 Physics 203 T10 June 3 lecture

Quantum mechanics on 1 page...

This precise, elegant and very strange mathematical theory has one thing really going for it: it agrees with experiments!

It totally predicts things we take for granted:

lasers → CD, DVD players, lasic surgery ... etc
superconductivity → MRI Magnets
superfluidity →
Photosynthesis (sunlight + CO_2 + H_2O → sugar)
Solar panels (sunlight → voltage + current)
Transistors → iPhones, all gadgets that need
Radioactivity & nuclear weapons
→ How the Sun "works" (Hans Bethe)
Creation of the Universe from 1 point.

AND... Why all of our electrons do not fall onto our nuclei and, therefore, we can exist and not shrink into 1 point.

The idea:

Any object is a wave which goes everywhere:

...But the observed trajectory is where all neighboring waves add in phase and interfere with each other constructively

(Just like light obeys Fermat's principle of shortest time, "matter" waves obey the principle of "Least action" $S = \sum \vec{p} \cdot \Delta\vec{x}$)

S is proportional to the phase $2\pi \frac{x}{\lambda}$ ⟶ $\boxed{\lambda = \frac{h}{p}}$

Consequences:

Bound trajectories (e.g. electron's orbits in an atom) must be standing waves → only discrete energy levels are possible ("modes")

Superposition of two different standing waves involves emission or absorption of light at a frequency $f = \frac{E_2 - E_1}{h}$

Same constant !!!

There fore, light can only be emitted and/or absorbed in discrete "chunks" of energy fh – these chunks are called "photons" and resemble particles, especially at high frequencies.

OK, all the waves we have studied so far are oscillations of some distributed quantity (air pressure for sound, string displacement for tight strings, $\vec{E}$ & $\vec{M}$ fields for light).

For "electron" wave, or "proton" wave, or "basketball" wave, what is the quantity that actually oscillates?

$$\boxed{\psi \sim \sqrt{P} \rightarrow \text{probability of "catching" the object}}$$

We can only calculate probabilities, not the actual events !!!

(3) Much more specific consequences, examples.

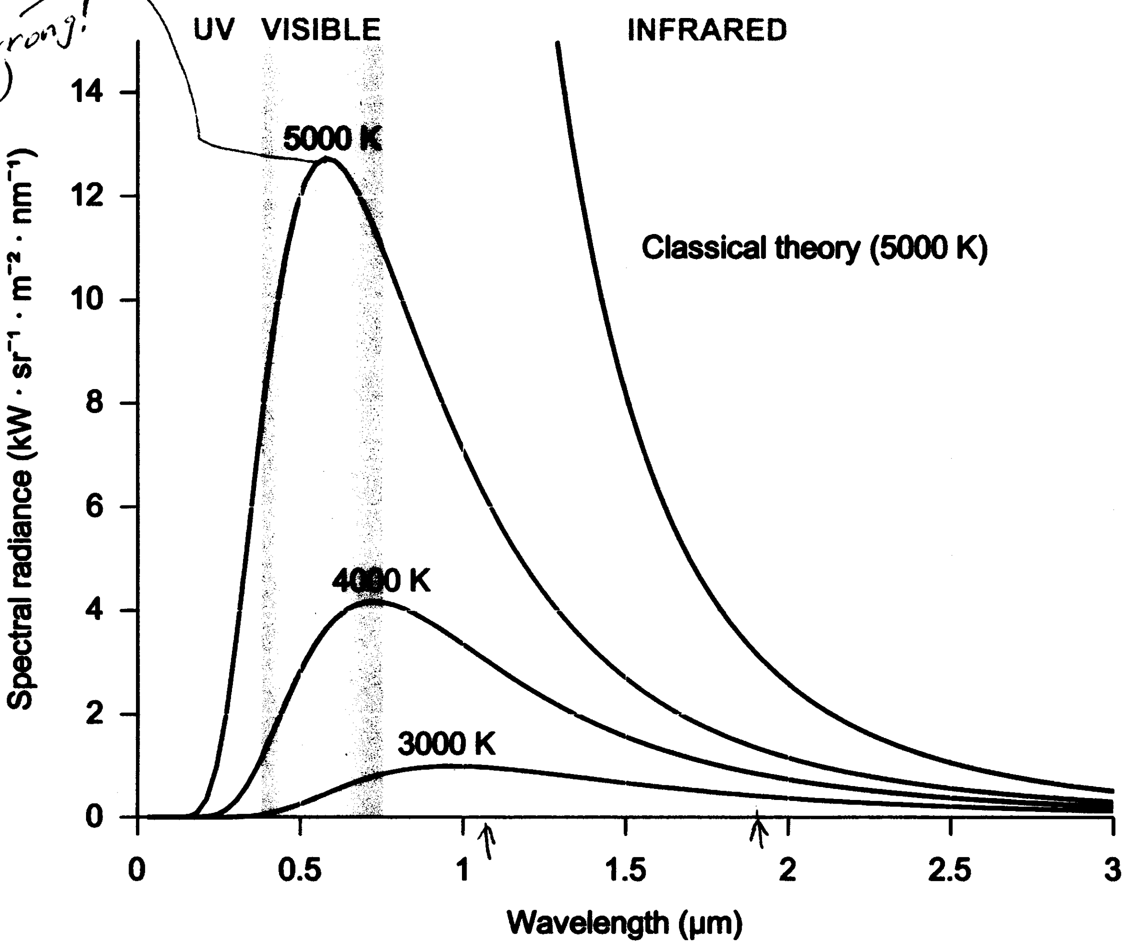
Black Body spectrum (e.g. sun surface)

