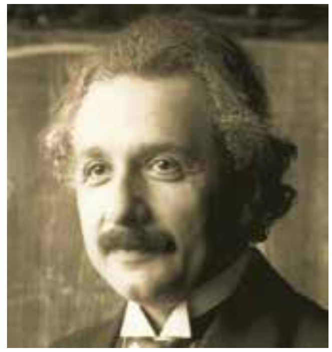
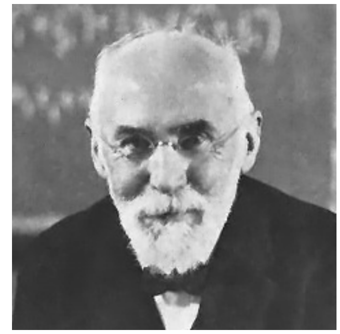
Physics 203 T10, May 27th lecture

Henri Poincaré

Relativity





Hendrik Lorentz

A story of space, time, and the speed of light

Space & Time – what are the differences?

1. Space 3D, but time is 1D
2. In space, can move back and forth.
   In time, unfortunately, only forth.
3. Units: space, measured in meters
   time, measured in seconds.

The last problem can be fixed by a "conversion factor" in the units of $\frac{m}{s}$ → speed.

So, some speed (which is fairly constant and independent of conditions)

can be used to convert space ⟷ time.

Example: he lives 2 hours away.

Can mean:
- 2-hour walk away
- 2-hour horse-ride away
- 2-hour car ride away
- 2-hour flight away.

really depends on speed ←

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The essence of travel: (hikers & bikers
"to convert time into distance"

Example: why not using the fastest known speed, with the added benefit of it being fairly constant and independent of conditions ...

From Orlov's novel: – "Demons can move with an incredible speed, faster than anything..."
– "Speed of light?"
– "No, faster. With the speed of thought".

1 light-year $\approx 9.5 \cdot 10^{15}$ m, ~¼ distance to the closest star system α-Centauri

1 light minute $\approx 1.8 \cdot 10^{10}$ m ~1/8 dist. to the Sun

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1 light second $= 3 \cdot 10^{8}$ m 75% dist to the Moon

1 light ms $= 3 \cdot 10^{5}$ m $=$ 186 miles - just beyond Seattle

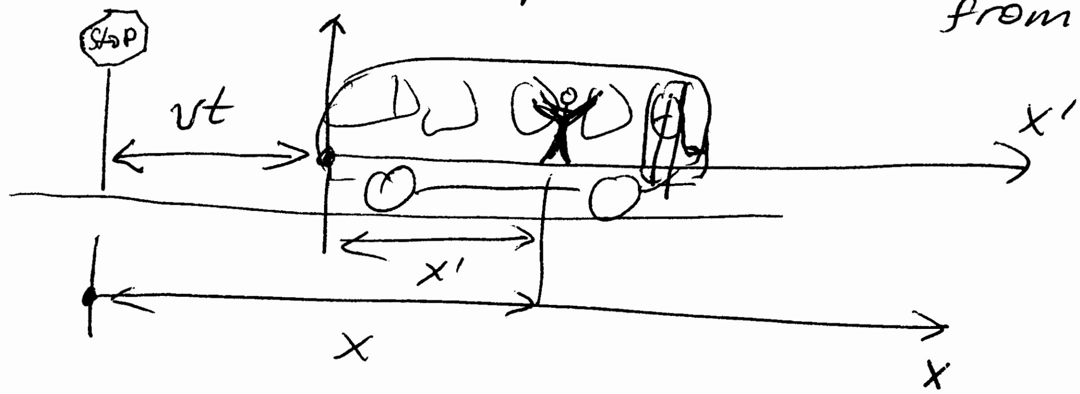
1 light μs $= 3 \cdot 10^{2}$ m $=$ to Phys departm. and back

1 light ns $= 0.3$ m ~ 1 ft.

$$c = 299792458 \ \frac{m}{s} \text{ exactly!}$$

③

What is happening when we measure the world from a bus or train?
(compared to the measurements from the ground.)

