Factorial Invariance Example[1]

To keep it simple, I illustrate a weak invariance test (loadings only) and I have requested no mean structure. The model is a one-factor CFA with five positive affect items, and widowed participants are compared to those who are not widowed. Degrees of freedom for a multigroup model are: $df = g\left[v(v+1)/2\right] - p$, where g is the number of groups, v is the number of measured variables, and p is the number of freely estimated parameters. So, for the free model (loadings allowed to be freely estimated in the two groups), with five measured variables and two groups, $df = 2\left[5(5+1)/2\right] - 16 = 2(15) - 20 = 10$. The 16 free parameters include 5 loadings in each group (10) and 5 residual variances in each group (10). Equality constraints are imposed by using the same label behind the loading in Mplus [e.g., `(ly1)`] and by referring to the same label for the two groups in parentheses preceding each variable in `lavaan` [e.g., (`ly1`)*]. When the five loadings are set equal across groups, four degrees of freedom are gained because a scaling constraint still has to be imposed to estimate the factor variance.

For invariance tests, I prefer to use the effects coding method of factor identification (Little, Slegers, & Card, 2006), which sets the average loading to 1 rather than setting either the first loading (referent or marker approach) or setting the factor variances. By setting a particular parameter to one, latter two methods always assume one additionally constrained parameter, either the first loading or the factor variance. To set the loadings to an average of 1, model constraints must be used, requiring that the first loading equal the number of loadings minus all of the other loadings, $\lambda_{11} = 5 - \lambda_{21} - \lambda_{31} - \lambda_{41} - \lambda_{51}$.

I also used listwise deletion in this example to keep the computation of the chi-square difference test straightforward, but I would usually recommend missing data estimation and robust estimation (e.g., MLR; the same size may be too small in the separate groups too). If robust adjustments are used, chi-square difference tests would need to weight by the scaling correction factor using the DIFFTEST process in Mplus or the Bryant-Satorra spreadsheet discussed earlier (see the handout "Examples of Chi-square Difference Tests with Nonnormal and Categorical Variables" for details).

All Parameters Free Across Groups

```
title:  Factorial invariance example;

  data:  file=C:\Jason\mplus\semclass\stack1.dat;  format=11f1.0;
      listwise=on;

  variable:  names = widow panas1 panas2 panas3 panas4 panas5
             panas6 panas7 panas8 panas9 panas10 ;
             grouping is widow (0=notwidow,1=widow);

             missing = blank;

             usevariables = panas1-panas5;

  analysis:  type=general; iterations = 200;
    model=nomeanstructure; information=expected;

  model:  posaff by panas1* panas2-panas5;

    !  Note:  by default in Mplus, measurement errors and factor correlations are not
    !   constrained to be equal across groups;

    Model notwidow:
      posaff by panas1* (ly1)
                panas2 (ly2)
                panas3 (ly3)
                panas4 (ly4)
                panas5 (ly5);
      posaff;
```