Definition: black body: any light that hits it equilibrates with the body temperature

any light that comes out was in thermal equilibrium with the body

$\boxed{\text{Wien's Displacement Law:}}$ $f_{peak} = (5.88 \cdot 10^{10} s^{-1} K^{-1}) T$ ← temperature

this is wrong! (see below)



There are two new types of LED lights in Home Depot:

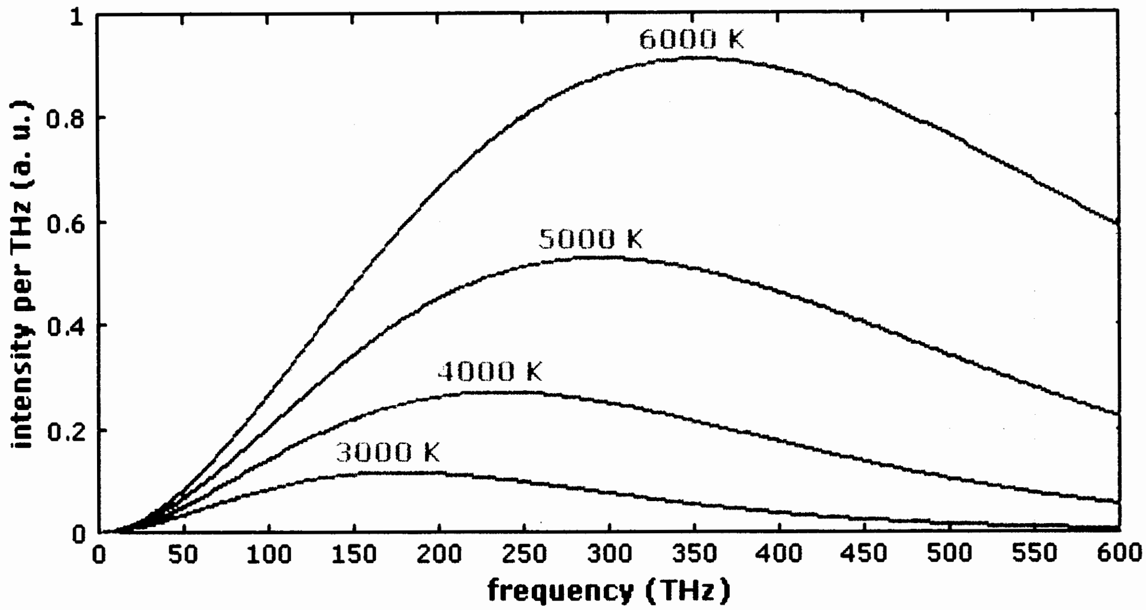
a "2700 K" warm white & an "outdoor" 4700 K light

"cool white" ???

