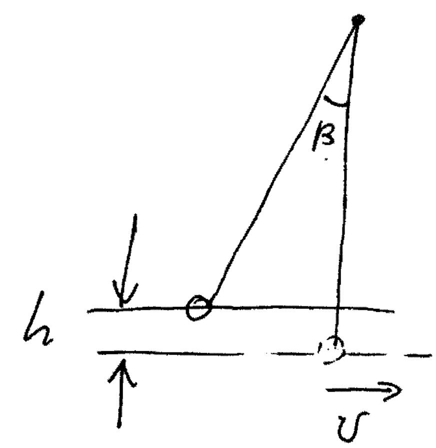
① Physics 203 T10 April 3 lecture

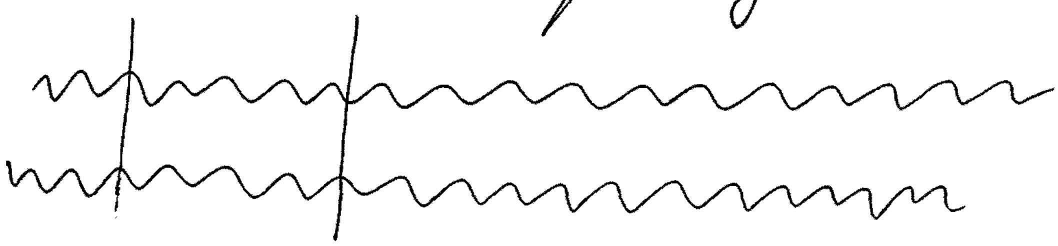
Finish up Ch. 13:

Energy considerations in a Pendulum



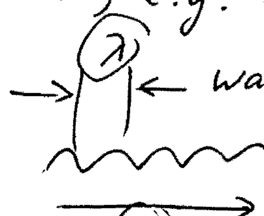
$$mgh + \frac{1}{2}mv^2 = const$$
$$\| $$
$$mgh_{max}$$
$$\|$$
$$\frac{1}{2}mv_{max}^2$$

Small differences in frequency,
Errors in frequency measurement



Begin Ch. 14

waves, e.g. sound:

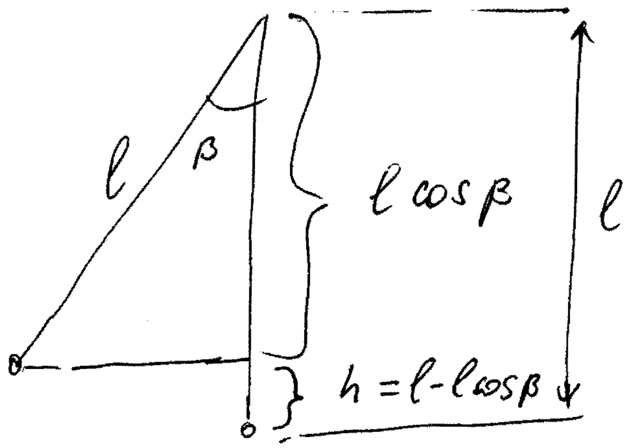


wavelength
& frequency (f)
(v) speed

What about "loudness" (volume)? → Power → per area
→ decibel levels

(2)

A Pendulum of length 1 m is deflected from vertical by 5° and released from rest.
Find velocities when it passes 4°, 3°, 2°, 1°, 0° positions.



$$= \ell(1-\cos\beta)$$

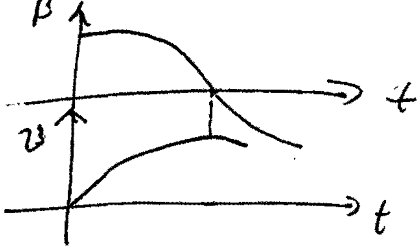
$$mgh + \frac{1}{2}mv^2 = const = mgh_{max} + 0$$
$$= 0 + \frac{1}{2}mv_{max}^2$$

$$\frac{1}{2}\cancel{m}v^2 = \cancel{m}g\,h_{max} - \cancel{m}gh$$

$$v^2 = 2g(h_{max} - h) =$$
$$= 2g\underbrace{\big(\ell(1-\cos\beta_{max}) - \ell(1-\cos\beta)\big)}_{\ell(\cancel{1} - \cos\beta_{max} - \cancel{1} + \cos\beta)}$$

