Wright's Rules of Tracing and the Fundamentals of Path Analysis

Sewall Wright (1918, 1934) developed a method of estimating causal path coefficients by decomposing the correlations among a set of variables. He articulated a set of rules for examining a path diagram that would allow for this mathematical decomposition. The correlation of any two variables in a path diagram can be expressed as the sum of coefficients that connect the two variables. The connection between one variable and another variable, then, can often be made through more than one route.

1) No loops are allowed. In tracing from one variable to another, you cannot pass through the same variable twice following a particular route.

2) No going forward and then backward. Once you have traveled along a route forward, you cannot travel backward to get to the variable at the end point. (Only use common causes to account for a correlation between two variables, not common outcomes).

3) Only one curved arrow is allowed in tracing from the first variable to the last variable in any route.

I present path analysis in terms of correlations and standardized path coefficients, but it is not necessary to use correlations and standardized values, only simpler. In fact, in actual practice, estimating structural models using covariances instead of correlations is standard (and better practice).

**Tracing Example**

In the above diagram, lower case letters represent values of the coefficients. For curved arrows, they represent correlations; for straight arrows, they represent regression coefficients; and for the short arrows, they represent the value of the error or disturbance.

By using Wright's tracing rules, one can obtain the value of the correlation between any two variables, by adding up the values of the coefficients that link them--provided the rules are followed. Every route between the two variables (acceptable according to the rules) is traced. The coefficients that make up each route are multiplied. If there are multiple routes that link the two variables, products for each route are added together.

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1 Note that these rules are often worded differently by different authors and I provide a paraphrased and reorganized version. Also, use of the RAM path model notation may be confusing at times (e.g., variables with circular arrows).

To find the correlation between \( B \) and \( D \) (or \( r_{BD} \)), two routes are possible: \( b \) and \( fa \). So, the value of \( r_{BD} \) is equal to \( b + fa \). If \( b = .3 \), \( f = .2 \), and \( a = .4 \), then 
\[
r_{BD} = .3 + (.2)(.4) = .38.
\]

- \( r_{CD} = gb + ha \).
- \( r_{AB} \) is simply equal to \( f \), because the curved arrow represents a correlation.
- \( r_{AE} = ad + fbd + hc \)

Some examples of rule violations:
- \( r_{CD} \neq cd \) (no forward and back—no common effects, only common causes)
- \( r_{AC} \neq fg \) (no two double head arrows)
- \( r_{AB} \neq abf \) (no loops)

**Decomposing the Correlation Matrix**

Although this seems like just a fun little game, it turns out to be immensely useful. Instead of computing the correlations between two variables, one can work backward from the correlations to derive the path coefficients. Taking a simple path diagram representing a two variable regression model, and assuming some values for the correlations between our three variables, we can derive the path coefficients.

Assume that \( r_{12} = .50 \), \( r_{1Y} = .65 \), and \( r_{2Y} = .70 \).

![Path Diagram](image)

From our tracing rules, we know \( r_{1Y} = \beta_1 + r_{12}\beta_2 \) and \( r_{2Y} = \beta_2 + r_{12}\beta_1 \). We can plug in the known values and solve for \( \beta_1 \) and \( \beta_2 \).

\[
.65 = \beta_1 + .50\beta_2 \\
.70 = \beta_2 + .50\beta_1
\]

by rearranging, substituting, and solving for \( \beta_1 \) and then \( \beta_2 \), we get \( \beta_1 = .4 \) and \( \beta_2 = .5 \). These coefficients are the standardized regression coefficients, because we started with a correlation matrix (i.e., standardized variables).

The disturbance term, \( e \), is the amount of unaccounted for variance in \( Y \) and is equal to \( \sqrt{1 - R^2} \). We know from regression analysis that \( R^2 = \beta_1^2 + \beta_2^2 + \beta_1\beta_2 r_{12} \). So, \( R^2 = .4^2 + .5^2 + .5(.4)(.5) = .61 \).

The disturbance term then equals \( e = \sqrt{1-.61} = .62 \). What we have just done is decompose the correlation matrix into unique values for the coefficients that are implied by the model we specified.
References and Further Reading