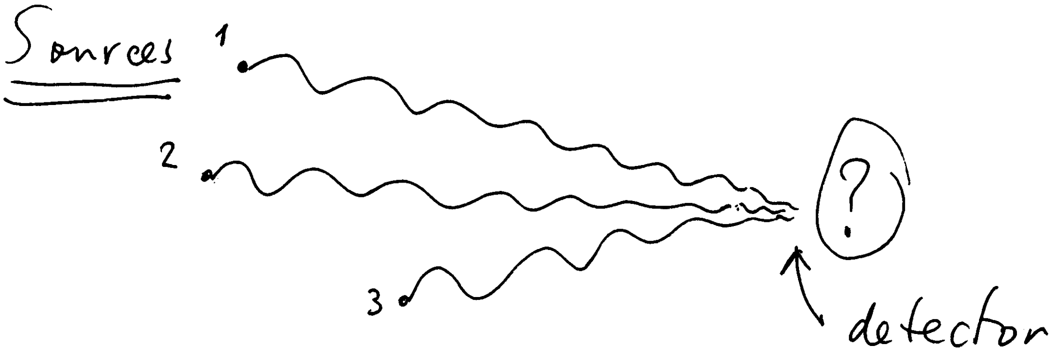
Physics 203 T10 May 20th lecture

Interference & Diffraction

Adding light or radio (sound, etc.) waves

Wares arrive at the detector from different sources.



Superposition principle:

Oscillation picked up by the detector is the sum (vector or scalar) of the individual oscillations caused by waves from each source.

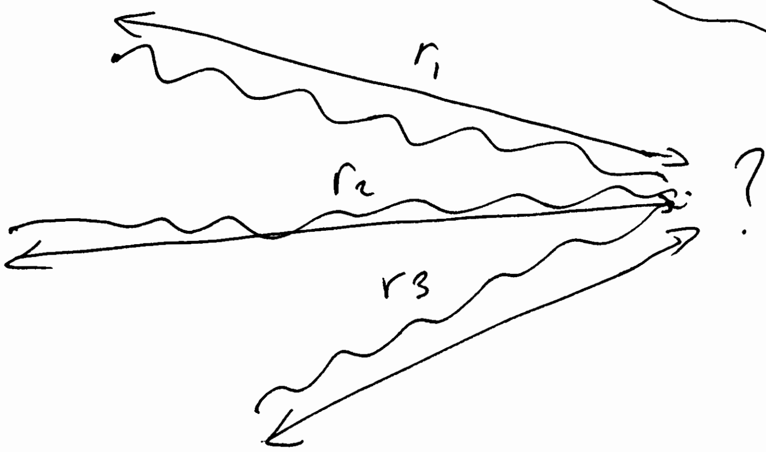
For Example, sound waves can be characterized by pressure deviations Δp (scalar)

light, as any E&M waves, can be characterized by $\vec{E}$ (vector).

Simplifications.

1. Consider all oscillations to be at the same frequency f. → "coherent sources"
2. In most examples, ignore the attenuation of intensity with distance
3. Ignore polarization (vector nature of $\vec{E}$), assume it's a scalar

General solution of the simplified problem



$$E_{tot}(t) = E_1 \cos\left(\frac{2\pi}{\lambda} r_1 - 2\pi f t + \theta_1\right) + E_2 \cos\left(\frac{2\pi}{\lambda} r_2 - 2\pi f t + \theta_2\right) + E_3 \cos\left(\frac{2\pi}{\lambda} r_3 - 2\pi f t + \theta_3\right)$$