$$v = \sqrt{2g\ell(\cos\beta - \cos\beta_{max})} =$$

β	v, $\frac{m}{s}$
5	0
4	0.16
3	0.22
2	0.25
1	0.27
0	0.27



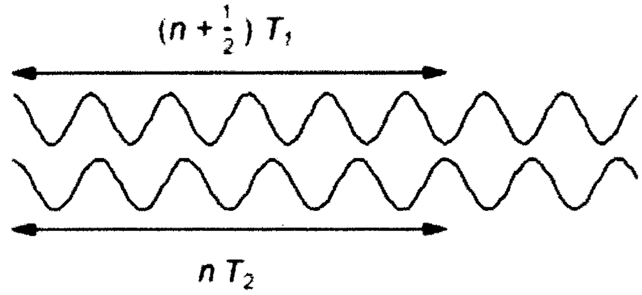
~~7~~ 3

Problem 1.4

Two pendulums (or, *pendula*) are made of identical 1 kg masses suspended on two weightless strings, 40.0 and 40.5 cm in length. If these pendulums are deflected from vertical by 5 cm and released at the same time, how long will it take for them to get completely "out of step" with each other?

Steps:

1. Convert **relevant** quantities to SI units:
   - l_1 = 40.0 cm = 0.400 m
   - l_2 = 40.5 cm = 0.405 m
   - masses and (small) deflections are irrelevant for finding periods and timing of oscillations
2. Find each period:
   - $T_1 = 2\pi\sqrt{\frac{l_1}{g}} \approx 6.283\sqrt{\frac{0.400\text{ m}}{9.8\ \frac{\text{m}}{\text{s}^2}}} = 1.269$ s
   - $T_2 = 2\pi\sqrt{\frac{l_2}{g}} \approx 6.283\sqrt{\frac{0.405\text{ m}}{9.8\ \frac{\text{m}}{\text{s}^2}}} = 1.277$ s
3. By the time t the two pendulums are "out of step", one would have completed an extra 1/2 periods compared to the other:
   - $t = \left(n + \frac{1}{2}\right)T_1 = nT_2$
4. Use this relationship to solve for n, then find t:
   - first, open the parentheses:
     $t = nT_1 + \frac{1}{2}T_1 = nT_2$
   - then, isolate n by combining terms:
     $\frac{1}{2}T_1 = nT_2 - nT_1 = n(T_2 - T_1)$
   - lastly, $n = \frac{T_1}{2(T_2 - T_1)} = \frac{1.269\text{ s}}{2 \cdot 8\times10^{-3}\text{ s}} = 79$
   - $t = nT_2 = 79 \cdot 1.277\text{ s} \approx 100$ s



5. Make sense of the answer:
   - This is much longer than the periods of oscillations, but short enough to observe the effect before the oscillations die down due to friction

Precision of frequency measurement

How does a doctor count the heart rate?

What is the error?

To distinguish two frequencies,
need to "listen" until they get out of step

$$t = nT_2 = \frac{T_1 T_2}{\cancel{n}\,2(T_2 - T_1)} = \frac{1}{2}\left(\frac{1}{\frac{1}{T_1} - \frac{1}{T_2}}\right)$$