[1] Note: I only illustrate some of the tests that may be needed, but there are other possible tests that may be desirable (e.g., all parameters equal vs. all parameters free, factor variance comparisons, or other specific paths). For brevity sake, I do not illustrate comparison of variances, covariances, or mean and intercept comparisons. See the handout "Invariance Tests in Multigroup SEM" for an overview. By default in Mplus Version 6 and later, analyses with mean structures set the intercepts to zero in the first group and allow them to be freely estimated in the second group.

```
    Model widow:
      posaff by panas1* (ly6)
                panas2 (ly7)
                panas3 (ly8)
                panas4 (ly9)
                panas5 (ly10);
      posaff;

  Model constraint:
      ly1 = 5 - ly2 - ly3 -ly4 - ly5;
      ly6 = 5 - ly7 - ly8 -ly9 - ly10;

  output:  stdyx ;

INPUT READING TERMINATED NORMALLY



Factorial invariance example ;

SUMMARY OF ANALYSIS

Number of groups                                                 2
Number of observations
   Group NOTWIDOW                                              161
   Group WIDOW                                                  41
   Total sample size                                           202

Number of dependent variables                                    5
Number of independent variables                                  0
Number of continuous latent variables                            1

Observed dependent variables

  Continuous
   PANAS1      PANAS2      PANAS3      PANAS4      PANAS5

Continuous latent variables
   POSAFF

Variables with special functions

  Grouping variable     WIDOW

Estimator                                                       ML
Information matrix                                        EXPECTED
Maximum number of iterations                                   200
Convergence criterion                                    0.500D-04
Maximum number of steepest descent iterations                   20

Input data file(s)
  C:\Jason\mplus\semclass\stack1.dat

Input data format
  (11F1.0)


MODEL FIT INFORMATION

Number of Free Parameters                       20

Loglikelihood

          H0 Value                        -921.811
          H1 Value                        -901.383

Information Criteria

          Akaike (AIC)                    1883.622
          Bayesian (BIC)                  1949.787
          Sample-Size Adjusted BIC        1886.423
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                             40.855
          Degrees of Freedom                    10
```

```
          P-Value                            0.0000

Chi-Square Contribution From Each Group

          NOTWIDOW                          31.578
          WIDOW                              9.277

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.175
          90 Percent C.I.                    0.121  0.232
          Probability RMSEA <= .05           0.000

CFI/TLI

          CFI                                0.969
          TLI                                0.938

Chi-Square Test of Model Fit for the Baseline Model

          Value                           1018.190
          Degrees of Freedom                    20
          P-Value                            0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                              0.028

MODEL RESULTS
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Group NOTWIDOW

 POSAFF   BY
    PANAS1             0.907      0.039     23.134      0.000
    PANAS2             1.061      0.054     19.703      0.000
    PANAS3             0.967      0.037     25.918      0.000
    PANAS4             1.048      0.037     28.394      0.000
    PANAS5             1.017      0.044     22.943      0.000

 Variances
    POSAFF             0.669      0.080      8.400      0.000

 Residual Variances
    PANAS1             0.179      0.025      7.310      0.000
    PANAS2             0.414      0.052      8.002      0.000
    PANAS3             0.152      0.023      6.716      0.000
    PANAS4             0.138      0.023      6.074      0.000
    PANAS5             0.249      0.033      7.470      0.000

Group WIDOW

 POSAFF   BY
    PANAS1             1.022      0.039     26.396      0.000
    PANAS2             1.172      0.062     18.856      0.000
    PANAS3             0.970      0.054     17.890      0.000
    PANAS4             0.988      0.056     17.786      0.000
    PANAS5             0.848      0.073     11.557      0.000

 Variances
    POSAFF             1.116      0.255      4.383      0.000

 Residual Variances
    PANAS1             0.042      0.022      1.913      0.056
    PANAS2             0.228      0.059      3.847      0.000
    PANAS3             0.157      0.041      3.851      0.000
    PANAS4             0.168      0.043      3.871      0.000
    PANAS5             0.325      0.076      4.295      0.000


STANDARDIZED MODEL RESULTS


STDYX Standardization

                                                     Two-Tailed
```

```
                    Estimate       S.E.  Est./S.E.    P-Value

Group NOTWIDOW

 POSAFF   BY
    PANAS1             0.868      0.022     38.769      0.000
    PANAS2             0.803      0.031     26.185      0.000
    PANAS3             0.897      0.019     47.818      0.000
    PANAS4             0.917      0.016     56.261      0.000
    PANAS5             0.858      0.024     36.066      0.000

 Variances
    POSAFF             1.000      0.000    999.000    999.000

 Residual Variances
    PANAS1             0.246      0.039      6.321      0.000
    PANAS2             0.355      0.049      7.214      0.000
    PANAS3             0.195      0.034      5.799      0.000
    PANAS4             0.159      0.030      5.301      0.000
    PANAS5             0.265      0.041      6.486      0.000

Group WIDOW

 POSAFF   BY
    PANAS1             0.982      0.010     98.103      0.000
    PANAS2             0.933      0.022     41.705      0.000
    PANAS3             0.933      0.022     41.541      0.000
    PANAS4             0.931      0.023     40.606      0.000
    PANAS5             0.844      0.047     18.073      0.000

 Variances
    POSAFF             1.000      0.000    999.000    999.000

 Residual Variances
    PANAS1             0.035      0.020      1.772      0.076
    PANAS2             0.130      0.042      3.105      0.002
    PANAS3             0.130      0.042      3.108      0.002
    PANAS4             0.133      0.043      3.124      0.002
    PANAS5             0.288      0.079      3.654      0.000


R-SQUARE

Group NOTWIDOW

    Observed                                        Two-Tailed
    Variable        Estimate       S.E.  Est./S.E.    P-Value

    PANAS1             0.754      0.039     19.384      0.000
    PANAS2             0.645      0.049     13.092      0.000
    PANAS3             0.805      0.034     23.909      0.000
    PANAS4             0.841      0.030     28.131      0.000
    PANAS5             0.735      0.041     18.033      0.000

Group WIDOW

    Observed                                        Two-Tailed
    Variable        Estimate       S.E.  Est./S.E.    P-Value

    PANAS1             0.965      0.020     49.052      0.000
    PANAS2             0.870      0.042     20.853      0.000
    PANAS3             0.870      0.042     20.771      0.000
    PANAS4             0.867      0.043     20.303      0.000
    PANAS5             0.712      0.079      9.037      0.000
```