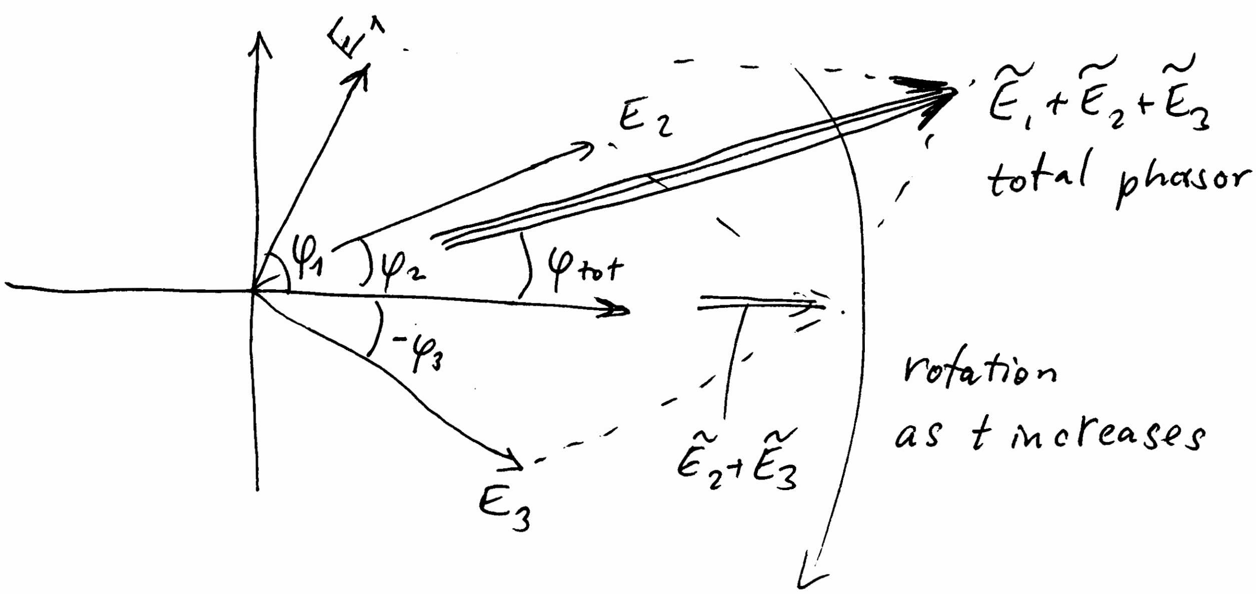
Define

$$\varphi_1 = \frac{2\pi r_1}{\lambda} + \theta_1$$

$$\varphi_2 = \frac{2\pi r_2}{\lambda} + \theta_2$$

$$\varphi_3 = \frac{2\pi r_3}{\lambda} + \theta_3$$

"Shadow analogy" again:



$$= E_{tot} \cos(\varphi_{tot} - 2\pi f t)$$

$$E_{tot} = \sqrt{(E_1 \cos\varphi_1 + E_2 \cos\varphi_2 + E_3 \cos\varphi_3)^2 + (E_1 \sin\varphi_1 + E_2 \sin\varphi_2 + E_3 \sin\varphi_3)^2}$$

$$\tan\varphi_{tot} = \frac{E_1 \sin\varphi_1 + E_2 \sin\varphi_2 + E_3 \sin\varphi_3}{E_1 \cos\varphi_1 + E_2 \cos\varphi_2 + E_3 \sin\varphi_3}$$

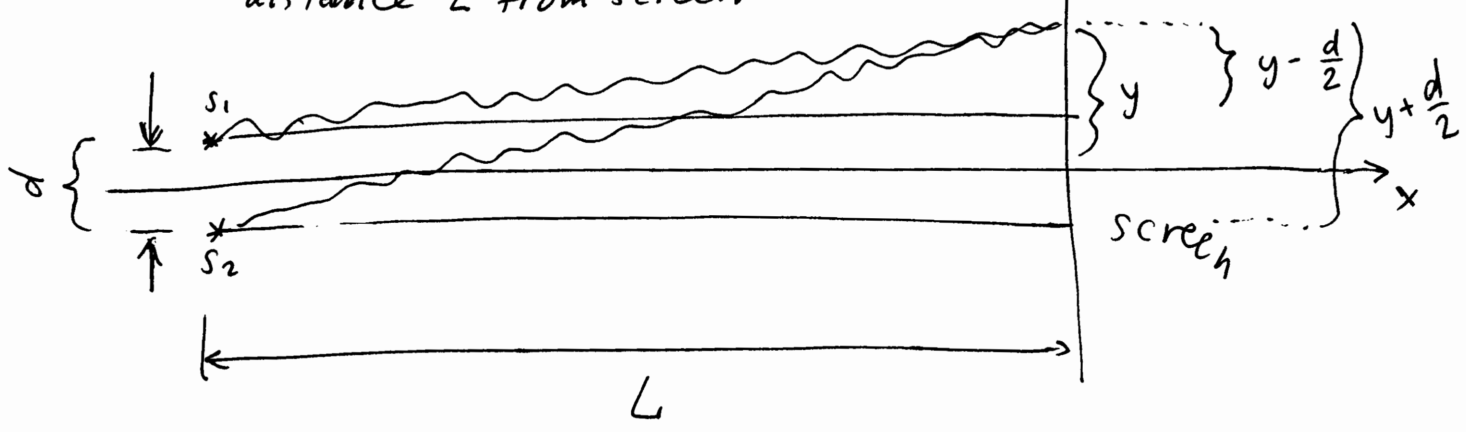
(3)

This still looks pretty complicated. To simplify further, we will only look for places where $E_{tot} = 0$ (dark spots) and where $E_{tot} = \max$ (bright spots).

