Physics 203 T10 Lecture May 22

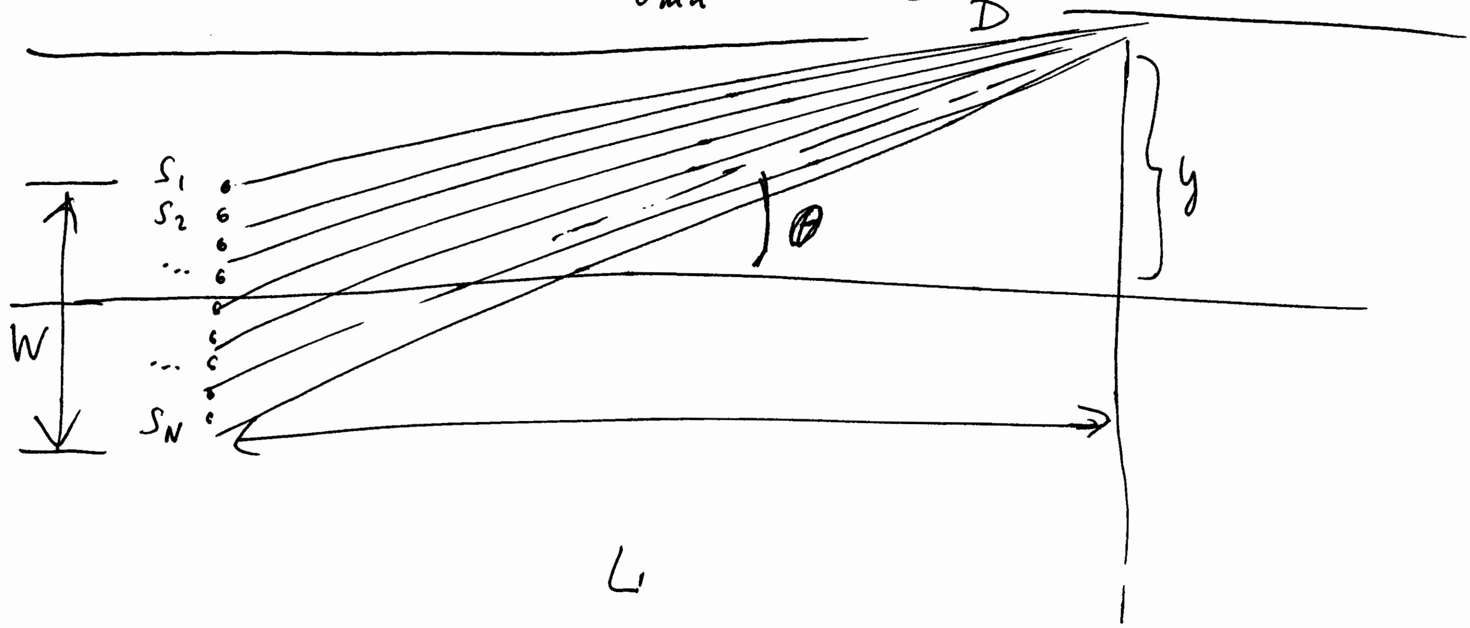
Diffraction:

Interference from a continuous distribution of sources.

