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Physics 203 T10 April 8 lecture

1 Intensity of sound → dB units

2 Transverse waves in a string / tight rope
↳ speed of sound, reflections, superposition, interference
↳ standing waves

$v = \sqrt{\frac{F_T}{\mu}}$ → Tension; → linear mass density

{ tight end
{ loose end

3 Wave equation for harmonic waves

$$y = A\cos\left(\frac{2\pi}{\lambda}x - \omega t + \theta_0\right)$$

~~Example 2.2~~

What determines "loudness"?

The amplitude – definitely.

The "effect" sometimes is determined by energy transfer

$$\text{Intensity} = \frac{\text{Power}}{\text{Unit Area}} \quad \left[\frac{W}{m^2}\right] \sim \boxed{\text{Amplitude}^2}$$

$$\left(= \frac{p_0^2}{2\rho v}\right)$$

Example:

Human "threshold of hearing" $10^{-12}\ \frac{W}{m^2}$

"threshold of pain" $1\ \frac{W}{m^2}$ → $p_0 = \sqrt{2\rho v I}$

$\doteq 30$ Pa,

or 0.03% of 1 atm!

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②

Units :

$\log_{10} \frac{I}{I_{t.h.}}$ → logarithmic intensity in "Bells"

too crude...

$10 \log_{10} \frac{I}{I_{t.h.}}$ → log intensity in "deciBells"

dB.

I	I log, dB	
10^{-12}	0	→ million times less Δp $(\sqrt{10^{12}})$
10^{-11}	10	
10^{-10}	20	
...		
10^{-1}	110 dB – rock concert	
$10^{0} = 1 \frac{W}{m^2}$	120 dB – Ouch!!!	

↳ only 0.03% change in air pressure.

Special meaning: 3 dB

$$10 \log_{10} \frac{I_1}{I_2} = 3$$

$$I_1 = I_2 \cdot 10^{0.3} = I_2 \cdot 1.995 \approx I_2 \cdot 2$$

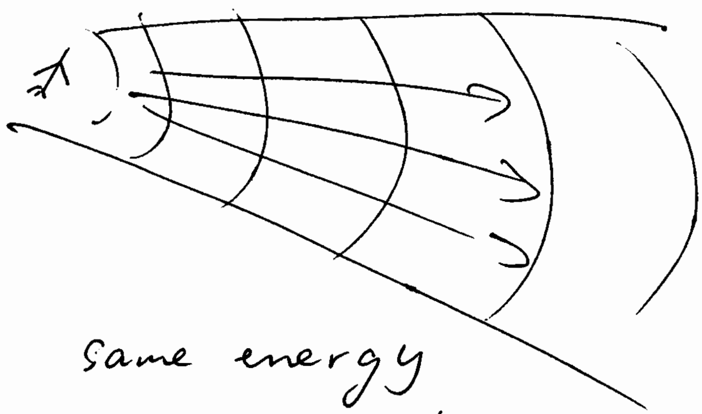
Special values

- 3 dB = "doubling of power"
- 6 dB = { "doubling of amplitude" / "quadrupling of power" }
- 20 dB = { x 100 power / x 10 amplitude } $(P \sim Amp^2)$
- 60 dB = { x million power / x thousand amplitude }

Example:

If one can't (almost) hear an airplane 35000 ft above, how close one can stand to one without feeling pain?

→ engines on!



same energy propagates outward, but the area $\sim r^2$

$$P = \text{const} = I \cdot A$$

$$I = \frac{P}{A} \sim \frac{1}{r^2} \quad \left(= \frac{\text{const}}{r^2}\right)$$

$$\frac{I_{T.P.}}{I_{T.H.}} = 10^{12} = \frac{\frac{\cancel{\text{const}}}{r_{T.P.}^2}}{\frac{\cancel{\text{const}}}{r_{T.H.}^2}} = \frac{r_{T.H.}^2}{r_{T.P}^2}$$

$$10^6 = \sqrt{10^{12}} = \sqrt{\frac{r_{T.H.}^2}{r_{T.P.}^2}} = \frac{r_{T.H.}}{r_{T.P}} = 1 \text{ million}$$

$$r_{T.P.} = \frac{r_{T.H.}}{10^6} = \frac{35000}{1000000} = 0.035 \text{ ft}$$

$$\approx 1 \text{ cm.}$$

(4)

String / Tight rope:

Tension: F_T [N] $\left\{ \right.$ the force with which each end is pulled apart (can vary along the string, but we neglect it or take an average)

Linear mass density $\mu \left[\frac{kg}{m}\right]$ $\left\{ \right.$ Mass of the string per unit length. Remember to convert to SI units!