Example: green light $550\ nm = \lambda$

f -?

is emitted by two equal sources in phase $(\theta_1 = \theta_2 = 0)$ located distance d apart distance L from screen:



Find the locations of bright and dark spots on the screen.

For a point y on the screen,

$$\Theta_1 = \frac{2\pi}{\lambda}\sqrt{L^2+\left(y-\frac{d}{2}\right)^2}$$

$$\Theta_2 = \frac{2\pi}{\lambda}\sqrt{L^2+\left(y+\frac{d}{2}\right)^2}$$

$$E_{tot}(y) = \underset{\overset{\shortparallel}{E}}{E_1} \cos\left(\frac{2\pi}{\lambda}r_1 - 2\pi f t\right) + \underset{\overset{\shortparallel}{E}}{E_2}\cos\left(\frac{2\pi r_2}{\lambda} - 2\pi f t\right)$$

$$\boxed{\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}$$

$$= E\left(\cos\left(\frac{2\pi}{\lambda}\sqrt{L^2+\left(y-\frac{d}{2}\right)^2} - 2\pi f t\right) + \cos\left(\frac{2\pi}{\lambda}\sqrt{L^2+\left(y+\frac{d}{2}\right)^2} - 2\pi f t\right)\right)$$

$$= E\left(\cos(\Theta_1 - 2\pi f t) + \cos(\Theta_2 - 2\pi f t)\right)$$

$$= 2E\cos\left(\frac{\Theta_1 - \Theta_2}{2}\right)\cos\left(\frac{\Theta_1+\Theta_2}{2} - 2\pi f t\right)$$

$$= 2E\cos\left(\frac{\theta_2-\theta_1}{2}\right)\cos\left(\frac{\theta_1+\theta_2}{2}-2\pi f t\right)$$

$\boxed{\cos\alpha = \cos(-\alpha)}$

$\underbrace{2E\cos\left(\frac{\theta_2-\theta_1}{2}\right)}$ amplitude that only depends on λ, L, d, and y

$\underbrace{\cos\left(\frac{\theta_1+\theta_2}{2}-2\pi f t\right)}$ simple oscillation with time t @ freq. f, with a constant delay

We want to find locations y such that

- **dark spots** $\cos\left(\frac{\theta_2-\theta_1}{2}\right) = 0$
- **bright spots** $\cos\left(\frac{\theta_2-\theta_1}{2}\right) = \pm 1$

That way, in dark spots amplitude $= 0$

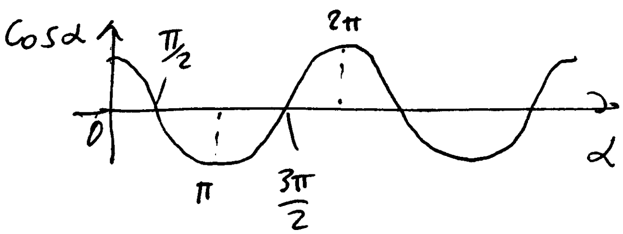
in bright spots $|\text{amplitude}| = 2E$ (doubles)

Question: when the amplitude doubles, what happens to the intensity?