Diffraction gratings

Rayleigh's Criterion of Optical Resolution

$$\theta_{min} = 1.22 \frac{\lambda}{D}$$



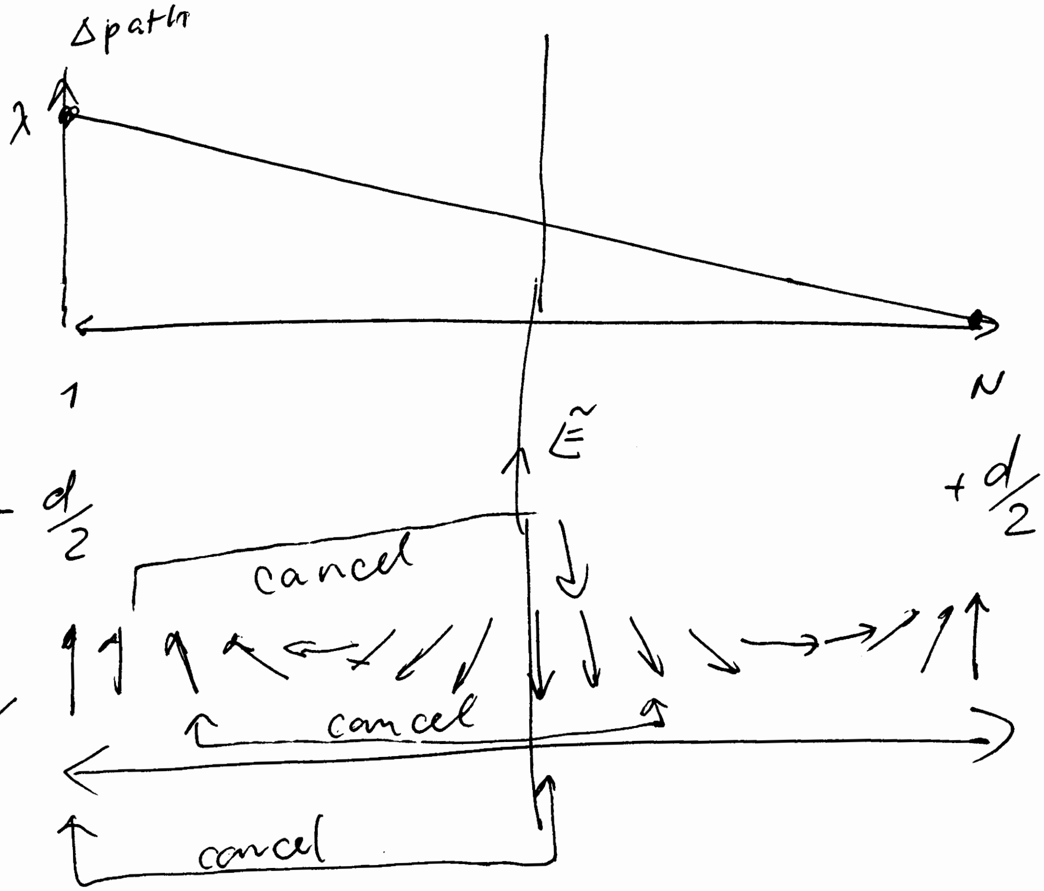
As before, let's consider multiple sources.

If only S_1 & S_N were present,
we would see the central bright spot,
and additional bright spots at $d \sin\theta = m\lambda$

$m = \pm 1, \pm 2$, etc

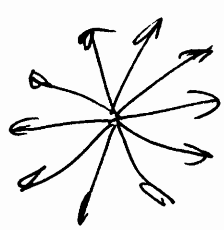
Lets plot the path length and the ~~vect~~ phasor $\tilde{E}$ as a function of source number n, (= position within the "slit")

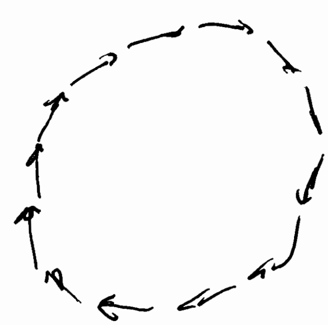
For $m=1$



→ rearrange all these $\tilde{E}$ phasors into a "chain":

or, into a daisy:



