Invariance Tests in Multigroup SEM

Factor invariance testing investigates the measurement properties of a factor or factors across groups.[1] Often the end goal is psychometric, to assess the equivalence of the factor structure or the quality of the items across groups for the aim of determining whether a measure is biased, will translate well, is equally interpretable, or is differentially reliable. Many possibilities exist for group comparisons that may be of interest to determine whether measurement properties are invariant, including comparisons based on race, nationality, sexual orientation, experimental conditions, or educational sectors, to name just a few. Invariance also is often seen as an initial step prior to investigating larger multigroup models, with the goal of ensuring equivalent measurement across groups to rule out measurement artifacts when comparing means or prediction across groups. The general approach to factorial invariance tests is to use likelihood ratio tests that compare nested models in which one model in which that set of parameters (e.g., loadings) is constrained to be equal across groups to another model in which that set of parameters is allowed to be estimated freely and separately across groups. A non-significant test established equality across groups, and thus, some aspect of factorial invariance.

Factorial invariance testing is a complex and detailed topic that requires more space for discussion than is available here. The issues discussed in this handout only scratch the surface of the relevant issues for multigroup SEM. Comparisons with categorical indicators (and threshold constraints), comparisons with nonnormal and missing data, and comparisons across more than two groups are applications that are relatively simple to employ drawing from other content discussed in the course. But given time limitations, I will not be able to go into much detail on these applications. Some of the issues have not been thoroughly considered in the literature yet, but many of them are discussed elsewhere (Millsap, 2011; Widaman & Olivera-Aguilar, 2023).

There are a plethora of systems and terminologies for classifying different aspects of invariance, which is often a great source of confusion for a complex topic to begin with. Essentially, all systems boil down to comparison of different sorts of parameters across groups—loadings (Λ), measurement errors (Θ_ε), factor variances and covariances (ψ), factor means (α), and measurement intercepts (ν). Although far from universally used, Meredith's (1993) classification terminology for these different sets of parameters is perhaps most widely used. He used *weak factorial invariance* to refer to establishing that loadings are equal across groups, *strong factorial invariance* to refer to establishing that loadings and measurement intercepts are equal across groups, *strict factorial invariance* to refer to establish that loadings, measurement intercepts, and measure residual variances are equal across groups, and *structural invariance* to refer to establishing that loadings, measurement intercepts, measurement residual variances, factor variances, and factor means are equal across groups. The figure on the next page is a visual illustration of each of the definitions.

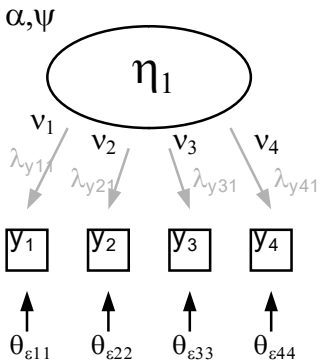
The question often arises about which level of factorial invariance is needed. The answer depends on what analyses are planned for the measure or what the measure will be used for in the future. To demonstrate that a measure has equal reliability across groups, one would need to establish that the loadings and the measurement residual variances are equal across groups. This goal is relevant to the extent that the measure is likely to be used as a composite index in future research. To the extent that either of these parameters differ and, hence, the reliability of the measure differs across groups, differences in prediction (e.g., interactions, multigroup models) across groups could be an artifact of differences in measurement reliability. If subsequent analyses use the measure as a latent variable, differences in measurement residual variances will not impact inferences about group differences in prediction as long as the loading are equal across groups. If subsequent analyses are to involve mean comparisons, intercepts should be equal across groups to avoid conclusions about group differences unduly influenced an individual or subset of items. Factor variance differences may lead to problems with heterogeneity of variance assumptions but may have no severe consequences for many structural models in which the assumption can be relaxed. Factor mean differences are typically involved in the substantive hypotheses when means are being compared, so do not seem like a psychometric concern.

[1] Longitudinal invariance testing is a related topic that shares many of the same rationales, strategies, and issues. See Bontempo and Hofer (2007), Millsap and Cham (2011), and Newsom (2024, Chapter 2) for introductions.

