Correlation Example[1]

Time since Ph.D.			**Publications**			
X	$X-\bar{X}$	$(X-\bar{X})^2$	Y	$Y-\bar{Y}$	$(Y-\bar{Y})^2$	$(X-\bar{X})(Y-\bar{Y})$
3	-4.67	21.81	**18**	-1.93	3.74	9.02
6	-1.67	2.79	**3**	-16.93	286.74	28.22
3	-4.67	21.81	**2**	-17.93	321.60	83.69
8	0.33	0.11	**17**	-2.93	8.60	-0.98
9	1.33	1.77	**11**	-8.93	79.80	-11.91
6	-1.67	2.79	**6**	-13.93	194.14	23.22
16	8.33	69.39	**38**	18.07	326.40	150.56
10	2.33	5.43	**48**	28.07	787.74	65.49
2	-5.67	32.15	**9**	-10.93	119.54	61.96
5	-2.67	7.13	**22**	2.07	4.27	-5.51
5	-2.67	7.13	**30**	10.07	101.34	-26.84
6	-1.67	2.79	**21**	1.07	1.14	-1.78
7	-0.67	0.45	**10**	-9.93	98.67	6.62
11	3.33	11.09	**27**	7.07	49.94	23.56
18	10.33	106.71	**37**	17.07	291.27	176.36

$\bar{X} = 7.67$ $\quad \sum(X-\bar{X})^2 = 293.33$ $\quad \bar{Y} = 19.93$ $\quad \sum(Y-\bar{Y})^2 = 2674.93$ $\quad \sum(X-\bar{X})(Y-\bar{Y}) = 581.67$

$$r = \frac{\sum(X-\bar{X})(Y-\bar{Y})}{\sqrt{\sum(X-\bar{X})^2\sum(Y-\bar{Y})^2}} = \frac{581.67}{\sqrt{(293.33)(2674.93)}} = .66$$

$$t = \frac{r\sqrt{N-2}}{\sqrt{1-r^2}} = \frac{.66\sqrt{13}}{\sqrt{1-.66^2}} = \frac{2.38}{.75} = 3.17$$

$df = N-2 = 15-2 = 13$, $t_{crit,\alpha=.05} = 2.160$

Confidence limits seem to be rarely reported, perhaps because software packages have often not computed them in the past and perhaps because they are a bit complicated to calculate than other types of confidence intervals.[2] They are useful for having a sense of sampling variability of the sample correlation estimate, however, and should probably be used more often. To compute them, the r value must be converted to a z value first, then the intervals are calculated, and then the interval values are converted back to the r scale. You will not need to compute these by hand, but just for illustrating, I have included the computation ($\ln$ is the natural log function and e is Euler's mathematical constant).

$$z = .5\left[\ln\left(\frac{1+r}{1-r}\right)\right] = .5\left[\ln\left(\frac{1+.66}{1-.66}\right)\right] = .787 \qquad SE_z = \sqrt{\frac{1}{n-3}} = \sqrt{\frac{1}{15-3}} = .289$$

$$z \pm (1.96)(SE_z) = .874 \pm (1.96)(.289) = .222, 1.353$$

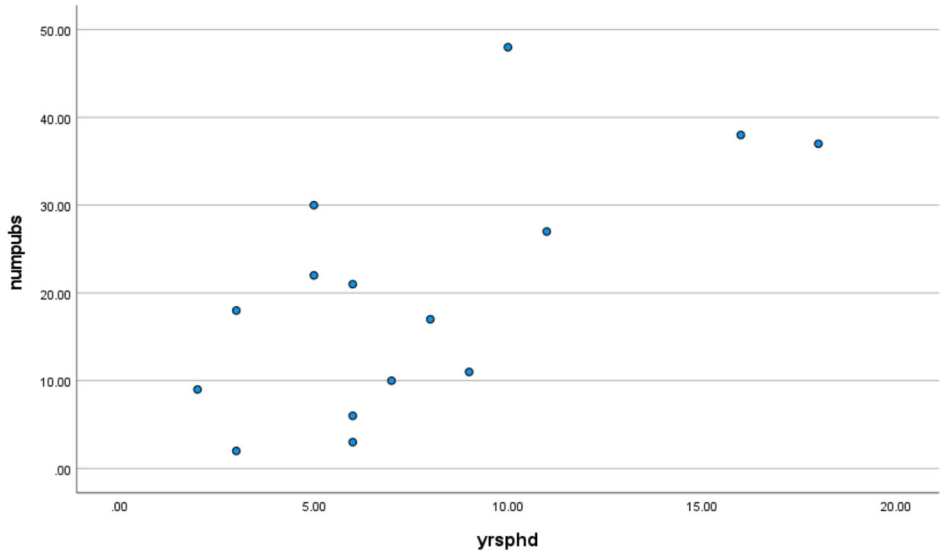
$$r = \frac{e^{2z}-1}{e^{2z}+1} = \frac{e^{2\times.222}-1}{e^{2\times.222}+1} = \frac{.559}{2.559} = .218 \qquad r = \frac{e^{2z}-1}{e^{2z}+1} = \frac{e^{2\times1.353}-1}{e^{2\times1.353}+1} = \frac{13.969}{15.969} = .875$$