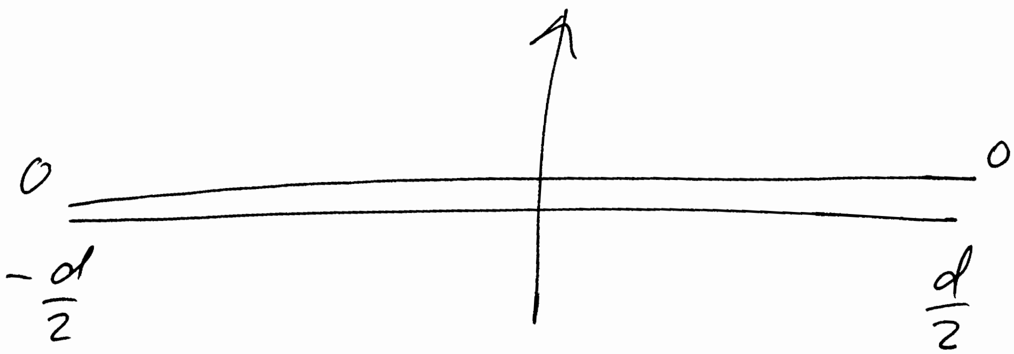


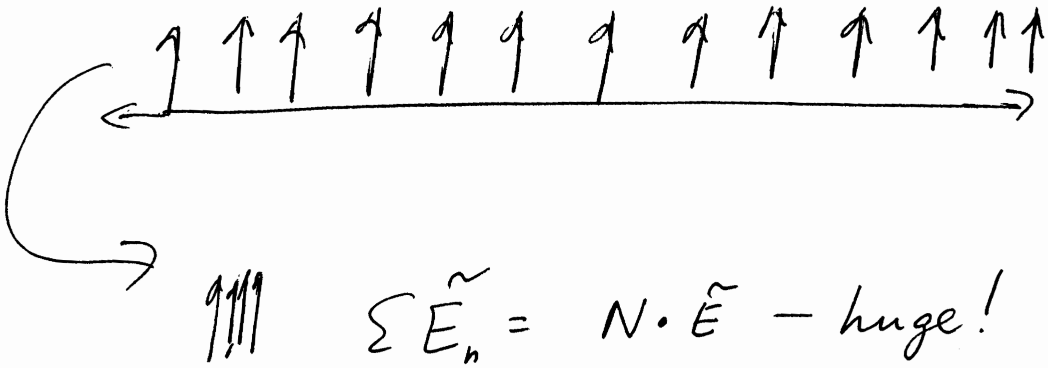
$\Sigma \tilde{E}_n = 0$!!!

$\Sigma \tilde{E} = 0$!!!

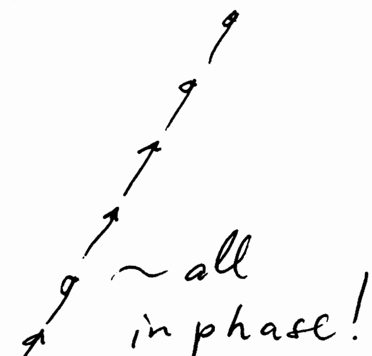
We see a dark spot where light used to be !!!

③ What happens for $m = 0$ ($y = 0$?)

