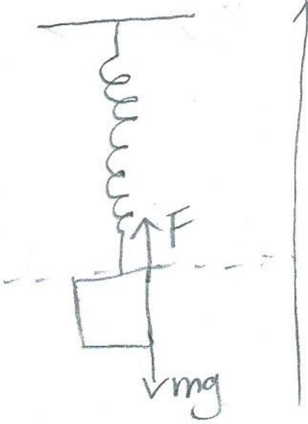
In the case of vertical spring.



$$F + mg = -kx$$

$x = 0$
at this point
$F = -mg$, so
$F + mg = 0$.

Energy Conservation in Oscillatory motion.

$$\text{Energy} = \text{Kinetic} + \text{Potential} = K + U$$
$$= \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

- In ideal system, the total energy is conserved, $K + U = \text{constant}$.
- When $K = \max$, $U = 0$. When $U = \max$, $K = 0$.

$$\text{Energy} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kx_{max}^2 + 0 = \frac{1}{2}mv_{max}^2 + 0.$$

So, $\frac{1}{2}mv^2 = \frac{1}{2}kx_{max}^2 - \frac{1}{2}kx^2$

$$v^2 = \frac{k}{m}(x_{max}^2 - x^2)$$

$x = x_{max}\cos(\omega t)$

or

$x = x_{max}\sin(\omega t)$

$$= \frac{k}{m}(x_{max}^2 - x_{max}^2\sin^2(\omega t))$$
$$= \frac{k}{m}x_{max}^2(\underbrace{1 - \sin^2(\omega t)}_{= \cos^2 \omega t})$$

or

$$= \frac{k}{m}(x_{max}^2 - x_{max}^2\cos^2(\omega t))$$
$$= \frac{k}{m}x_{max}^2(1 - \cos^2(\omega t))$$
$$= \sqrt{\frac{k}{m}x_{max}^2} \cdot \sin(\omega t)$$

$$v = \sqrt{\frac{k}{m}} \cdot x_{max} \cdot \cos(\omega t)$$
$$= \omega x_{mas} \cos(\omega t)$$