$$= \frac{1}{2}\left(\frac{1}{f_1 - f_2}\right) = \frac{1}{2\Delta f}$$

Example

middle A 440 Hz

A# 466 Hz

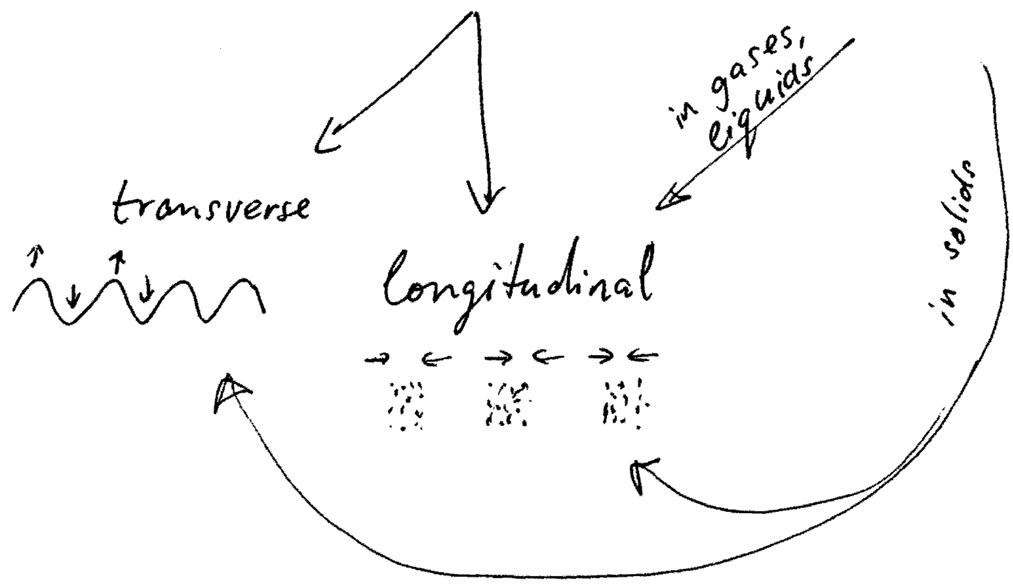
$$t = \frac{1}{2}\left(\frac{1}{26\text{ Hz}}\right) = 0.02\text{ s}$$

left-most A 27.5 Hz

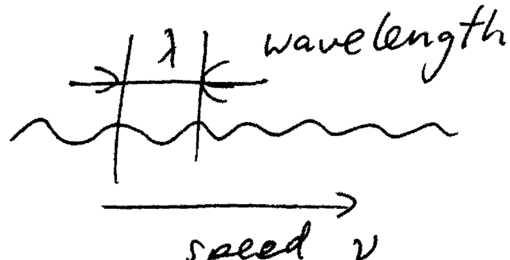
A# 29.135 Hz

$$t = \frac{1}{2}\left(\frac{1}{1.635\text{ Hz}}\right) = 0.3\text{ s}$$

Waves & Sound



Any quantity that changes both in space & time (e.g. pressure, temperature, E-field, color, gravitation) in such away that these changes oscillate, and there is propagation of energy — is a wave.



frequency f

the "pattern" propagates over λ in time $T = \frac{1}{f}$ with velocity v, so that

$$\boxed{\frac{\lambda}{T} = \lambda \cdot f = v}$$

Usually the "pattern" is a sin (cos) wave, — in this case it is a simple harmonic wave

Examples Ch.14

Problem 2.1

What is the wavelength of the lowest and the highest frequency you can hear?

Steps:

1. Determine (or look up) the range of audible frequencies [http://en.wikipedia.org/wiki/Hearing range]
    - my lower range is $f_1 = 35$ Hz (some people can hear down to $f_{\min} = 20$ Hz)
    - my higher range is $f_2 = 15.5$ kHz (some people can hear up to $f_{\max} = 20$ kHz)
2. Look up the speed of sound [http://en.wikipedia.org/wiki/speed of sound] at room temperature
    - $v = 343\ \frac{\text{m}}{\text{s}}$
3. Calculate the wavelength from $v = \lambda \cdot f$:
    - $\lambda_{f\max} = \frac{v}{f_{\max}} = \frac{343\ \frac{\text{m}}{\text{s}}}{20 \times 10^3\ \text{Hz}} = 0.017\ \text{m} = 1.7\ \text{cm}$
    - $\lambda_2 = \frac{v}{f_2} = \frac{343\ \frac{\text{m}}{\text{s}}}{15.5 \times 10^3\ \text{Hz}} = 0.022\ \text{m} = 2.2\ \text{cm}$
    - $\lambda_1 = \frac{v}{f_1} = \frac{343\ \frac{\text{m}}{\text{s}}}{35\ \text{Hz}} = 9.8\ \text{m}$
    - $\lambda_{f\min} = \frac{v}{f_{\min}} = \frac{343\ \frac{\text{m}}{\text{s}}}{20 \times 10^3\ \text{Hz}} = 17\ \text{m}$

4. Make sense out of the answer:
    - This is an impressive range from about an inch to 30-50 feet!

~~Example 2.2~~ What determines "loudness"?