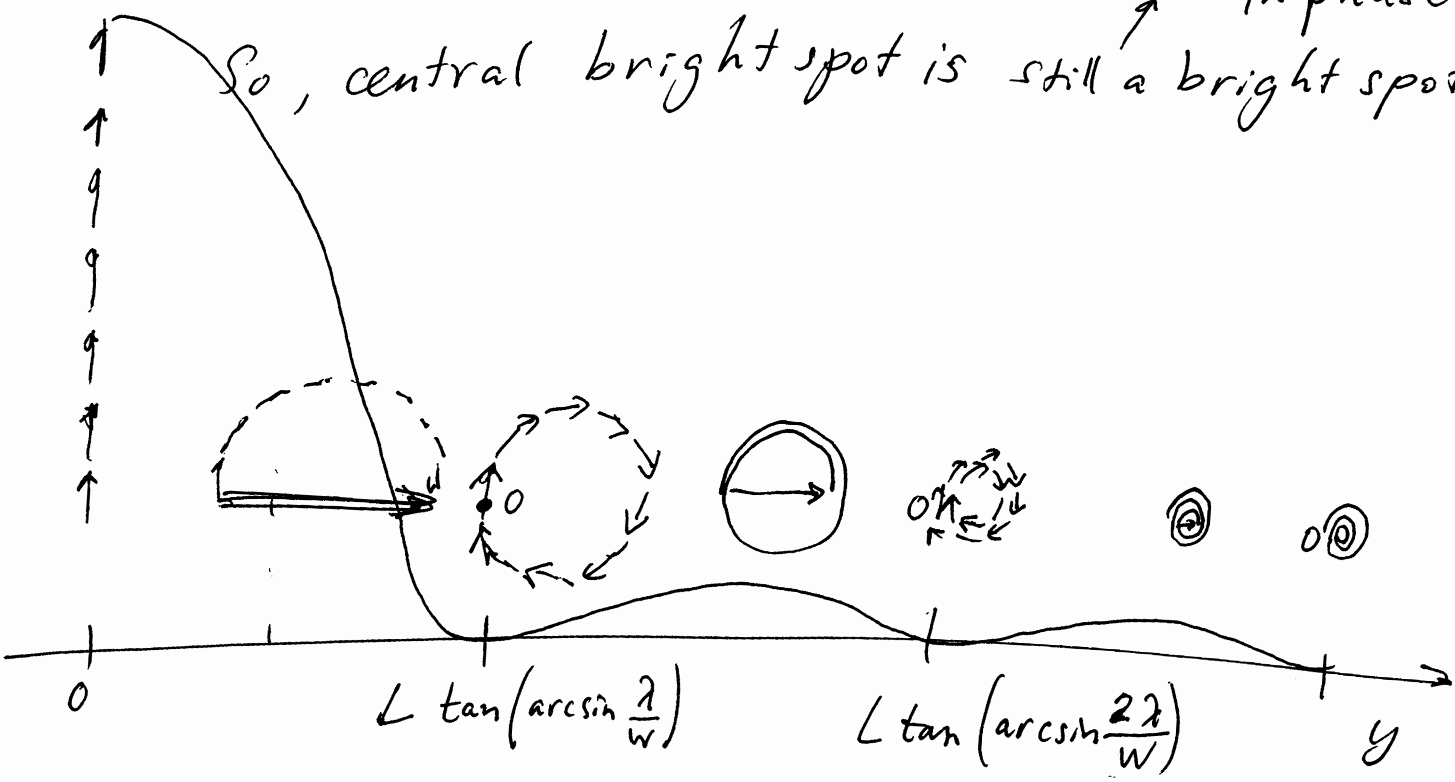




$$\sum \tilde{E}_n = N \cdot \tilde{E} \text{ — huge!}$$



So, central bright spot is still a bright spot!



Summary:

Dark fringes at $W \sin\theta = m\lambda$

$m = \pm 1, \pm 2, \pm 3,$ etc.

But, a bright fringe at $m = 0$.

Example:

Use a caliper with $\frac{1}{1000}$ inch gap,

$L = 15$ m. $W = \frac{0.0254 \text{ m}}{1000} = 2.54 \cdot 10^{-5}$ m

$\lambda = 450$ nm (blue light)

First dark fringe:

$W \sin\theta = \lambda$ (↑ $m = 1$) $\quad \sin\theta = \frac{\lambda}{W} = \frac{4.5 \cdot 10^{-7}}{2.54 \cdot 10^{-5}}$

$= 0.0177 = 1°$

$y = L \tan\theta = 15\text{ m} \cdot \tan 1° = 0.27$ m

Note, that as $W \downarrow$, $y \uparrow$, and vice versa.

Math note:
$\tan(\arcsin x) = \frac{x}{\sqrt{1-x^2}} \approx x$

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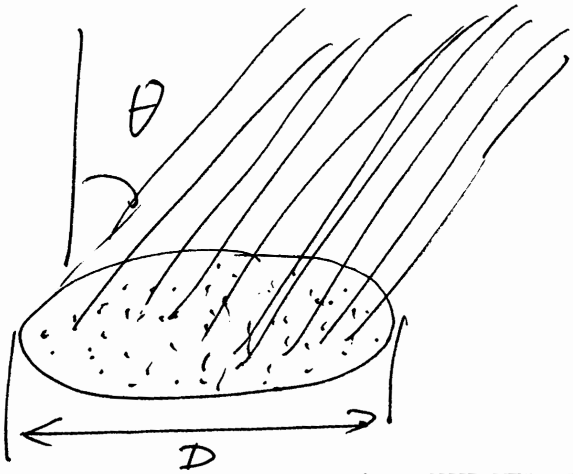
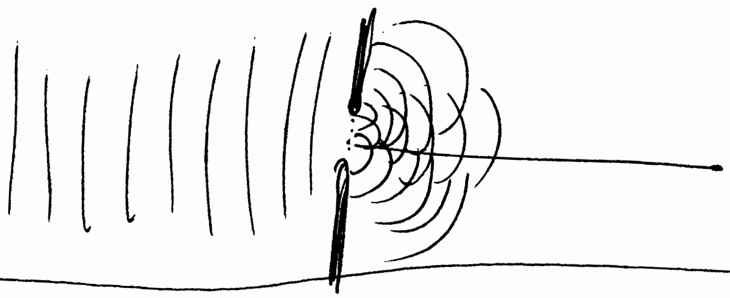
Now consider the sources occupying a circle:



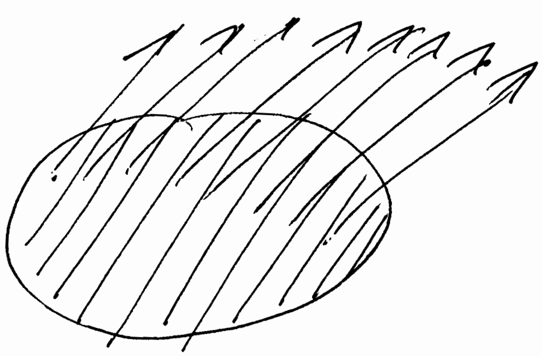
want to find out intensity at some angle θ.

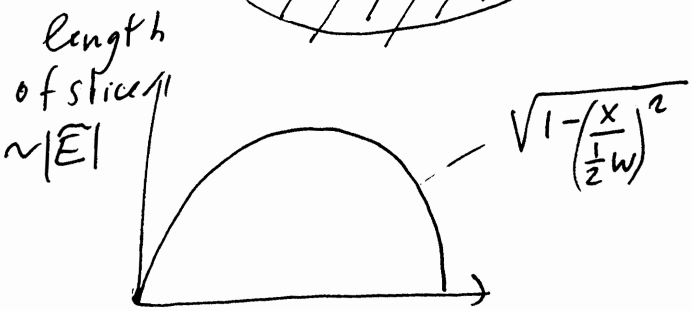
(Huygens-Fresnel principle:

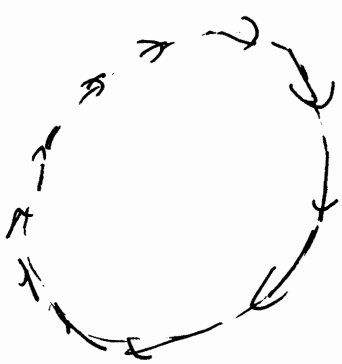


each point becomes a secondary point source!

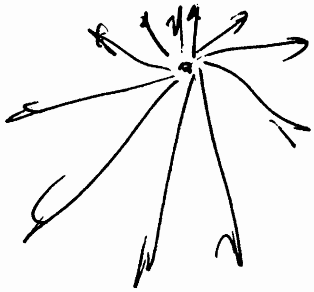
Slice the circle:





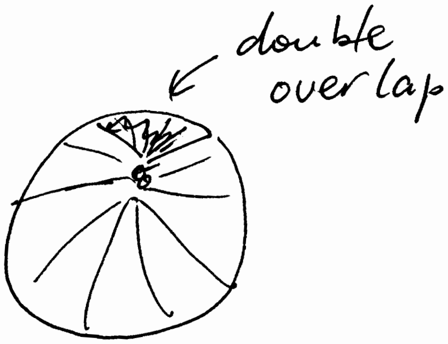


or, if we bring all the phasors to the same origin,

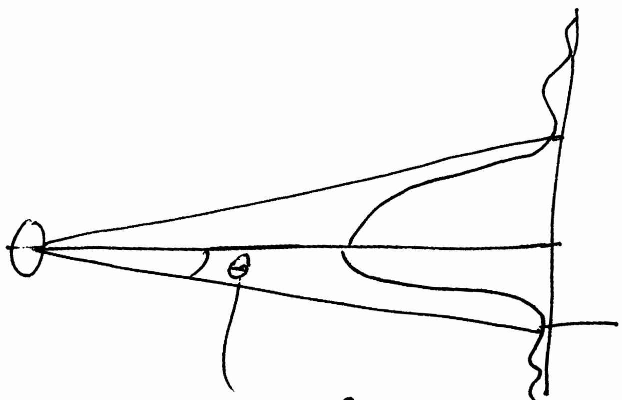


$\neq 0$

So, need to "bend" the phasors a bit more,

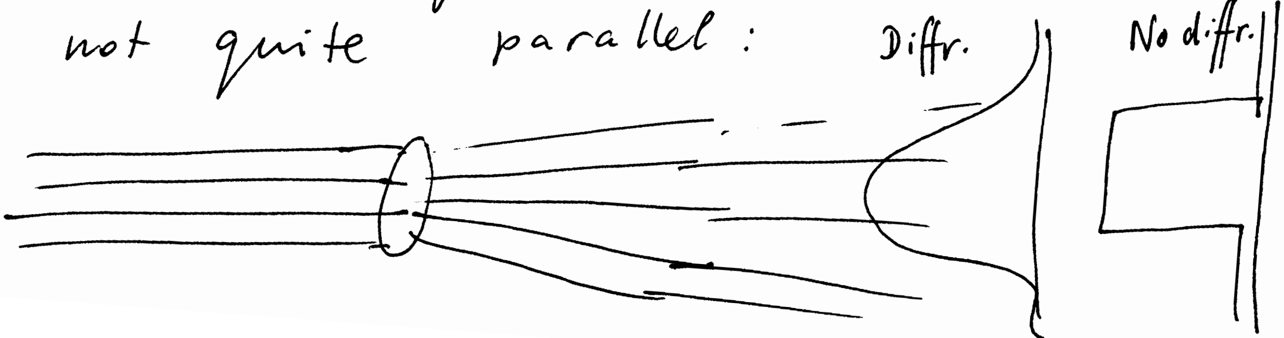


can be calculated exactly, by 1.21967 ~ 1.22 times more.



$\sin\theta = 1.22 \frac{\lambda}{D}$

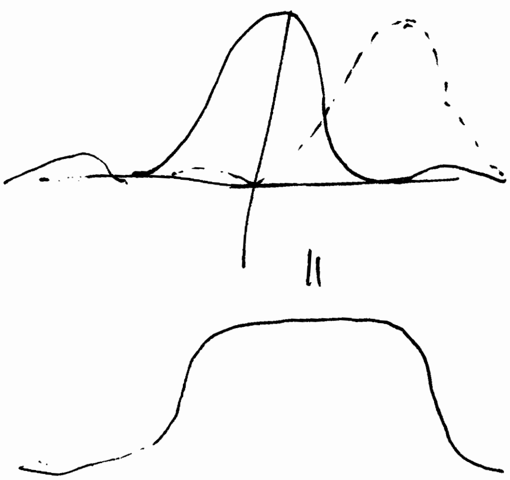
That means, a parallel beam passing through a hole becomes not quite parallel:



Rayleigh's criterion:

Observing two objects an angle θ apart through an objective of Diameter D, they become indistinguishable if

$$\theta < 1.22 \frac{\lambda}{D}$$



The two peaks merge into a plateau.

Example:

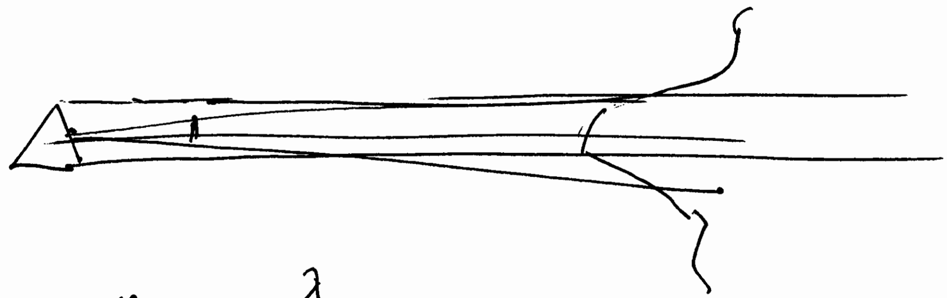
Find diffraction limit for a 10-cm diameter telescope, $\lambda = 600$ nm.

$$\theta_{lim} = 1.22 \cdot \frac{600 \cdot 10^{-9} m}{10^{-1} m} = 7.3 \cdot 10^{-6} \text{ rad}$$

$$= 1.5 \text{ arcsec.}$$

Can see 1.7 mile-object on the moon.