Find peak wavelength of the corresponding blackbody spectrum

$$\lambda_{peak} = \frac{c}{f_{peak}} = \frac{c}{5.88 \cdot 10^{10} \cdot \begin{cases} 2700\,K \\ 4700\,K \end{cases}} = \begin{cases} 1.89\ \mu m = 1890\ nm \\ 1.09\ \mu m = 1090\ nm \end{cases}$$ — hm...

④



The problem is $\lambda_{peak} \neq \dfrac{c}{f_{peak}}$...

Because the $\dfrac{\Delta I}{\Delta \lambda}(\lambda)$ curve is showing a different quantity than the $\dfrac{\Delta I}{\Delta f}(f)$

Intensity per nm peaks at a different point than the intensity per Hz.

$\boxed{\text{When using Wien's law, stick to } \dfrac{\Delta I}{\Delta f}(f)}$

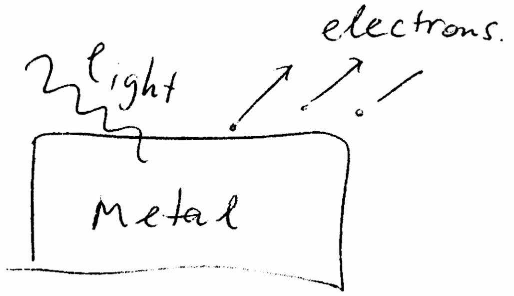
So, the question should ask, find the peak frequencies

$$f_{peak} = 5.88 \cdot 10^{10} \begin{Bmatrix} 2700\,K \\ 4700\,K \end{Bmatrix} = \begin{Bmatrix} 159\ THz \\ 276\ THz \end{Bmatrix}$$

<u>All</u> the experimentallists plot spectrum vs. λ.

WHAT WAS HE THINKING???

(5) Photoelectric effect: First eye-opener on quantum!



Classical:

$I_{light} \uparrow \rightarrow$ # of e's $\uparrow$, $E_{kin} \uparrow$

$f_{light} \uparrow \rightarrow$ # of e's $\downarrow$ $E_{kin} \downarrow$

Quatum: at $f < f_{cutoff}$, # = 0
no matter what Intensity
at $f \geq f_{cutoff}$,

$\# \sim I$, $\left(\# = \frac{U}{hf}\right)$

$E_{kin\,max} \sim f - f_{cutoff}$

$\left(E_{kin} = h(f - f_{cutoff})\right)$

It takes some energy to rip electron out of the metal
($-U_{pot} = W_0$ ("work function"))
Each chunk of light (photon) "kicks" just 1 electron
Some of the hf energy goes to pay "the workfunction"
The rest becomes kinetic energy.

Example: Potassium work function $W_0 = 2.29$ eV $= 3.67 \cdot 10^{-19}$ J
What happens when blue (450 nm) an red (750 nm) light strikes potassium?

1 Convert W_0 to wavelength.

$W_0 = hf_0 \rightarrow f_0 = \frac{W_0}{h} = \frac{3.67 \cdot 10^{-19}\,J}{6.62 \cdot 10^{-34}\,J \cdot s} = 5.537 \cdot 10^{14}$ Hz

$\lambda_0 = \frac{c}{f_0} = \frac{3 \cdot 10^8}{5.537 \cdot 10^{14}} = 541$ nm

red light has lower photon energy ($4 \cdot 10^{14}$ Hz, 1.7 eV) – no electrons come out!

(6) Blue light has enough energy ($6.662 \cdot 10^{14}$ Hz; 2.8 eV) to kick out some electrons.