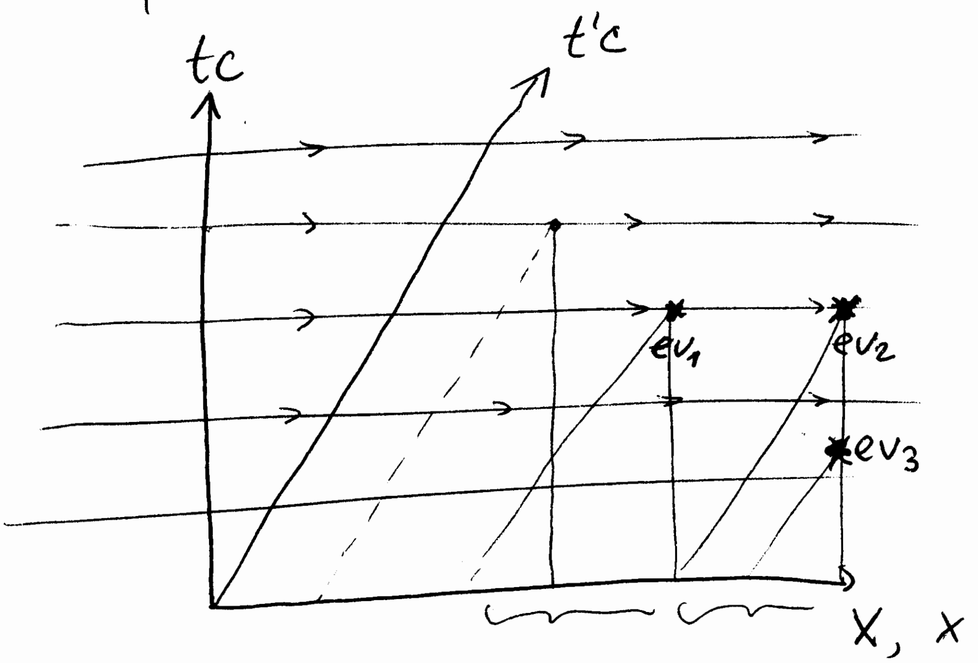


Ground system $\begin{cases} \text{space} & x \\ \text{time} & tc \end{cases}$ (light meters)

Bus system $\begin{cases} \text{space} & x' \\ \text{time} & t'c \end{cases}$

$$\begin{cases} x' = x - \left(\frac{v}{c}\right)tc \\ ct' = ct \end{cases}$$

(distances are the same @ $t=0$, and if $x' = \text{const}$, x keeps increasing)