Speed of "sound", or transverse waves:

$$\left.\begin{array}{ll} \mu \uparrow & v \downarrow \\ T \uparrow & v \uparrow \end{array}\right\} \quad v \sim \frac{F_T}{\mu}\ ?$$

But the units of $\frac{F_T}{\mu}$ are

$$\frac{N}{\left[\frac{kg}{m}\right]} = \frac{N\,m}{kg} = \frac{kg\,\frac{m}{s^2}\,m}{kg} = \left(\frac{m}{s}\right)^2$$

$v \sim \sqrt{\frac{F_T}{\mu}}$? Turns out

$$\boxed{v = \sqrt{\frac{F_T}{\mu}}}$$

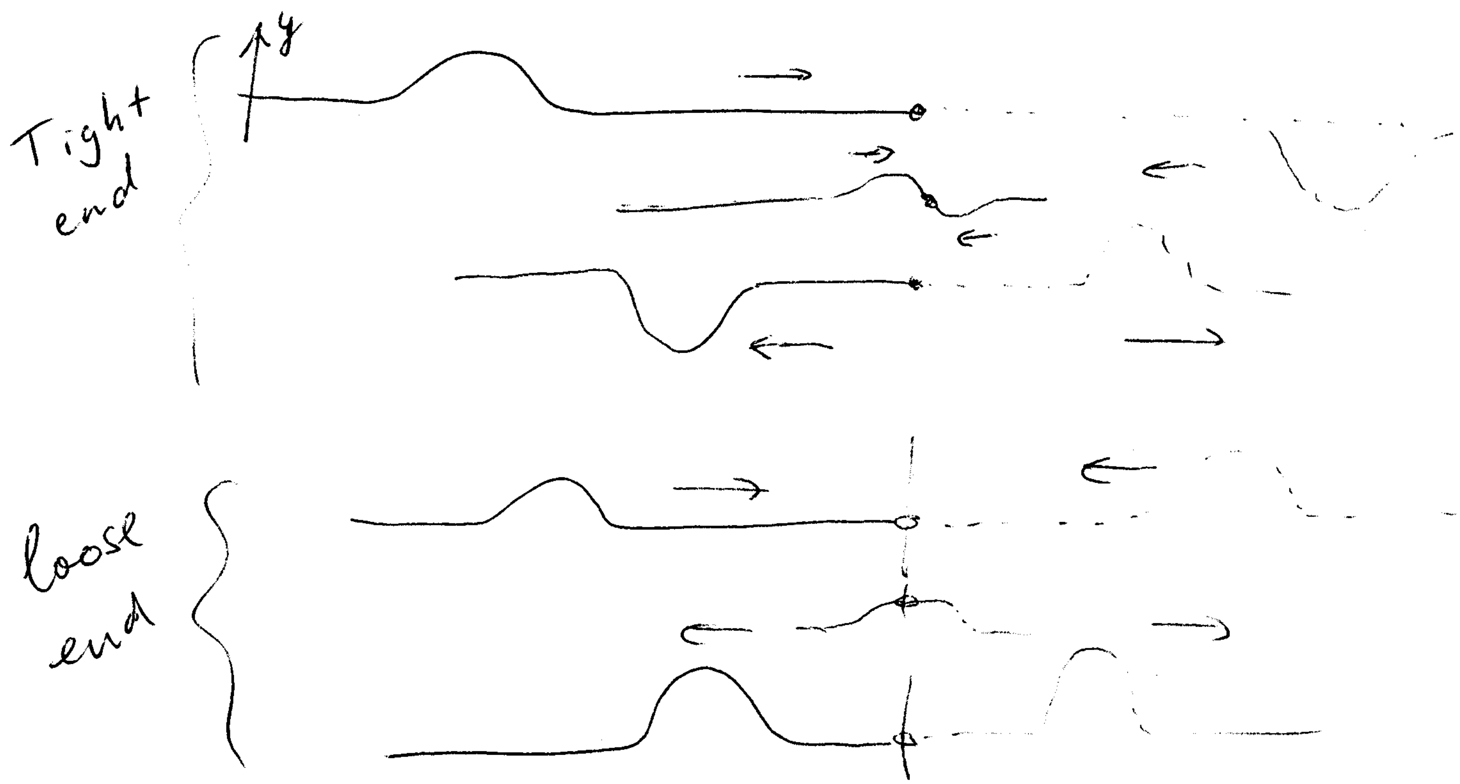
Example 2.4

A mountaineer, whose mass (including gear) is 250 lb, is hanging next to the face of a cliff in total fog, suspended by a 24-lb, 600-ft \\ kernmantle rope, tied to his buddy above. What is the delay in perception of the buddy's movements (hammering nails into the rock) sensed by the mountaineer through the rope?