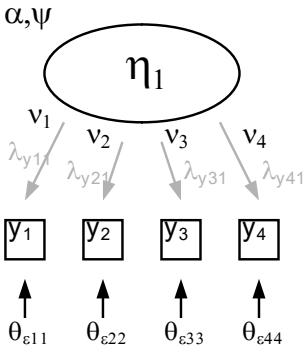
Illustration using Meredith's (1993) Terminology

Weak Factorial Invariance

Group A

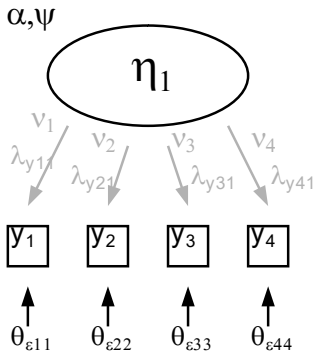


Group B

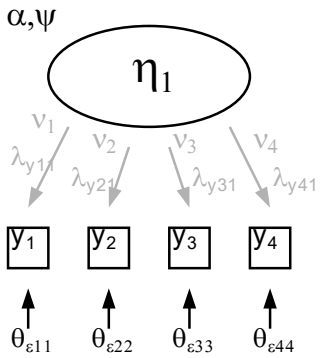


Strong Factorial Invariance

Group A

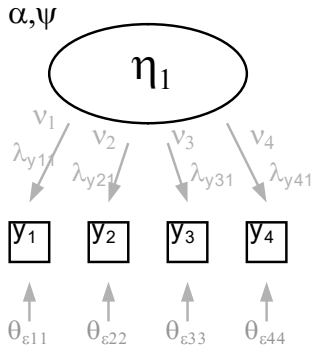


Group B

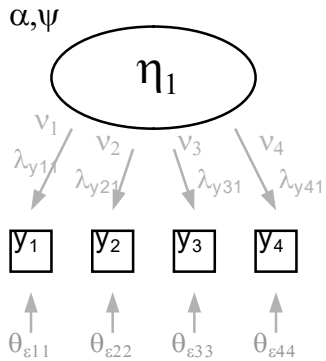


Strict Factorial Invariance

Group A

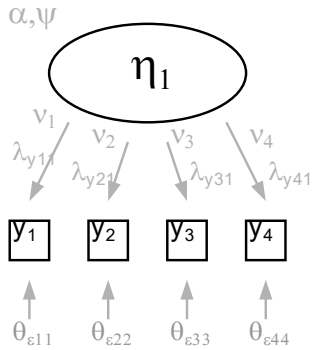


Group B

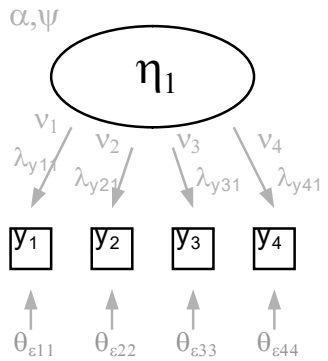


Structural Invariance

Group A



Group B

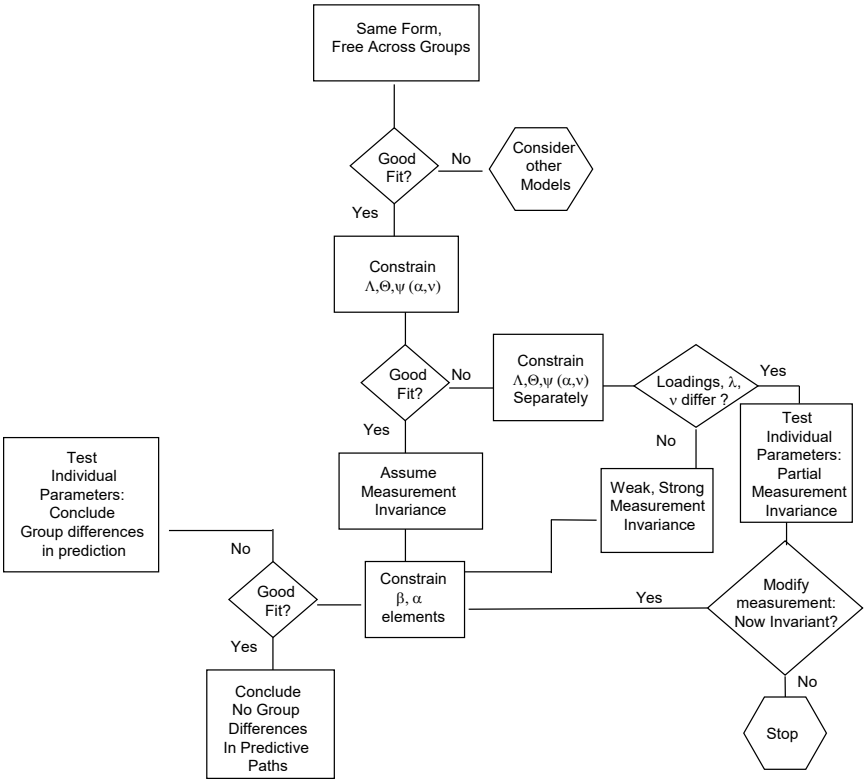


Note: Grayed elements represent equality constraints across groups. η is the factor, α is the factor mean, ψ is the factor variance, ν ("nu") is the loading intercept, λ is the factor loading, θ_ε is the measurement residual variance (I use the matrix element symbol instead of just ε in my figure to emphasize that the equality test is a comparison of variances).