⑧. Example: A triangular mirror ~ 10 cm size is used to project sun onto the wall. At what distance the spot becomes round?



$$\frac{\text{size}}{L} = \theta = \frac{\lambda}{\text{size}}$$

$$L = \frac{\text{size}^2}{\lambda} = \frac{(0.1\ m)^2}{500 \cdot 10^{-9} m} = \frac{10^{-2}}{5 \cdot 10^{-7}}$$

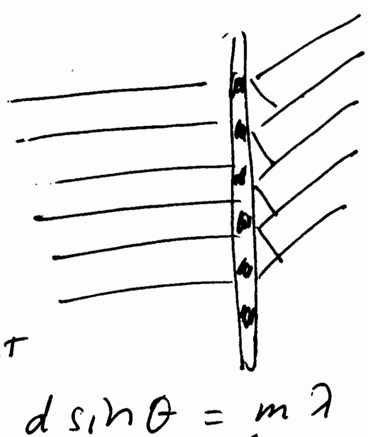
$$= 2 \cdot 10^4\ m$$

$$= 20\ km$$

For 1 cm mirror,

$$\frac{10^{-4}}{5 \cdot 10^{-7}} = 200\ m$$

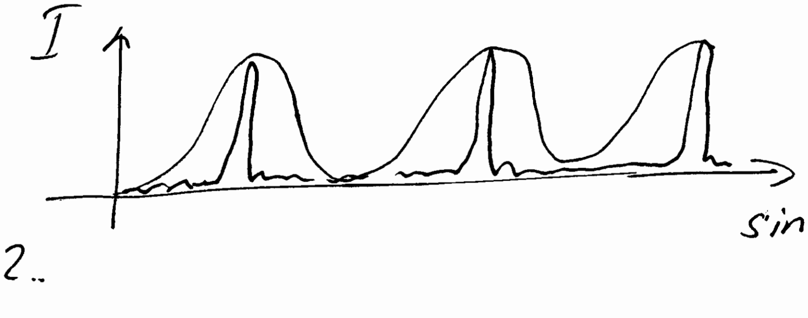
Gratings



Bright spots: $d \sin\theta = m\lambda$

$m = 0, \pm 1, \pm 2 ..$

Same idea as two-slit, but the bright spots are much sharper:



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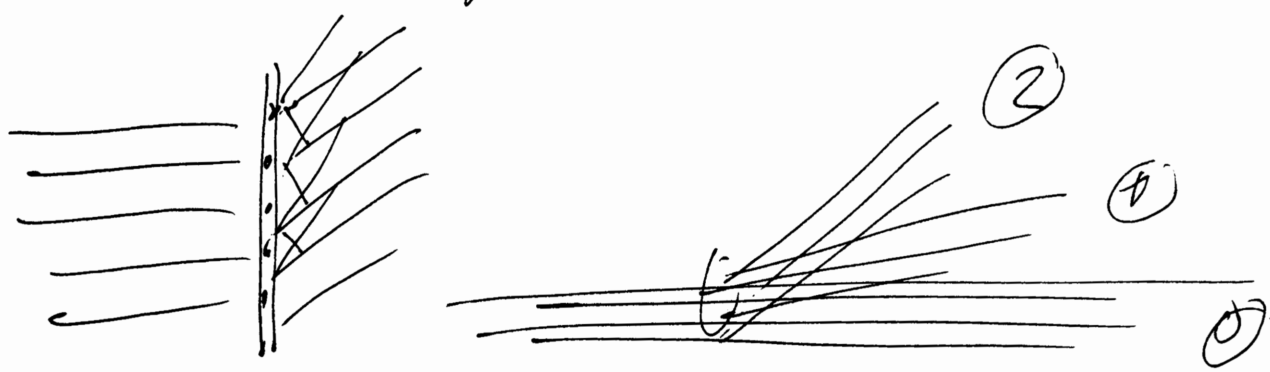
Example: a laser (633 nm) shines normally at a grating with 2 μm spacings.

Find the angles of 1st, 2nd peaks.



1st peak: $d \sin\theta = \lambda$

$$\sin\theta = \frac{\lambda}{d} = \frac{633 \cdot 10^{-9}}{2 \cdot 10^{-6}} = 0.3165$$

$$\theta = 0.322 \text{ rad} = 18.5^\circ$$

2nd peak: $d \sin\theta = 2\lambda$

$$\sin\theta = \frac{2 \cdot 633 \cdot 10^{-9}}{2 \cdot 10^{-6}} = 0.633$$

$$\theta = 0.685 \text{ rad} = 39.3^\circ$$