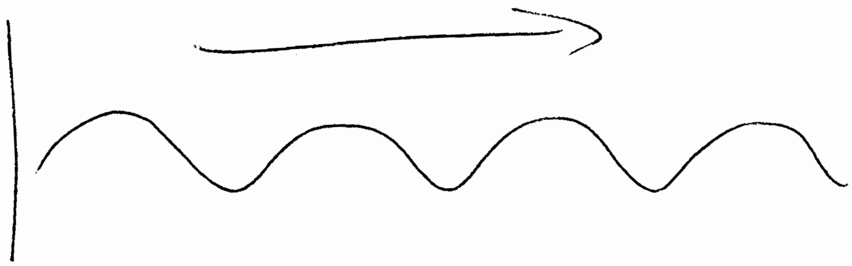
Steps:

1. Convert units to SI:
   - m_m = 250 lb = 113 kg
   - m_r = 24 lb = 10.9 kg
   - L = 600 ft = 183 m
2. Find average tension in the rope
   - $F_T = \left(m_m + \frac{1}{2} m_r\right) g = 118\,\text{kg} \cdot 9.8\,\frac{\text{m}}{\text{s}^2} = 1160\,\text{N}$
3. Find the linear mass density of the rope
   - $\mu = \frac{m_r}{L} = 0.060\,\frac{\text{kg}}{\text{m}}$
4. Find the speed of transverse waves in the rope
   - $v = \sqrt{\frac{F_t}{\mu}} = \sqrt{\frac{1160\,\text{N}}{0.060\,\frac{\text{kg}}{\text{m}}}} = 140\,\frac{\text{m}}{\text{s}}$
5. Find the time delay over the length of the rope
   - $\Delta t = \frac{L}{v} = \frac{183\,\text{m}}{140\,\frac{\text{m}}{\text{s}}} = 1.3\,\text{s}$

Reflections: Superposition of "images"



⑥ Standing waves:



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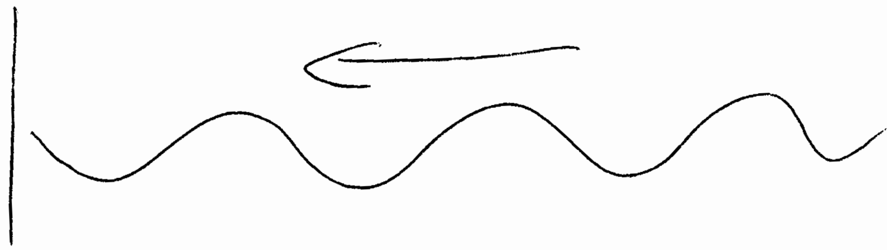

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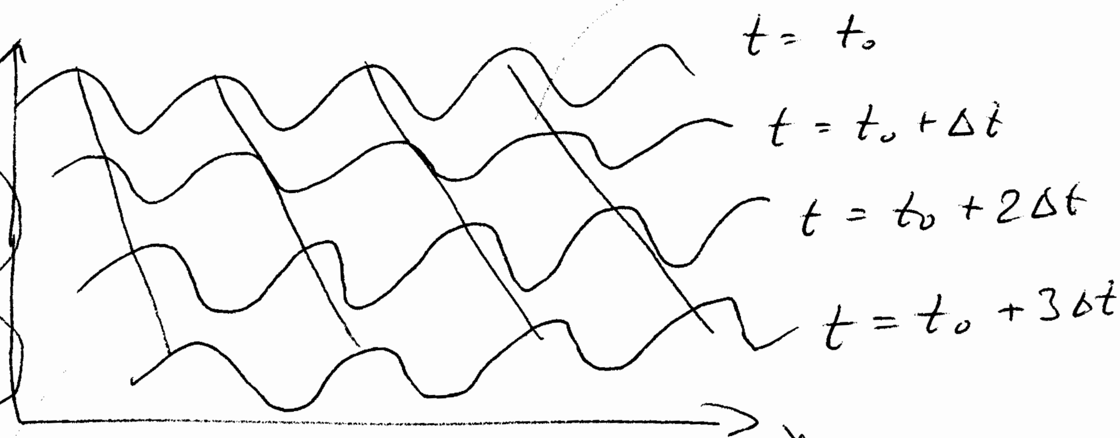
?