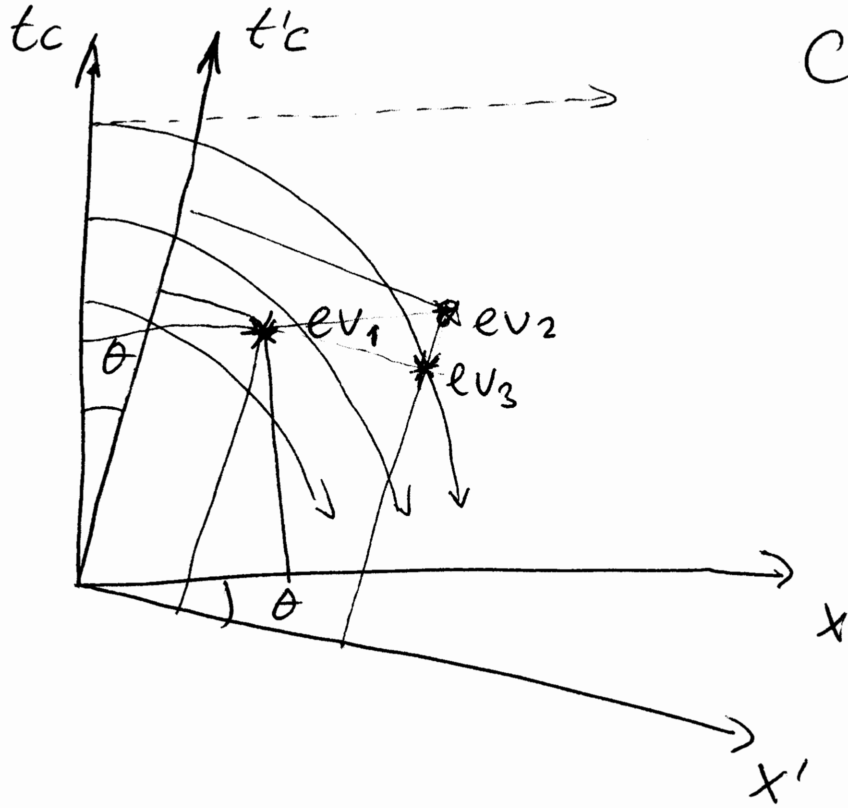


"Shear transformation"

Distances between synchronous events are the same in both systems, but the distances between past/future events are not.

④

Poincaré's & Lorentz's idea:
instead of "shear", why not try the rotation?



Crazy or what?

idea:

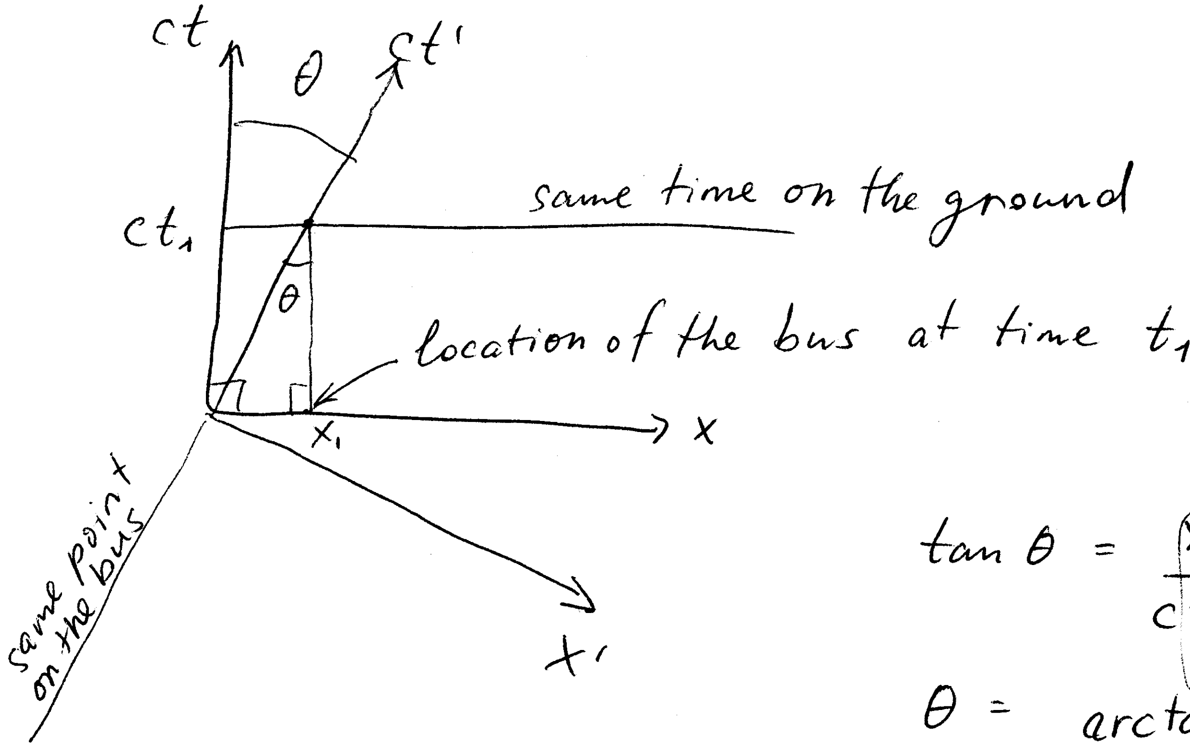
$$\begin{cases} x' = x\cos\theta - tc\sin\theta \\ ct' = \underline{x}\cdot\sin\theta + tc\cos\theta \end{cases}$$

Crazy Consequences: events that are at the same time on the ground are <u>**not**</u> at the same time on the bus!!!

