^3$ keV$^{-1}$, $\delta\varepsilon \approx 10^{-3}$ keV, and $\Delta\varepsilon \approx 1$ keV on the assumption that single-particle energy-level difference is $\approx 1$ keV and the level density increases to a value $10^3$ times larger than that of single particle motion:
\[I \approx \rho_n(E) \Delta\varepsilon/\delta\varepsilon = 10^6. \quad (eV^{-1})\]

Thus, an order of magnitude of $v_{np}(ii')$ in a crystal PdH becomes
\[v_{np}(ii') \approx 1 \times 10^{-24} (eV).\]

Including the enhancement factors $\alpha$ and $\beta$ defined to take into several effects [4, 5], we obtain the final result:
\[v_{np}(ii') \approx 1 \times 10^{-24} \alpha^2 \beta^2 (eV).\]

The enhancement factor $\alpha$ is for the level density of neutrons in a nucleus and another enhancement factor $\beta$ is for the probability amplitude of a proton (deuteron) wavefunction at a lattice nucleus with $\alpha_{max}^2 = 10^{12}$ and $\beta_{max}^2 = 10^{20}$.

(b) Neutron Valence Band in PdH

To show briefly the crystal-structure dependence of the bandwidth $\Delta$ of the neutron valence band, we will make a simplification of the super-nuclear interaction (6) between adjacent nuclei at $a_i$ and $a_i'$ assuming that it depends only on the magnitude of the vector $a_i \equiv a_i - a_i'$.

Then, we can rewrite the total energy (4) and have the energy spectrum of the neutron Bloch waves in the fcc lattice ($a$ is the side of the lattice cube):
\[\varepsilon_k = E_{ln,pl} - y_{n,p} - 2 \times 4 y_{n,p} (\cos k_x a/4 \cos k_y a/4 + \cos k_y a/4 \cos k_z a/4 + \cos k_z a/4 \times \cos k_x a/4) - 2 y_{n,p} (\cos k_x a + \cos k_y a + \cos k_z a)\]
\[E_{ln,pl} = E_{ln} + \sum_i \varepsilon_{pj},\]
\[- y_{n,p} = v_{np}(0),\]
\[ -y_{n,p}' = \nu_{np}(ii') \]  

The shifts \( y_{n,p} \) of energy level \( E_{(n,p)} \) may be negative in appropriate situations. A characteristic of this energy band formation is the contributions not only from nearest neighbors \((0, \pm a/2, \pm a/2), \text{etc.}\) but also from next nearest neighbors \((\pm a, 0, 0), \text{etc.}\)

Putting the value \( \nu_{np}(ii') \approx 1 \times 10^{-24} \alpha^2 \beta^2 \) in (17), we obtain the bandwidth \( \Delta \) from (14) as follows:

\[ \Delta \approx 2.5 \times 10^{-23} \alpha^2 \beta^2 \text{ (eV)}. \]

The value of the bandwidth \( \Delta \approx 2.5 \text{ GeV} \) for \( \alpha_{\text{max}} = 10^6 \) and \( \beta_{\text{max}} = 10^{10} \) is obtained by the assumptions that the level density at excited levels around the evaporation energy is similarly strongly magnified by collective modes as observed by neutron resonance absorption experiments and that the excited proton states are expressed by plane waves. The first assumption has no experimental basis at present and seems too exaggerated the situation to be real. We may assume rather small value as \( \alpha = 10^2 \) or \( 10^3 \) instead of \( 10^6 \). Then, the bandwidth may have a value about \( \Delta \approx 1 - 10^2 \text{ eV} \) for \( \beta = \beta_{\text{max}} \).

The assumption, on the other hand, of a plane wave for the proton (deuteron) Bloch state \((\beta_{\text{n}} = 10^{10})\) may overestimate the matrix elements (8) by several orders of magnitude making the bandwidth down to \( \Delta \approx 10^{-2} - 1 \text{ eV} \) if we take \( \beta = 10^8 - 10^9 \).

On the other hand, if the bandwidth \( \Delta \) is less than the thermal energy \( \approx 25 \text{ meV} \) of ions at room temperature, the concept of the neutron band may be not realistic and the neutrons should be treated as localized at lattice nuclei. This requires \( \alpha^2 \beta^2 > 10^{21} \) for a realistic neutron valence band. This means that the parameters to realize the neutron valence band should be \( \alpha > 10^{5.5} \) for \( \beta = 10^5 \) and \( \beta > 10^7 \) for \( \alpha = 10^3 \).

(c) Accumulation of Neutrons at Surface/Boundary Regions

When there is a neutron valence band in a crystal, accumulation of neutrons at surface/boundary regions occurs by coherent reflection of neutron Bloch wave at these region as shown in Fig. 5 [4].
3. Complexity in the cold fusion phenomenon (CFP)

In this Chapter, we investigate a phase of the CFP as a complexity based on the experimental data sets.