After paying off the workfunction "debt", still have $2.8 - 2.29 = 0.51$ eV left

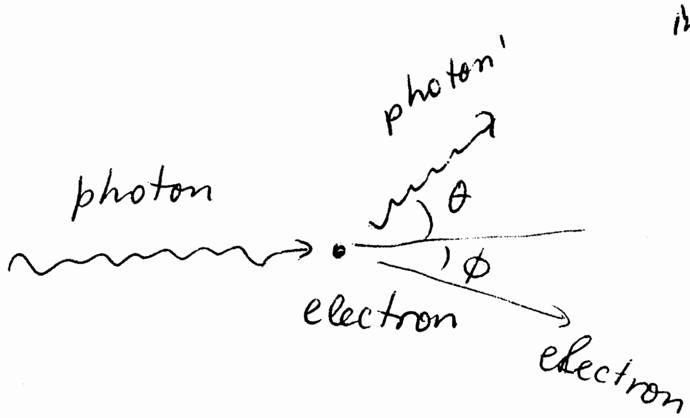
Max energy of electrons $\sim 8 \cdot 10^{-20}$ J

Photon momentum & Compton scattering.

E = Photon Energy $= hf$

Photon momentum $= \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$

(By de way, de Broglie wavelength $\lambda = \frac{h}{p}$ implies same is true for matter as well: $p = \frac{h}{\lambda}$!!!)



Energy and momentum are conserved!

$E_{ini} = E_{fin}$

$$\begin{cases} \frac{h}{\lambda} = \frac{h}{\lambda'} \cos\theta + p_e \cos\phi \\ 0 = \frac{h}{\lambda'} \sin\theta + p_e \sin\phi \end{cases}$$

$hf = hf' + K$

$K \to \frac{mv^2}{2}$ or $m_0 c^2 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)$

slow — fast

Compton shift:

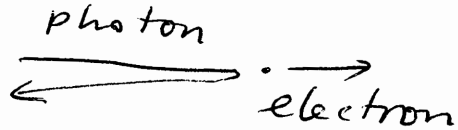
$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

True for both !!!

Example:

Find the maximum change in λ for blue $\lambda = 450$ nm light and X-rays $\lambda = 0.1$ nm (10^{-10} m)

Solution: Max change is the same, when $\theta = 180°$ (π)



$$\Delta\lambda = \frac{h}{m_e c}\left(1-(-1)\right) = \frac{2h}{m_e c} = 4.86 \cdot 10^{-12}\ \text{m}$$

only 10^{-5} (0.001%) change for blue ligh, ~5% change for this X-ray.

Heisenberg uncertainty principle

$$\Delta p_x \Delta x \geq \frac{h}{2\pi}$$

$$\Delta p_y \Delta y \geq \frac{h}{2\pi}$$

$$\Delta E \Delta t \geq \frac{h}{2\pi}$$ ← similar to beats!!!

$$\frac{2\pi\Delta E}{h} = 2\pi\Delta f = \Delta\omega, \quad \boxed{\Delta\omega\Delta t \geq 1}$$

(8) Example:

in a classical picture,

an electron is orbiting the proton in a hydrogen atom with velocity $v = \sqrt{\dfrac{k_c q_e^2}{m_e r}} = 2.2 \cdot 10^6 \frac{m}{s}$ on a radius $r = 5.3 \cdot 10^{-11} m$

Find the uncertainty of energy and momentum.

$$\text{Energy} = \frac{1}{2} m_e v^2 = 2.176 \cdot 10^{-18} J$$

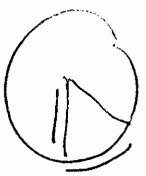
$$\text{Momentum} = m_e v = 2 \cdot 10^{-24} \, kg \frac{m}{s}$$

$$\Delta x \sim r = 5.3 \cdot 10^{-11} m$$

$$\Delta t \sim \frac{1}{2\pi} T = \frac{r}{v} = 2.4 \cdot 10^{-17} s$$

$$\Delta E = \frac{h}{2\pi \Delta t} = 4.3 \cdot 10^{-18} J$$

$$\Delta p = 2 \cdot 10^{-24} \, kg \frac{m}{s}$$



$\sim \frac{1}{2\pi} T$

Uncertainties are as big as the quantities, classical orbit is not calculable.