Let's write down the wave equation

$$y = A\cos\left(\frac{2\pi}{\lambda}x \pm \omega t + \theta_0\right)$$

to cover initial shift

Different snapshots



At any t, ripple in x, but shifted

At any x, ripple in t, but shifted

$y = \text{const}$, therefore $\frac{2\pi}{\lambda}x \pm \omega t = \text{const}$!

7

If x changes by Δx

t changes by Δt

$$\frac{2\pi}{\lambda}\Delta x \pm \omega \Delta t = 0 \quad (\text{change of const} = 0)$$

$$v = \frac{\Delta x}{\Delta t} = \mp \frac{\omega \lambda}{2\pi} = \mp \lambda f = \mp \frac{\lambda}{T} \quad \text{of course!}$$

Same sign of "$\frac{2\pi}{\lambda}x$" and "ωt" terms : direction $\leftarrow$ $-x$

opposite sign of —— " —— : direction $\rightarrow$ $+x$

Now, we can do a standing wave:

$$y_{st} = A_0 \cos\left(\frac{2\pi}{\lambda}x + \omega t\right) + A_0 \cos\left(\frac{2\pi}{\lambda}x - \omega t\right)$$

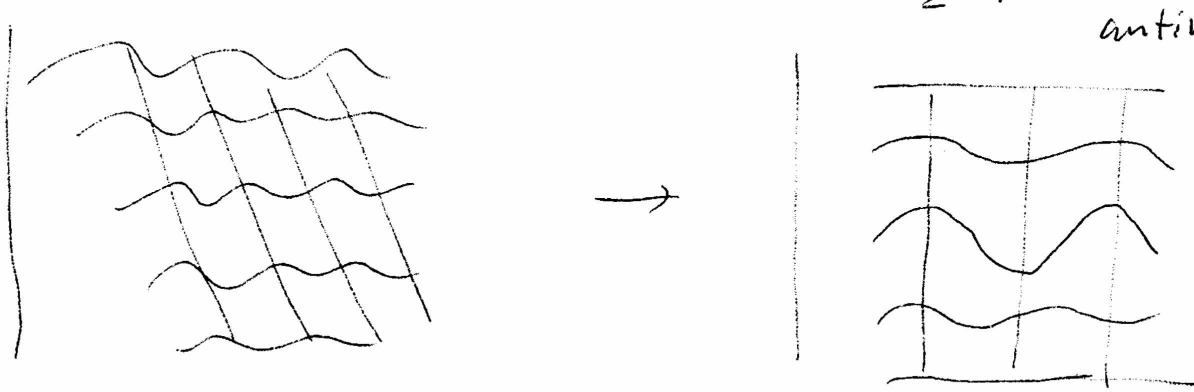
$$= 2A_0 \cos\left(\frac{2\pi x}{\lambda}\right) \cos \omega t$$

$\hookrightarrow$ At some x $\left(x = \frac{n\lambda}{2} + \frac{\lambda}{4} \right)$, no change in time!!!

$\hookrightarrow$ nodes, destructive interference

At other x $\left(x = \frac{n\lambda}{2} \right)$, double the change!

antinodes, constructive interference



— the wave is standing!

(8) Distance between nodes: $\frac{\lambda}{2}$

Tight end: can provide F_{Ty}, but $y=0$ – node

Loose end: cannot provide F_{Ty}, but $y \updownarrow$ has max amplitude – anti node

Resonances of standing waves:

Same type of end on both sides:

$$L = n\left(\frac{\lambda_n}{2}\right),$$ where λ_n – harmonic wavelength.

$$\lambda_n = \frac{2L}{n}$$

$$f_n = \frac{v}{\lambda_n} = \frac{nv}{2L}, \quad n = 1, 2, 3 \ldots$$

Different type of end on each side:

node – antinode,

$$L = \left(m+\frac{1}{2}\right)\frac{\lambda_m}{2}, \quad m = 0, 1, 2 \ldots$$

$$= \frac{n}{2}\frac{\lambda_n}{2} = \frac{n\lambda_n}{4}, \quad n = 1, 3, 5 \text{ – odd}$$

$$\lambda_n = \frac{4L}{n}$$

$$f_n = \frac{v}{\lambda_n} = \frac{nv}{4L}, \quad n = 1, 3, 5 \ldots \text{ – odd}$$