3.1 Agents and Interaction

In many-particle systems including agents (constituents responsible for appropriate effects) interacting nonlinearly each other, there occur various events composing a phenomenon called complexity as a whole. The complexity includes such events as bifurcations of a stable state to several ones by variation of a parameter in the system, chaos, fractals, self-organization (automatic appearance of new organized state) or emergence, and so forth.

In the CFP, we have used complex systems composed of host metal atoms, occluded hydrogen isotopes, unavoidably included ambient neutrons, adsorbed/absorbed active elements in surface/boundary regions during experiment, and others. Based on our insight into physics of the CFP, we can choose agents from particles composing the CF system for explanation of the CFP.

When we notice such characteristics of the CF materials as open, non-equilibrium, unstable, many-particle systems interacting nonlinearly, we understand why the CFP has only qualitative reproducibility [4, 5]. Furthermore, to overcome the theoretical barrier against the CFP pointed out in Introduction, we have to take neutrons as a member of agents in our treatment. Then, our agents for the CFP are supposed to be transition-metal
nuclei with occluded hydrogen isotopes, neutrons included in CF materials (in the state discussed in §2), and minor element nuclei on/in surface/boundary regions of the samples.

3.2 Parameters – loading ratio (D/Pd, H/Ni), Temperature, Electric Field, Density Gradients

It is well known that characteristics of the complexity in a complex system depend on parameters of the system. A typical example of the feature is illustrated in Fig. 6 for the logistic difference equation (l.d.e.) with a parameter $\lambda$: $x_{n+1} = \lambda x_n(1 - x_n)$ ($0 < x_0 < 1$, $\lambda > 0$). Bifurcations and finally chaos appear in the diagram of $x_\infty$ vs. $\lambda$ [12].

Fig. 6  Period-doubling and Chaos (From “Chaos” by J. Gleick [12, p.71]). Four small figures inserted depict variation of $x_n$ in terms of the increase of $n$ for different values of $\lambda$. The main figure depicts $x_\infty$ ($x_n$ at $n = \infty$) vs. the parameter $\lambda$ (abscissa) of the logistic difference equation (l.d.e.) $x_{n+1} = \lambda x_n(1 - x_n)$ ($0 < x_0 < 1$).

In contrast to the simple structure of l.d.e., CF systems are tremendously complicated; we can list up such parameters responsible to the CFP on the basis of our experience as the loading ratio $\eta$ (and its gradient) of D (or H) to the host metal ($\eta = D/Pd$, H/Ni and so on), temperature $T$ (and its gradient) of the sample, electric field applied to the sample, and so on.

It is possible to have various events in the CFP more complex than those appeared in Fig. 6 in l.d.e. One example of chaotic behavior in the CFP as a function of the parameter $\eta = D/Pd$ was obtained by McKubre et al. [13] (c.f. also Fig. 6 of [8]).

Another example of complexity in the CFP was illustrated in Figs. 4 and 5 in [8] from the data obtained by Dash et al. In their data sets obtained in
precision experiments, we could read out bifurcation and the positive feedback behavior of the excess power generation as a function of temperature $T$. Furthermore, data of the neutron emission from TiDx by De Ninno et al. may be another example of the bifurcation in the CFP as explained in our paper [8].

Based on the conjecture about the positive feedback in the excess power generation, we can give unified explanation of explosions observed in the CFP hitherto. Examples in the following paragraphs may be explained by the positive feedback mechanism observed in the experiment as analyzed in [8] but without appropriate negative feedback to stop the explosion.

There are several reports of explosions occurred in CFP experiments. The two experiences by Zhang et al. [14] and Biberian [15] were reported in conferences and we have concrete data. In these cases, they used electrolytic system with similar cathodes of Pd pipes to the one used in [8]. Another example of the explosions in CFP experiments was reported by T. Mizuno [16]. In this case, the experimental system was also electrolytic but a tungsten plate as a cathode with light water and electrolyte K$_2$CO$_3$.

There are further more two cases of explosions which are legendary. The first of them was written in the pioneering paper by Fleischmann, et al. [1]. In the Table 1 of their paper, there are words “Warning! Ignition?” for a cubic sample of $1 \times 1 \times 1$ cm$^3$. Details of this explosion was not made clear due to scant data reserved after the accident in the confused situation induced by the extraordinary event.

The second of them occurred in the SRI (Stanford Research Institute) [17] resulting in fatal human damage. This case was ascribed to an explosion of mixture gas of hydrogen and oxygen. If it is the case, we can exclude this case from the explosion of CF samples due to uncontrolled positive feedback of energy production by cold fusion reactions.