Loadings Only Constrained Equal Across Groups

Only unstandardized loadings shown to save space

```
 title:  Factorial invariance example ;

  data:  file=C:\Jason\mplus\semclass\stack1.dat;  format=11f1.0;
      listwise=on;

  variable:  names = widow panas1 panas2 panas3 panas4 panas5
             panas6 panas7 panas8 panas9 panas10 ;
             grouping is widow (0=notwidow,1=widow);
```

```
             missing = blank;

             usevariables = panas1-panas5;

  analysis:  type=general; iterations = 200;
    model=nomeanstructure; information=expected;

  model:  posaff by panas1* panas2-panas5;

    !  Note:  by default in Mplus, measurement errors and factor correlations are not
    !   constrained to be equal across groups;

    Model notwidow:
      posaff by panas1* (ly1)
                panas2 (ly2)
                panas3 (ly3)
                panas4 (ly4)
                panas5 (ly5);
      posaff;


    Model widow:
      posaff by panas1* (ly1)
                panas2 (ly2)
                panas3 (ly3)
                panas4 (ly4)
                panas5 (ly5);
      posaff;

  Model constraint:
      ly1 = 5 - ly2 - ly3 -ly4 - ly5;

  output:  stdyx ;

Factorial invariance example ;

SUMMARY OF ANALYSIS

Number of groups                                                 2
Number of observations
   Group NOTWIDOW                                              161
   Group WIDOW                                                  41
   Total sample size                                           202

Number of dependent variables                                    5
Number of independent variables                                  0
Number of continuous latent variables                            1

Observed dependent variables

  Continuous
   PANAS1      PANAS2      PANAS3      PANAS4      PANAS5

Continuous latent variables
   POSAFF

Variables with special functions

  Grouping variable     WIDOW

Estimator                                                       ML
Information matrix                                        EXPECTED
Maximum number of iterations                                   200
Convergence criterion                                    0.500D-04
Maximum number of steepest descent iterations                   20

Input data file(s)
  C:\Jason\mplus\semclass\stack1.dat

Input data format
  (11F1.0)

MODEL FIT INFORMATION

Number of Free Parameters                        16

Loglikelihood
```

```
          H0 Value                        -926.082
          H1 Value                        -901.383

Information Criteria

          Akaike (AIC)                    1884.164
          Bayesian (BIC)                  1937.096
          Sample-Size Adjusted BIC        1886.405
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                             49.397
          Degrees of Freedom                    14
          P-Value                           0.0000

Chi-Square Contribution From Each Group

          NOTWIDOW                          35.010
          WIDOW                             14.388

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.158
          90 Percent C.I.                    0.112  0.207
          Probability RMSEA <= .05           0.000

CFI/TLI

          CFI                                0.965
          TLI                                0.949

Chi-Square Test of Model Fit for the Baseline Model

          Value                           1018.190
          Degrees of Freedom                    20
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                              0.065



MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Group NOTWIDOW

 POSAFF   BY
    PANAS1             0.960      0.027     35.266      0.000
    PANAS2             1.100      0.040     27.695      0.000
    PANAS3             0.959      0.030     31.654      0.000
    PANAS4             1.018      0.031     33.315      0.000
    PANAS5             0.963      0.038     25.269      0.000

 Variances
    POSAFF             0.671      0.080      8.426      0.000

 Residual Variances
    PANAS1             0.172      0.024      7.051      0.000
    PANAS2             0.402      0.051      7.900      0.000
    PANAS3             0.152      0.022      6.770      0.000
    PANAS4             0.147      0.023      6.393      0.000
    PANAS5             0.260      0.034      7.686      0.000

Group WIDOW

 POSAFF   BY
    PANAS1             0.960      0.027     35.266      0.000
    PANAS2             1.100      0.040     27.695      0.000
    PANAS3             0.959      0.030     31.654      0.000
    PANAS4             1.018      0.031     33.315      0.000
    PANAS5             0.963      0.038     25.269      0.000
```

```
 Variances
    POSAFF            1.176      0.267      4.410      0.000

 Residual Variances
    PANAS1            0.052      0.022      2.353      0.019
    PANAS2            0.229      0.059      3.865      0.000
    PANAS3            0.155      0.041      3.778      0.000
    PANAS4            0.166      0.044      3.737      0.000
    PANAS5            0.340      0.081      4.197      0.000
```

