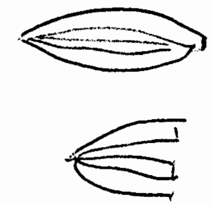
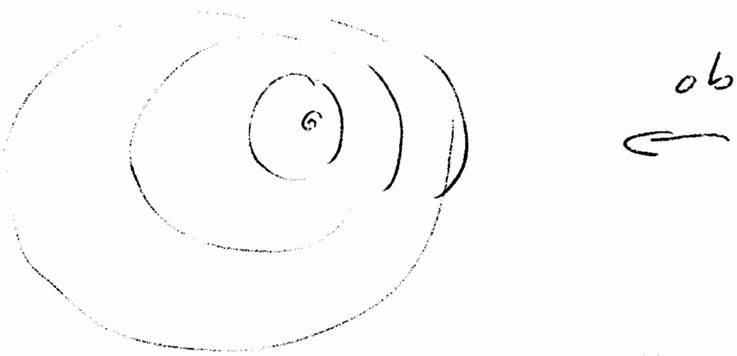
Physics 203 T10 Apr 10 lecture

Plan: Standing waves: resonances

Examples...



Doppler effect, examples



Begin Electromagnetic Waves...

Speed of light

$\lambda \to f$ conversion

Colors, wave ranges

2 Distance between nodes: $\frac{\lambda}{2}$

Tight end: can provide F_{Ty}, but $y = 0$ – node

Loose end: cannot provide F_{Ty}, but $y \updownarrow$ has max amplitude $\Big\}$ – antinode

Resonances of standing waves:

Same type of end on both sides:

$L = n\left(\frac{\lambda_n}{2}\right)$, where λ_n – harmonic wavelength.

$\lambda_n = \frac{2L}{n}$

$f_n = \frac{v}{\lambda_n} = \frac{nv}{2L}$, $n = 1, 2, 3 \ldots$

Different type of end on each side:

node – antinode,

$L = \left(m + \frac{1}{2}\right)\frac{\lambda_m}{2}$, $m = 0, 1, 2 \ldots$

$= \frac{n}{2}\frac{\lambda_n}{2} = \frac{n\lambda_n}{4}$, $n = 1, 3, 5$ – odd

$\lambda_n = \frac{4L}{n}$

$f_n = \frac{v}{\lambda_n} = \frac{nv}{4L}$, $n = 1, 3, 5 \ldots$ – odd

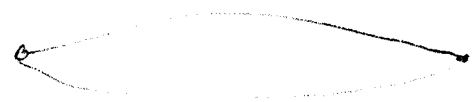
(3) Example:

60 cm long

The string of a violine is tuned to 440 Hz by applying 14 lb of force

Find the mass of the string

Steps: f_1 (fundamental harmonic) = 440 Hz



1 Convert units: $L = 60\ cm = 0.60\ m$

$$F_T = 14\ lb \cdot 0.454 \frac{kg}{lb} \cdot 9.8 \frac{m}{s^2}$$

$$= 62.3\ N$$

2. Find speed of the wave

$$\boxed{f_1 = \frac{1 \cdot v}{2L}}$$

$$v = \sqrt{\frac{F_T}{\mu}} = \lambda f_1 = 2L f_1$$

$$= 1.2\ m \cdot 440\ Hz$$

$$= 528\ m/s$$

$$\frac{F_T}{\mu} = v^2$$

$$\mu = \frac{F_T}{v^2} = \frac{62.3\ N}{(528\ \frac{m}{s})^2} = 0.00022\ kg/m$$

$$= 0.22\ g/m$$

4 Does it make sense?

Let's find cross-section, diameter assuming steel vs plastic

$7.8 \frac{g}{cm^3}$ $\quad$ $1 \frac{g}{cm^3}$

$$\pi \left(\frac{d}{2}\right)^2 \cdot L \cdot \rho = m = \mu L \Rightarrow d = \sqrt{\frac{4m}{\pi L \rho}} = \begin{cases} \cancel{0.7}\ 0.53\ mm\ \text{plastic} \\ \cancel{1}\ 0.19\ mm\ \text{steel} \end{cases}$$

(4) Another example:

Organ pipe (closed + open ends)
is tuned to 440 Hz at 20°C
(fundamental)

(a) Find its length

(b) How much does it detune at 0°C ?

Constants:
$v(20°C) = 343 \frac{m}{s}$
$v(0°C) = 331 \frac{m}{s}$

Half-open system:

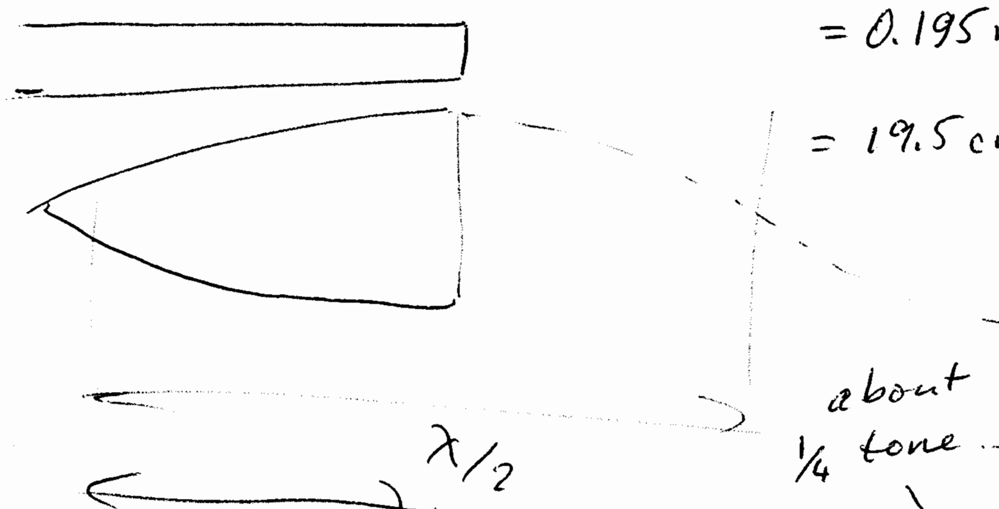
fundamental

$f_n = \frac{nv}{4L}$, $n = 1, 3, 5$

$f_1 = \frac{v}{4L} \rightarrow L = \frac{v}{4f_1} = \frac{343 \frac{m}{s}}{4 \cdot 440 Hz}$