Solution

$\cos\left(\frac{\theta_2-\theta_1}{2}\right) = 0$ when $\frac{\theta_2-\theta_1}{2} = \frac{\pi}{2} + \pi m$

$m = 0, \pm 1, \pm 2, \ldots$



$\cos\left(\frac{\theta_2-\theta_1}{2}\right) = \pm 1$ when $\frac{\theta_2-\theta_1}{2} = \pi m$

$m = 0, \pm 1, \pm 2, \ldots$

(5)

$$\frac{\theta_2-\theta_1}{2} = \frac{\cancel{2}\pi}{\lambda}\left(\frac{\sqrt{L^2+(y+\frac{d}{2})^2}-\sqrt{L^2+(y-\frac{d}{2})^2}}{\cancel{2}}\right) = \begin{cases} \frac{\pi}{2}+\pi m & \text{dark} \\ \pi m & \text{bright}\end{cases}$$

"x" both sides by $\frac{\lambda}{\pi}$:

$$\left(\sqrt{L^2+(y+\frac{d}{2})^2}-\sqrt{L^2+(y-\frac{d}{2})^2}\right) = \begin{cases} \frac{\lambda}{2}+\lambda m & \text{dark} \\ \lambda m & \text{bright.}\end{cases}$$

Interpretation is simple:

Path difference = { Half-integer # of λ's (arrival out-of-phase, destructive interference); Integer # of λ's (arrival in phase, constructive interference) }

To simplify further,

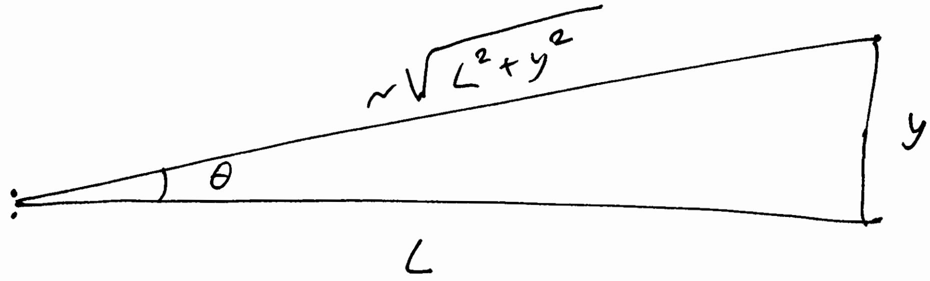
assume $d \ll y$, $d \ll L$

$$\sqrt{L^2+y^2+\frac{d^2}{4}+dy} - \sqrt{L^2+y^2+\frac{d^2}{4}-dy}$$

$$= \sqrt{L^2+y^2+\frac{d^2}{4}}\underbrace{\left(\sqrt{1+\frac{dy}{L^2+y^2+\frac{d^2}{4}}}-\sqrt{1-\frac{dy}{L^2+y^2+\frac{d^2}{4}}}\right)}_{\approx\left(1+\frac{1}{2}\frac{dy}{L^2+y^2+\frac{d^2}{4}}-\left(1-\frac{1}{2}\frac{dy}{L^2+y^2+\frac{d^2}{4}}\right)\right)}$$

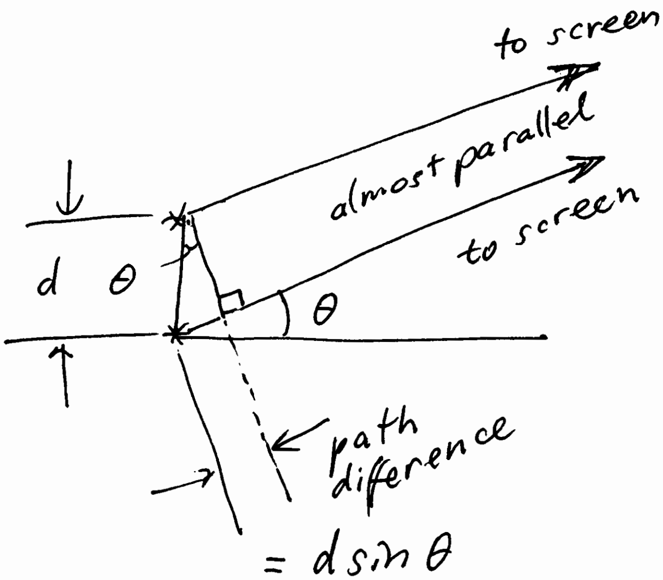
$$\sqrt{L^2+y^2+\frac{d^2}{4}} \times \left(\frac{dy}{L^2+y^2+\frac{d^2}{4}}\right) = \frac{dy}{\sqrt{L^2+y^2+\frac{d^2}{4}}}$$

⑥



$$\sin\theta = \frac{y}{\sqrt{L^2+y^2}}$$

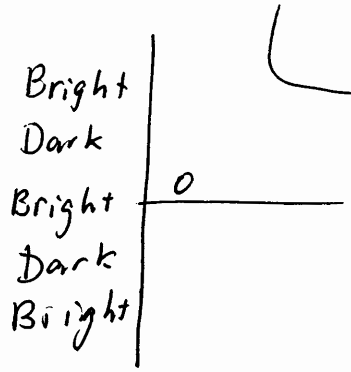
Path difference $\approx d\sin\theta$



Summary:

Bright fringes: $d\sin\theta = m\lambda$, $m = 0, \pm1, \pm2 \ldots$

Dark fringes: $d\sin\theta = \cancel{\frac{\lambda}{2} + m\lambda}$, $\cancel{m = 0, \pm1, \pm2 \ldots}$



$\left(m-\frac{1}{2}\right)\lambda$, $m = 1, 2, 3$
(above central bright)

$\left(m+\frac{1}{2}\right)\lambda$, $m = -1, -2, -3$
(below central bright)

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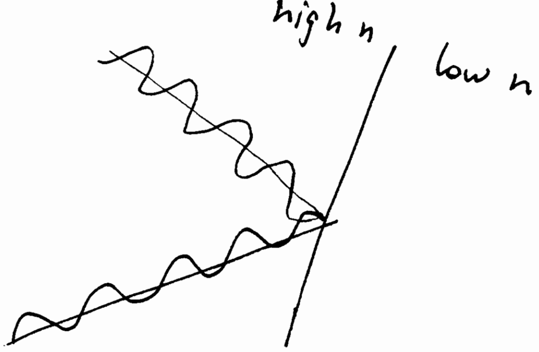
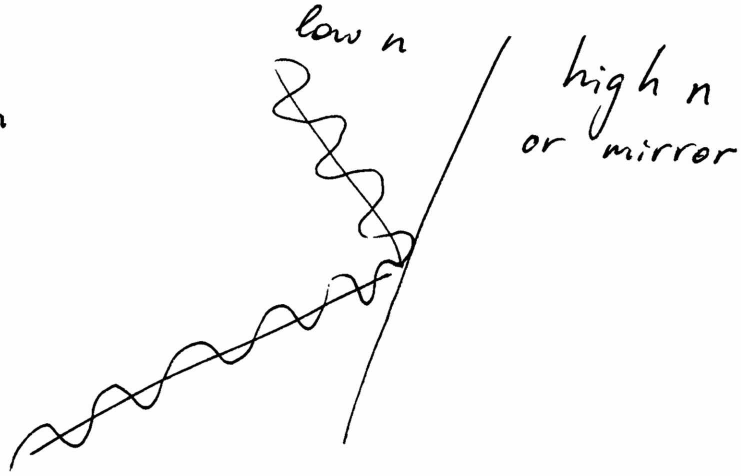
Phase change due to reflection:

"internal" reflections (off of lower n region back into higher n):
no phase change

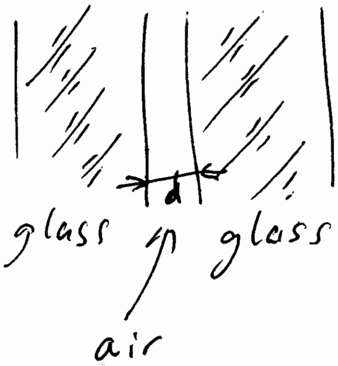




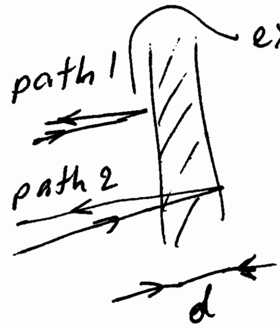
Extra π of phase
(extra $\frac{\lambda}{2}$ effective path)

Normal incidence:

"air gap"

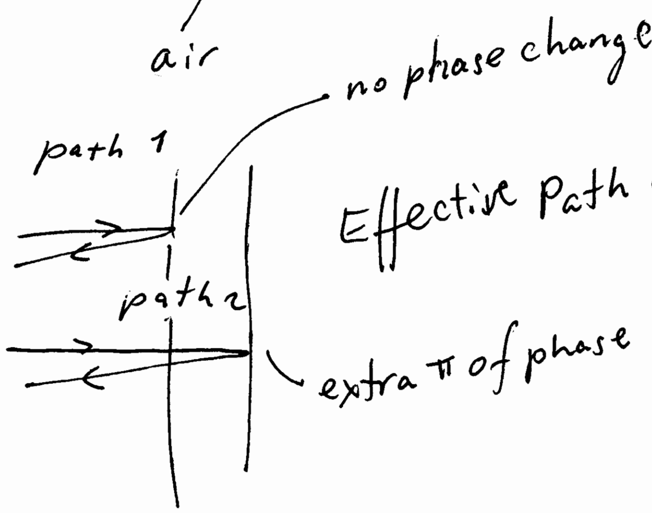


Thin film (highest n)