The amplitude – definitely.

The "effect" sometimes is determined by energy transfer

$$\text{Intensity} = \frac{\text{Power}}{\text{Unit Area}} \quad \left[\frac{W}{m^2}\right] \sim \boxed{\text{Amplitude}^2} \quad \left(= \frac{p_0^2}{2\rho v}\right)$$

Example:

Human "threshold of hearing" $10^{-12}\ \frac{W}{m^2}$

"threshold" of pain $1\ \frac{W}{m^2}$ $\rightarrow p_0 = \sqrt{2\rho v I} \doteq 30\ \text{Pa}$,

or 0.03% of 1 atm!

(7) Units :

$\log_{10} \frac{I}{I_{t.h.}}$ → logarithmic intensity in "Bells"
too crude...

$10 \log_{10} \frac{I}{I_{t.h.}}$ → log intensity in "decibells"
dB.

I	I log, dB	
10^{-12}	0	→ million times less Δp $(\sqrt{10^{12}})$
10^{-11}	10	
10^{-10}	20	
...		
10^{-1}	110 dB	– rock concert
$10^{0} = 1 \frac{W}{m^2}$	120 dB	– ouch!!!

↳ only 0.03% change in air pressure.

Special meaning: 3 dB

$$10 \log_{10} \frac{I_1}{I_2} = 3$$

$$I_1 = I_2 \cdot 10^{0.3} = I_2 \cdot 1.995 \approx I_2 \cdot 2$$

Special values

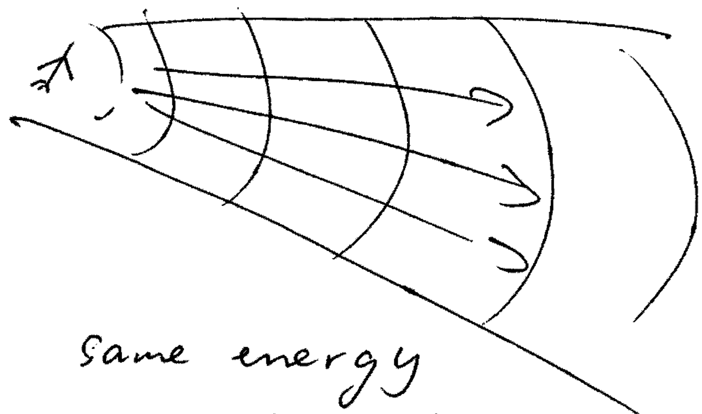
- 3 dB = "doubling of power"
- 6 dB = { "doubling of amplitude" / "quadrupling of power" }
- 20 dB = { ×100 power / ×10 amplitude } $(P \sim Amp^2)$
- 60 dB = { × million power / × thousand amplitude }

(8)

Example:

If one can't (almost) hear an airplane 35000 ft above, how close one can stand to one without feeling pain?

engines on!



same energy propagates outward, but the area $\sim r^2$

$$P = \text{const} = I \cdot A$$

$$I = \frac{P}{A} \sim \frac{1}{r^2} \quad \left(= \frac{\text{const}}{r^2} \right)$$

$$\frac{I_{T.P.}}{I_{T.H.}} = 10^{12} = \frac{\frac{\cancel{\text{const}}}{r_{T.P.}^2}}{\frac{\cancel{\text{const}}}{r_{T.H.}^2}} = \frac{r_{T.H.}^2}{r_{T.P}^2}$$

$$10^6 = \sqrt{10^{12}} = \sqrt{\frac{r_{T.H.}^2}{r_{T.P.}^2}} = \frac{r_{T.H.}}{r_{T.P}} = 1 \text{ million}$$

$$r_{T.P.} = \frac{r_{T.H.}}{10^6} = \frac{35000}{1000000} = 0.035 \text{ ft}$$

$$\approx 1 \text{ cm.}$$