The bus will appear shorter/longer to people on the ground vs. on the bus

And, they will not even be able to agree on who is right!!!

⑤ Let's find θ:



$$\tan\theta = \frac{x_1}{ct_1} = \frac{v}{c}$$

$$\theta = \arctan\frac{v}{c}$$

Plug this value of θ into our equations:

$$\begin{cases} x' = x\cos\arctan\frac{v}{c} - tc\sin\arctan\frac{v}{c} \\ ct' = x\sin\arctan\frac{v}{c} + ct\cos\arctan\frac{v}{c} \end{cases}$$

Math:

$$\cos\arctan x = \frac{1}{\sqrt{1+x^2}}$$

$$\sin\arctan x = \frac{x}{\sqrt{1+x^2}}$$

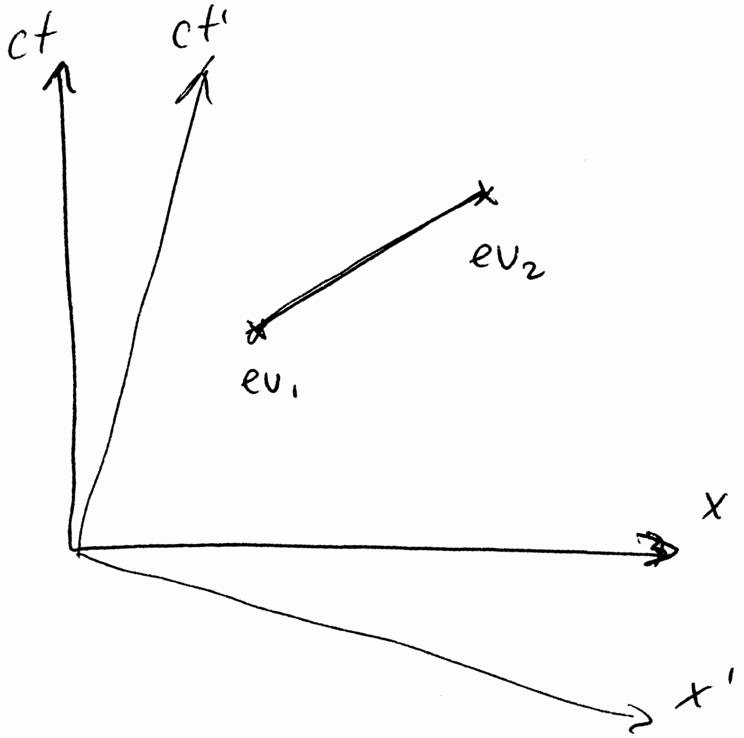
$$\begin{cases} x' = \dfrac{x}{\sqrt{1+\left(\frac{v}{c}\right)^2}} - \dfrac{vt}{\sqrt{1+\frac{v^2}{c^2}}} \\ ct' = \dfrac{\frac{v}{c}x}{\sqrt{1+\frac{v^2}{c^2}}} + \dfrac{ct}{\sqrt{1+\frac{v^2}{c^2}}} \end{cases}$$

in the limit $v \ll c$ of bus being slow compared to light,

$$\begin{cases} x' = x - vt \\ ct' = ct \end{cases} \quad !!!$$

(6)

The invariant



the "distance" between any two events in the "space-time", given by

$$\sqrt{(x_1-x_2)^2+(ct_1-ct_2)^2} =$$ is the same:

$$= \sqrt{(x_1'-x_2')^2+(ct_1'-ct_2')^2}$$

For the slow bus, this distance is approximately $\sim \sqrt{(ct_1'-ct_2')^2} = c|\Delta t'| \approx c|\Delta t|$

$$c|\Delta t| = \sqrt{(ct_1-ct_2)^2}$$

or simply the time difference.

(7) Unfortunately, this "theory" was not confirmed by the measurements. Poor Poincaré and Lorentz went back to their equations, and tweaked them...