$$2d + \frac{\lambda_{film}}{2} = \begin{cases} m\lambda_{film} & \text{bright} \\ (m+\frac{1}{2})\lambda_{film} & \text{dark} \end{cases}$$

$$\lambda_{film} = \frac{\lambda_{vac}}{n}$$



Effective Path difference $2d + \frac{\lambda}{2} = \begin{cases} m\lambda - \text{bright} \\ (m+\frac{1}{2})\lambda - \text{dark} \end{cases}$

(8) Examples:

(1) Two slits are 1 mm apart, $\lambda = 750\,nm$ (dark-red), the screen is $L = 15\,m$ away.

Find where the fringes are.

Bright fringes: $d \sin\theta_b = m\lambda$

$$\sin\theta_b = \frac{m\lambda}{d} = m\,\frac{750 \cdot 10^{-9}\,m}{10^{-3}\,m} =$$

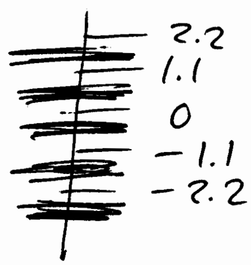
$$= m \cdot (7.5 \cdot 10^{-4}) \approx \theta_b \approx \tan\theta_b$$

$$y_b = L \tan\theta_b \approx L\,\theta_b = 15\,m \cdot m \cdot (7.5 \cdot 10^{-4})$$

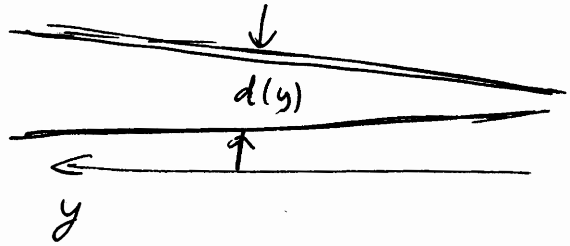
$$= 0.01125\,m \cdot m$$

$$\approx 1.1\,cm \cdot m$$

$\boxed{1.1 \text{ cm apart}}$



(2) An airgap is getting wider along y,



$d(y) = \beta \cdot y$

$\beta \to 10^{-5}$ (rad)

$\lambda = 600\,nm$

Find the spacing of the fringes along y.

$2d \pm \frac{\lambda}{2} = m\lambda$, $\quad d = m\frac{\lambda}{2} \mp \frac{\lambda}{4}$.

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Changing d by $\frac{\lambda}{2}$ gives the next fringe

Corresponding change in y is $\frac{1}{\beta}\cdot\frac{\lambda}{2}$

$$\left(y = \frac{d}{\beta} \right)$$

Fringes are $\frac{1}{\beta}\cdot\frac{\lambda}{2}$ apart in y!

$$10^5 \cdot \frac{600 \cdot 10^{-9}\ m}{2} = 3 \cdot 10^{-2}\ m = 3\ cm \text{ apart.}$$

3

A camera lens ($n = 1.42$) is coated with a thin film ($n = 1.55$) to prevent reflections at $\lambda_{vac} = 600$ nm. Find d (thickness)

Assume this in the "first" dark spot.

$$2d + \frac{\lambda_{film}}{\cancel{2}} = \left(m + \cancel{\tfrac{1}{2}}\right) \lambda_{film}$$

1

$$2d = \lambda_{film} = \frac{\lambda_{vac}}{n}$$

$$d = \frac{\lambda_{vac}}{2n} = \frac{600\,nm}{2 \cdot 1.55} = 194\ nm$$

If the camera lens were higher n than the film, then both reflections give π of extra phase, and the condition would be

$$2d + \frac{\lambda_{film}}{2} + \frac{\lambda_{film}}{\cancel{2}} = \left(m + \cancel{\tfrac{1}{2}}\right) \lambda_{film}$$

in this notation, choose $m=0$

$$d = \frac{\lambda_{vac}}{4n} = 97\ nm$$