lavaan

All Parameters Free Across Groups *(some output deleted)*

```
> #all parameters free across groups
> model1 = '
+                posaff =~ c(NA,NA)*panas1 + c(ly1,ly6)*panas1 + c(ly2,ly7)*panas2 + c(ly3,ly8)*panas3 + c(ly4
,ly9)*panas4 + c(ly5,ly10)*panas5
+   panas1~~panas1
+        panas2~~panas2
+        panas3~~panas3
+        panas4~~panas4
+        panas5~~panas5
+        posaff~~posaff
+
+        #complex constraints
+       ly1 == 5 - ly2 - ly3 - ly4 - ly5
+       ly6 == 5 - ly7 - ly8 - ly9 - ly10
+       '
>
> #I resort back to listwise deletion to simplify examples and LR tests
> fit1 <- sem(model=model1, data = stack1, missing="listwise", estimator="ML", group="widow", meanstructure=FALS
E)
> summary(object=fit1,fit.measures=TRUE, rsquare=TRUE, standardized=TRUE)
lavaan 0.6.15 ended normally after 37 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        22
  Number of equality constraints                     2

  Number of observations per group:               Used       Total
    0                                              161         162
    1                                               41          43

Model Test User Model:
  Test statistic                                40.855
  Degrees of freedom                                10
  P-value (Chi-square)                           0.000
  Test statistic for each group:
    0                                           31.578
    1                                            9.277

Model Test Baseline Model:
  Test statistic                              1018.190
  Degrees of freedom                                20
  P-value                                        0.000

User Model versus Baseline Model:
  Comparative Fit Index (CFI)                    0.969
  Tucker-Lewis Index (TLI)                       0.938

Loglikelihood and Information Criteria:
  Loglikelihood user model (H0)               -921.811
  Loglikelihood unrestricted model (H1)       -901.383

  Akaike (AIC)                                1883.622
  Bayesian (BIC)                              1949.787
  Sample-size adjusted Bayesian (SABIC)       1886.423

Root Mean Square Error of Approximation:

  RMSEA                                          0.175
  90 Percent confidence interval - lower         0.121
  90 Percent confidence interval - upper         0.232
  P-value H_0: RMSEA <= 0.050                    0.000
  P-value H_0: RMSEA >= 0.080                    0.997

Standardized Root Mean Square Residual:
  SRMR                                           0.028

Parameter Estimates:
  Standard errors                             Standard
  Information                                 Expected
  Information saturated (h1) model         Structured
```

```
Group 1 [0]:

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  posaff =~
    panas1   (ly1)    0.907    0.039   23.134    0.000    0.742    0.868
    panas2   (ly2)    1.061    0.054   19.703    0.000    0.867    0.803
    panas3   (ly3)    0.967    0.037   25.918    0.000    0.791    0.897
    panas4   (ly4)    1.048    0.037   28.394    0.000    0.857    0.917
    panas5   (ly5)    1.017    0.044   22.943    0.000    0.832    0.858

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .panas1            0.179    0.025    7.310    0.000    0.179    0.246
   .panas2            0.414    0.052    8.002    0.000    0.414    0.355
   .panas3            0.152    0.023    6.716    0.000    0.152    0.195
   .panas4            0.138    0.023    6.074    0.000    0.138    0.159
   .panas5            0.249    0.033    7.470    0.000    0.249    0.265
    posaff            0.669    0.080    8.400    0.000    1.000    1.000

R-Square:
                   Estimate
    panas1            0.754
    panas2            0.645
    panas3            0.805
    panas4            0.841
    panas5            0.735


Group 2 [1]:

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  posaff =~
    panas1   (ly6)    1.022    0.039   26.396    0.000    1.080    0.982
    panas2   (ly7)    1.172    0.062   18.856    0.000    1.238    0.933
    panas3   (ly8)    0.970    0.054   17.890    0.000    1.025    0.933
    panas4   (ly9)    0.988    0.056   17.786    0.000    1.043    0.931
    panas5  (ly10)    0.848    0.073   11.557    0.000    0.896    0.844

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .panas1            0.042    0.022    1.913    0.056    0.042    0.035
   .panas2            0.228    0.059    3.847    0.000    0.228    0.130
   .panas3            0.157    0.041    3.851    0.000    0.157    0.130
   .panas4            0.168    0.043    3.871    0.000    0.168    0.133
   .panas5            0.325    0.076    4.295    0.000    0.325    0.288
    posaff            1.116    0.255    4.383    0.000    1.000    1.000

R-Square:
                   Estimate
    panas1            0.965
    panas2            0.870
    panas3            0.870
    panas4            0.867
    panas5            0.712

Constraints:
                                               |Slack|
    ly1 - (5-ly2-ly3-ly4-ly5)                    0.000
    ly6 - (5-ly7-ly8-ly9-ly10)                   0.000
```