$= 0.195 m$

$= 19.5 cm$



$\lambda/2$

$\lambda/4$

about ¼ tone ..

Detuning:

$f_1(0°C) = \frac{v(0°C)}{4L} = \frac{331}{4 \cdot 0.195} = \frac{331}{343} \cdot 440 = 425$ Hz

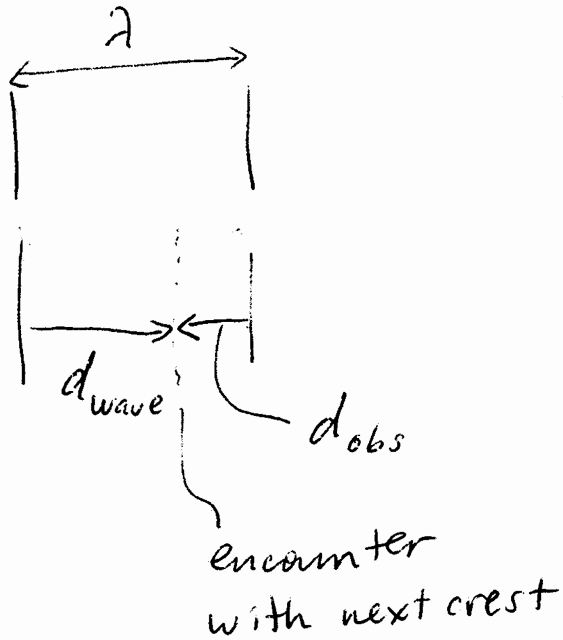
5 Doppler effect

3 types:

v_o pos, if $v_o \uparrow\uparrow v$

v_o neg, if $v_o \uparrow\downarrow v$

observer moving:



$$\overset{-v_oT_o}{\overset{\parallel}{d_{obs}}} + \overset{vT_o}{\overset{\parallel}{d_{wave}}} = \lambda = Tv$$

$$(-v_o + v)T_o = Tv$$

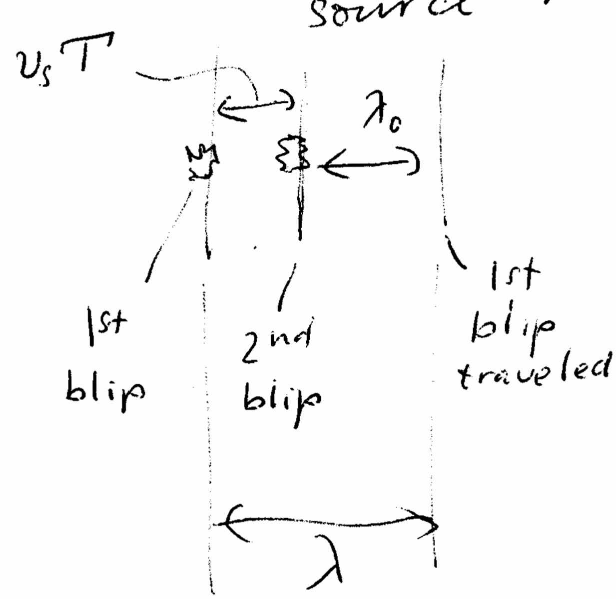
$$T_o = T\frac{v}{-v_o+v}$$

$$f_o = f\frac{v-v_o}{v}$$

$$= f\left(1 - \frac{v_o}{v}\right)$$ ← $v_o > 0$ moving away

$$f\left(1 + \left|\frac{v_o}{v}\right|\right)$$ – $v_o < 0$ moving towards

source moving.



$$\lambda_o = \lambda - v_sT = \lambda - \frac{v_s}{f}$$

$$f_o = \frac{v}{\lambda_o} = \frac{v}{\lambda - \frac{v_s}{f}} = \frac{v}{v-v_s}f$$

$$= \frac{f}{1-\frac{v_s}{v}}$$

6

Combined:

$$f_o = \frac{1 - \frac{v_o}{v}}{1 - \frac{v_s}{v}} f$$ relative to the medium (air)

If $v_o = v_s$ (in the same direction),

$f_o = f$!!!

Example:

Listening to echo while playing a 440 Hz note and biking 20 mph toward a wall.

$$20 \frac{mi}{h} = \frac{20\,mi \cdot \cancel{1.6}\frac{km}{mi} \cdot 1000\frac{m}{km}}{3600\frac{s}{h}} =$$

Convert units: $= 8.94 \frac{m}{s}$

$$f_{wall} = f \frac{1}{1 - \frac{8.94 \frac{m}{s}}{343 \frac{m}{s}}}$$

$$f_{obs} = \underset{440}{f} \frac{1 + \frac{8.94}{343}}{1 - \frac{8.94}{343}} = 463.5\ Hz$$

↳ almost ~~fa~~ half tone

(A# = 466 Hz)