One of them said, "What if time is not real, we are just imagining it?"

The other answered, mumbling:

"You mean, time should be ict, not ct?"

$i = \sqrt{-1}$

$\cos(i\theta) = \cosh\theta$

$\sin(i\theta) = i\sinh\theta$

"But then we should rotate by an imaginary angle, too!!!"

"Ta-daaa!!!"

Everyone thought these guys were crazy, but Einstein said, "wait a minute"...

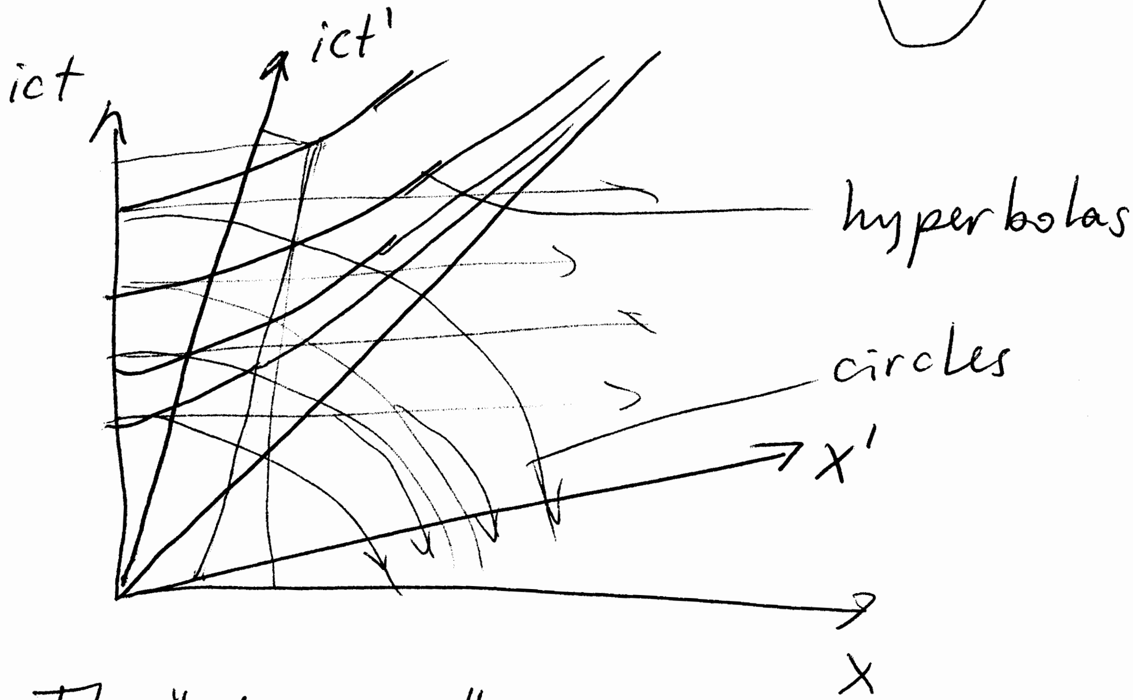
So, here we go:

$$\begin{cases} x' = x\cos i\theta - tc\sin i\theta \\ ct' = x\sin i\theta + ct\cos i\theta \end{cases} \rightarrow$$

$$\begin{cases} x' = x\cosh\theta - itc\sinh\theta \\ ct' = ix\sinh\theta + ct\cosh\theta \end{cases}$$

$\times i$ $\times i$ $\times i$

$$\begin{cases} x' = x\cosh\theta - itc\sinh\theta \\ ict' = -x\sinh\theta + ict\cosh\theta \end{cases}$$



The "distance" between events

$$\sqrt{(i\Delta tc)^2 + (\Delta x)^2} = \sqrt{-(tc)^2 + x^2}$$

or $\sqrt{(tc)^2 - x^2}$

9 Same way, can express θ as velocity

$$\tanh\theta = \frac{v}{c}$$

$$\cos \operatorname{arctanh}\frac{v}{c} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\sin \operatorname{arctanh}\frac{v}{c} = \frac{v/c}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\begin{cases} x' = \dfrac{x}{\sqrt{1-\frac{v^2}{c^2}}} - \dfrac{vt}{\sqrt{1-\frac{v^2}{c^2}}} \\[2ex] ct' = \dfrac{-\frac{v}{c}x}{\sqrt{1-\frac{v^2}{c^2}}} + \dfrac{ct}{\sqrt{1-\frac{v^2}{c^2}}} \end{cases}$$

Done !!!