[1] Numerical example from Cohen, Cohen, West, and Aiken (2003) Table 2.2.2.
[2] Until the most recent version of SPSS, confidence intervals for correlations were not available through the CORRELATIONS procedure. They are available now, so I don't anticipate that you will ever need to compute confidence intervals manually. But just in case the occasion arises, I have created an Excel spreadsheet that will do these computations automatically here: http://web.pdx.edu/~newsomj/uvclass/correlation confidence intervals.xlsx.

SPSS Syntax

```
graph
  /scatterplot(bivar)=numpubs with yrsphd.

 correlations vars=yrsphd numpubs
       /ci.
```

Correlations



Correlations

		yrsphd	numpubs
yrsphd	Pearson Correlation	1	.657
	Sig. (2-tailed)		.008
	N	15	15
numpubs	Pearson Correlation	.657	1
	Sig. (2-tailed)	.008	
	N	15	15

Confidence Intervals

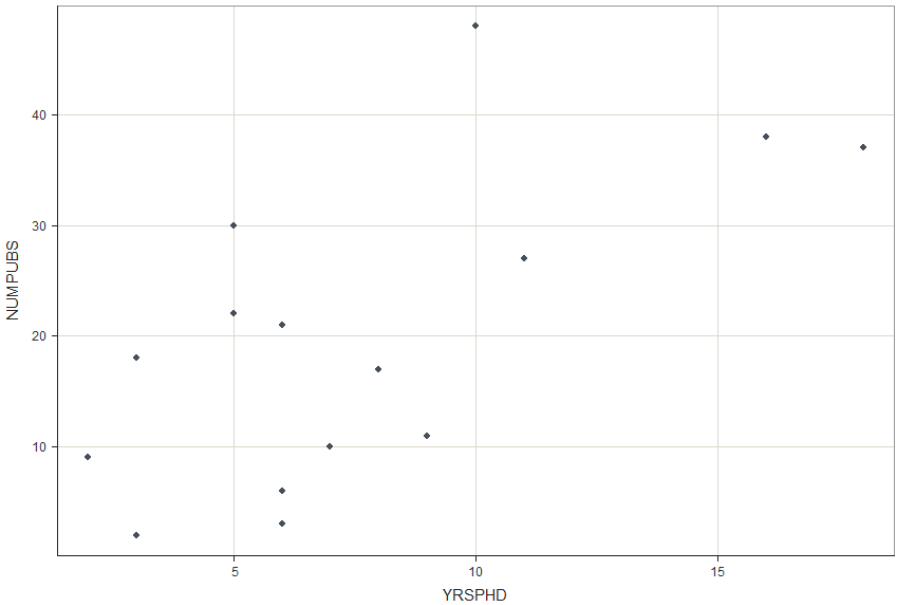
	Pearson Correlation	Sig. (2-tailed)	95% Confidence Intervals (2-tailed)[a]	
			Lower	Upper
yrsphd - numpubs	.657	.008	.218	.875

a. Estimation is based on Fisher's r-to-z transformation.

R

```
> #clear active frame from previous analyses
> rm(d)
> #clear console
> cat("\014")
> library(haven)
> d = read_sav("c:/jason/spsswin/uvclass/ccwa2_2_2.sav")
>
> #add digits to get more digits for CIs
> Correlation(YRSPHD, NUMPUBS, digits_d = 3, data=d)
Correlation Analysis for Variables YRSPHD and NUMPUBS

--- Pearson's product-moment correlation ---

Number of paired values with neither missing, n = 15
Number of cases (rows of data) deleted: 0

Sample Covariance: s = 41.548

Sample Correlation: r = 0.657
```

```
Hypothesis Test of 0 Correlation:  t = 3.139,  df = 13,  p-value = 0.008
95% Confidence Interval for Correlation:  0.218 to 0.875

> #an alternative method
> #base R function
> #cor.test(d$YRSPHD,d$NUMPUBS)
>
> #scatterplot (this function also reports the correlation)
> ScatterPlot(YRSPHD, NUMPUBS)
```



Example Write-up

A Pearson correlation coefficient was computed to examine the relationship between the amount of time since a faculty member received a PhD and the number of peer-reviewed publications. There was a significant positive correlation between the time since receiving the PhD and the number of publications, r = .66, p = .008, 95% [CI = .22, .88]. Approximately 44% of the variance was shared between the two variables, r^2 = .44.