One Suggested Process

One source of debate or inconsistency in the literature involves what the best process for conducting tests should be. There may be more than one "good" process for conducting variances tests. The general strategy illustrated in the flow chart below follows what Stark and colleagues (Stark, Chernyshenko, & Drasgow, 2006) call a *free baseline approach*, which makes sense to me. The logic of this approach is that if the model does not fit when there are no cross-group constraints placed on it, rejection or modification of the general model is required. In contrast, one can propose a *constrained baseline approach* in which all parameters are constrained across groups first (which is the same as a single group model). Neither approach is right or wrong per se, but they have different rationales and strengths.

Following the free baseline approach, comparisons should be made between a more constrained model and the baseline model (Bentler, 2000). An omnibus approach is usually used in which classes of parameters (e.g., loadings) are constrained simultaneously. The general overall idea is to establish measurement equivalence before comparing predictive paths across groups to avoid confounding group differences in measurement properties with substantive differences in means or predictive paths across groups. As long as SEM is used to assess differences in prediction across groups (and not mean comparisons), weak invariance across groups should be sufficient, because the differences in measurement residuals across groups should not affect relations among latent variables as long as the measurement residual variances are allowed to freely vary across groups. (And, in fact, constraining measurement residuals to be equal across groups when they are truly not equal will lead to biases in the predictive paths). On the other hand, invariance of loadings and measurement residual variances will be required if the goal is to compare groups in subsequent analyses using a composite index of the items, because group differences in the amount of measurement error across groups can impact the results (Millsap & Kwok, 2004). Measurement intercept invariance should be established if the goal is to compare means across groups, but because factor means are a function of item intercepts as well as item loadings, both loadings and intercepts (strong factorial invariance) should be invariant to fairly compare factor means and loadings, intercepts, and measurement residual variances (strict factorial invariance) would be need to compare means with a composite of the items.



Finding the Specific Source of Measurement Invariance

It is important to realize that testing only a subset of loadings or intercepts for measurement invariance (sometimes "partial invariance"; Byrne, Shavelson, & Muthen, 1989) can be problematic, because there is an interdependence of loadings and factor variances (or similarly factor intercepts and factor means). This makes sense if you remember that the factor variance is altered when a different indicator is chosen as the referent. For testing factorial invariance on only a subset of loadings (or other parameters) for a factor, special procedures are needed to be ensure that the choice of referent indicator does not obscure the correct identification of the specific indicators that differ (see Cheung & Lau, 2012; Yoon & Millsap, 2007). Cheung and Lau propose a complex but logical system of equal constraints using ratios of loadings. The method requires software that allows for complex equality constraints to be defined when the model is tested.

Means

Mean comparisons (of factors, α, or intercepts, ν) may not always be of interest, but when they are important, factorial invariance of the measurement intercepts should be examined. Two cases in which intercept invariance should be established: 1) bias may be introduced because groups are combined or assumed equivalent in later analyses, and 2) mean differences between groups are of substantive interest (analogous to t-test or ANOVA comparisons). In other cases, in which the researcher is interested in examining predictive differences between groups, one would not necessarily assume that group differences on the mean of an independent or dependent variable would affect associations with other variables within a group. Loadings should always be constrained equal across groups when comparing means, because factor means are a function of the measurement intercept and the loading (see Newsom, 2024, Chapter 1).

Multigroup SEMs with Multiple Factors

Multifactor models present additional complications. Structural relations between a set of predictors and an outcome will depend on correlations among the predictors, for example. So, in order to meaningfully interpret differences in prediction across groups, one would normally want to assume equivalence in the correlations among the predictors. Finally, with large sample sizes, significant differences may be found for very small magnitude differences, and the researcher needs to decide which differences are important. Calculating magnitude of the difference is encouraged (Cheung & Rensvold, 2002; Fan & Sivo, 2010). As outlined in the handout *Nested Models, Model Modifications, and Correlated Errors*, one can use relative fit indices such as the CFI or McDonald's noncentrality index or Cohen's w to evaluate how large the chi-square difference is. As a separate matter, the mean differences across groups can be evaluated in terms of effects size as well, using Cohen's standardized effect size d, for instance (Hancock, 2001).

References and Recommended Readings

Bentler, P.M. (2000). Rites, wrongs, and gold in model testing. *Structural Equation Modeling, 7*, 82-91.