Example: event 1 – light emitted

event 2 – light absorbed

"world space-time distance"

$$\mathcal{D} = \sqrt{(c\Delta t)^2 - (\Delta x)^2} = \sqrt{(c\Delta t)^2 - (c\Delta t)^2} = 0,$$

no matter from what platform we observe the light. Speed of light is the same!!!

Example 2:

Event 1: a clock departed Station A at time 0, with velocity v

Event 2: same clock arrived at Station B at earth time t, clock's own time t'.

$$\mathcal{D} = \sqrt{(c\Delta t)^2 - (\Delta x)^2} =$$

in Earth frame, $\Delta x = v\Delta t$

$$= \sqrt{(c\Delta t')^2 - (\Delta x')^2}$$

in clock's own frame, $\Delta x' = 0$

$$c|\Delta t'| = \sqrt{(c\Delta t)^2 - (v\Delta t)^2} =$$

$$= \Delta t \sqrt{c^2 - v^2}$$

$$\Delta t' = \Delta t \sqrt{1 - \frac{v^2}{c^2}}$$

Clock's time is less than the Earth time: "Time dilation".

More concrete example:

Twin paradox:

one twin goes to α centaury (4.3 ly away) and back with $0.99c$ speed.

How old is he when he gets back?

Earth frame: $\Delta t = 2\frac{d}{v} = 2 \cdot \frac{4.3\,y}{0.99} =$

$= 8.7$ years later

Ship frame; going there and back

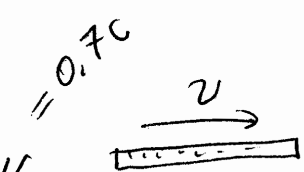
$$\Delta x = 0$$

$$\Delta t' = \Delta t \sqrt{1 - 0.99^2} = 1.22 \text{ year}$$

The astronaut twin will be $7\frac{1}{2}$ years younger!!!

<u>Example 3</u>

<u>Length contraction</u>



A meterstick is moving at v, How long does it appear on Earth.

Event 1: left end at Earth time 0

Event 2: right end at Earth time 0

(12)

$$\mathcal{D} = \sqrt{(\Delta x)^2 - (c\underset{\substack{\uparrow \\ 0}}{\Delta t})^2} = |\Delta x|$$

$$\mathcal{D} = \sqrt{(\underset{\substack{\uparrow \\ 1m}}{\Delta x'})^2 - (c\Delta t')^2}$$

$$\mathcal{D} = \mathcal{D}$$

$$(\Delta x')^2 - c^2(\Delta t')^2 = (\Delta x)^2$$

Let event 1 be (0, 0) in both frames

$$c^2(\Delta t')^2 = (c t_2')^2 = \left(\frac{-\frac{v}{c} x_2}{\sqrt{1-\frac{v^2}{c^2}}}\right)^2 = \left(\frac{-\frac{v}{c}\Delta x}{\sqrt{1-\frac{v^2}{c^2}}}\right)^2$$

$$(\Delta x')^2 - \frac{\frac{v^2}{c^2}(\Delta x)^2}{1-\frac{v^2}{c^2}} = \Delta x^2 \frac{1-\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}}$$

$$(\Delta x')^2 = \Delta x^2 \frac{\cancel{\frac{v^2}{c^2}} + 1 - \cancel{\frac{v^2}{c^2}}}{1-\frac{v^2}{c^2}}$$

$$\Delta x = \Delta x' \left(\sqrt{1-\frac{v^2}{c^2}}\right) = 1m\sqrt{1-0.7^2} = 0.71m$$