4. Conclusion

We have found three laws or rules in the CFP: 1) The stability effect in production of new nuclides: the more the nuclides stable, the more often they are generated in CF materials [4]. 2) The inverse-power law for excess power productions: the number of observation $N(P)$ of an excess power $P$ in a CF material is expressed as an inverse-power of $P$ with an exponent $-1$ to $-2$, i.e. $N(P) = C/P^b$ ($C$: constant, $b = -1 \sim -2$) [4, 5]. 3) The bifurcation of excess
power generation $P$ as a function of a sample temperature $T$: the magnitude of excess power $P$ generated in PdD$_x$ ($x = 0.8 \sim 1.0$) samples becomes finite above a critical temperature $T_0$ increasing with $T$ and experiences bifurcation to two branches of higher and lower excess power. In the higher $P$ branch, there appears a positive feedback mechanism to increase $P$ radically.

These laws are essential to many-particle systems with own characteristics. The first law (the stability effect) is discovered by comparison of the number of observation $N_{ob}(Z)$ of a nucleus with a proton number $Z$ and the relative abundance $H(Z)$ of the nucleus in the universe. $H(Z)$ is a result of many-particle interactions in stars and this law suggests existence of similar interactions in CF materials at surface/boundary regions where nuclear transmutation occurs.

The second law (the inverse-power law) is frequently observed in complex systems in non-equilibrium condition. One of the well-known examples of this law is the Gutemberg-Richter’s formula [4 (§ 2.12), 5] describing a relation between the energy $E$ of an earthquake and its frequency $P(E)$:

$$P(E) = AE^{-b}, \ (A: \text{constant}).$$

It is told that the exponent $b$ has been supposed to be equal to 2 at first (when it was proposed in 1942) and converged to 1.7 at present [8].

The third law (the bifurcation of the excess power production as a function of the sample temperature $T$) is a typical manifestation of complexity. The most simple example of this behavior is demonstrated by the logistic difference equation (l.d.e.): $x_{n+1} = \lambda x_n(1 - x_n) \ (\lambda: \text{positive parameter, } 0 < x_0 < 1)$ [12]. If we plot $x_n$ as a function of the parameter $\lambda$, the graph shows bifurcations and chaos. We have shown the bifurcation of the excess power in the CFP as a function of the sample temperature [8]. The chaotic behavior depicted in Fig. 6 suggests that the excess power production observed by McKubre et al. [13] can be interpreted as a chaos as explained in [8].

In addition to the bifurcation, we have noticed the occurrence of the positive feedback to increase excess power in PdD$_x$ samples [8]. Fortunately, the positive feedback in the data analyzed by us [8] did not last too long and terminated at the maximum power less than about 20 W by some negative feedback mechanisms.

However, there were several reports of explosion in PdD$_x$ [1, 14, 15] and WH$_x$ [16] systems that may be results of the positive feedback in the production of excess energy not restrained by effective negative feedback.
mechanisms.

Thus, the CFP characterized by these laws in addition to such various events summarized in Table 2.1 of Reference [4] has to be treated as a complexity occurring in open, non-equilibrium, inhomogeneous, multi-component systems with nonlinear interactions between agents.

In this paper, we have given first few steps to understand fundamental properties of agents in CF materials participating in the complexity when there are optimum arrangements of host atoms and occluded hydrogen isotopes in the surface/boundary regions of T-MH/D under influence of thermal neutrons: 1) neutron wavefunctions of host lattice nuclei (Ni, Ti, Pd, - - - ) might be responsible for the CFP, especially when these nuclei become exotic with a large excess number of neutrons; 2) wavefunctions of occluded protons/deuterons at interstices of fcc/hcp transition-metal hydrides/deuterides (T-MH/D) should be responsible for the CFP in these materials. Correlation between the CFP and known physical properties of T-MH/D is explained consistently by the nature of these wavefunctions; 3) the super-nuclear interactions between neutrons in adjacent lattice nuclei mediated by proton/deuteron wavefunctions in 2) explains qualitatively fundamental features of the CFP summarized in References [3 – 5] and especially in Chapter 2 of Reference [4].

We may expect that the characteristics of the CFP will be understood as a complexity including atomic and nuclear interactions in compound systems composed of exotic lattice nuclei of transition metals (Ni, Ti, Pd, - - - ) occluding hydrogen isotopes (H and/or D) with high loading ratio ( \( \eta > 0.8 \)) under irradiation of ambient or artificial thermal and epithermal neutrons.

If we establish such physics of the CFP on the line outlined in this paper, we can then develop possible application of the CFP as energy sources, generation of new or rare elements, elimination of hazardous radioactive waste, and so forth.

This work is supported by a grant from the New York Community Trust.

References