Bontempo, D. E., & Hofer, S. M. (2007). Assessing factorial invariance in cross-sectional and longitudinal studies. In A.D. Ong & M. van Dulmen (Eds.), *Handbook of methods in positive psychology* (pp. 153-175). Oxford University Press.

Byrne, B.M., Shavelson, R.J., & Muthen, B. (1989). Testing for the equivalence of factorial covariance and mean structures: The issue of partial measurement invariance. *Psychological Bulletin, 105*, 456-466.

Cheung, G. W. & Lau, R. S. (2012). A direct comparison approach for testing measurement invariance. Organizational

Cheung, G.W. & Rensvold, R.B. (1999). Testing factorial invariance across groups: A reconceptualization and proposed new method. Journal of Management, 25, 1-27.

Cheung, G W., & Resnvold, R. B (2002). Evaluating Goodness of fit indexes for testing measurement invariance. Structural Equation Modelling: A Multidisciplinary Journal, 9, 233–255.

Choi, J., Fan, W., & Hancock, G. R. (2009). A note on confidence intervals for two-group latent mean effect size measures. *Multivariate behavioral research, 44(*3), 396-406.

Fan, X., & Sivo, S. (2009). Using _goodness-of-fit indices in assessing mean structure Invariance. *Structural Equation Modeling: A Multidisciplinary Journal, 16*, 54–69.

Gabler. Millsap, R.E. (2011). Statistical Approaches to Measurement Invariance. New York, NY: Routledge Academic.

Hancock, G. R. (200 I). Eflect size, power, and sample size determination for structured means modeling and mimic approaches to between-groups hypothesis testing of means on a single latent construct. *Psychometrika, 66*, 373-388.

Kim, E.S., & Yoon, M. (2011). Testing Measurement Invariance: A Comparison of Multiple-Group Categorical CFA and IRT. Structural Equation Modeling, 18(2), 212-228.

Kim, J. O., & Ferree, G. D., Jr. (1981). Standardization in causal analysis. *Sociological Methods and Research, 10*, 187-210.

Meredith, W. (1993). Measurement invariance, factor analysis, and factorial invariance. *Psychometrika, 58*, 525-543.

Millsap, R. E. (1998). Group differences in regression intercepts: Implications for factorial invariance. *Multivariate Behavioral Research, 33,* 403-424.

Millsap, R.E. (2011). *Statistical Approaches to Measurement Invariance.* New York: Routledge. Research Methods, 15(2), 167-198

Millsap, R. E. & Cham, H. (2011). Investigating factorial invariance in longitudinal data. In B. Laursen, T. D. Little, and N. A. Card (Eds.), *Handbook of developmental research methods*. New York: Guilford.

Millsap, R. E., & Kwok, O.M. (2004). Evaluating the impact of partial measurement invariance on selection in two populations. *Psychological Methods, 9*, 93-115.

Millsap, R. E, & Yun-Tein, J. (2004). Assessing Factorial Invariance in Ordered-Categorical Measures. *Multivariate Behavioral Research., 39, 479-515.*
Newsom, J.T. (2024). *Longitudinal Structural Equation Modeling: A Comprehensive Introduction, second edition*. New York: Routledge.
Sass, D.A. (2011). Testing Measurement Invariance and Comparing Latent Factor Means Within a Confirmatory Factor Analysis Framework. Journal of Psychoeducational Assessment, 29(4), 347-363.
Stark, S., Chernyshenko, O. S., & Drasgow, F. (2006). Detecting differential item functioning with confirmatory factor analysis and item response theory: Toward a unified strategy. *Journal of Applied Psychology, 91*, 1292.
Temme, D. (2006). Assessing measurement invariance of ordinal indicators in cross-national research. In S. Diehl and R.
Terlutter (Eds.), International Advertising and Communication: Current Insights and Empirical Findings (pp. 455-472).
Vandenberg, R.J., & Lance, C.E. (2000). A review and synthesis of the measurement invariance literature: Suggestions, practices, and recommendations for organizational research. *Organizational Research Methods, 3,* 4-69.
Werts, C.E., Rock, D.A., Linn, R.L., & Joreskog, K.G. (1977). Validating psychometric assumptions within and between several populations. *Educational and Psychological Measurement, 37, 863-872.*
Widaman, K.F., & Olivera-Aguilar, M. (2023). Investigating measurement invariance using confirmatory factor analysis *(pp. 367-384).* In R.H. Hoyle, *Handbook of structural equation modeling, second edition.* Guilford.
Yoon, M., & Millsap, R. E. (2007). Detecting Violoations of Factorial Invariance Using Data-9 Based Specification Searches: A Monte Carlo Study. *Structural Equation Modeling:A Multidisciplinary Journal, 143, 435–463.*