Loadings Only Constrained Equal Across Groups *(some output deleted)*

```
> # test of all loadings equal (weak invariance)
> model2 = '
+        posaff =~ NA*panas1 + (ly1)*panas1 + (ly2)*panas2 + (ly3)*panas3 + (ly4)*panas4 + (ly5)*panas5
+   panas1~~panas1
+        panas2~~panas2
+        panas3~~panas3
+        panas4~~panas4
+        panas5~~panas5
+        posaff~~posaff
+        #complex constraints
+       ly1 == 5 - ly2 - ly3 - ly4 - ly5
+  '
>
> #I resort back to listwise deletion to simplify examples and LR tests
> fit2 <- sem(model=model2, data = stack1, missing="listwise", estimator="ML", group="widow", group.equal=c("loa
dings"),meanstructure=FALSE)
> summary(object=fit2,fit.measures=TRUE, rsquare=TRUE, standardized=TRUE)
lavaan 0.6.15 ended normally after 33 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        22
```

```
  Number of equality constraints                     6

  Number of observations per group:               Used       Total
    0                                              161         162
    1                                               41          43

Model Test User Model:

  Test statistic                                49.397
  Degrees of freedom                                14
  P-value (Chi-square)                           0.000
  Test statistic for each group:
    0                                           35.010
    1                                           14.387

Model Test Baseline Model:

  Test statistic                              1018.190
  Degrees of freedom                                20
  P-value                                        0.000

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    0.965
  Tucker-Lewis Index (TLI)                       0.949

Loglikelihood and Information Criteria:
  Loglikelihood user model (H0)               -926.082
  Loglikelihood unrestricted model (H1)       -901.383

  Akaike (AIC)                                1884.164
  Bayesian (BIC)                              1937.096
  Sample-size adjusted Bayesian (SABIC)       1886.405

Root Mean Square Error of Approximation:
  RMSEA                                          0.158
  90 Percent confidence interval - lower         0.112
  90 Percent confidence interval - upper         0.207
  P-value H_0: RMSEA <= 0.050                    0.000
  P-value H_0: RMSEA >= 0.080                    0.996

Standardized Root Mean Square Residual:
  SRMR                                           0.061

Parameter Estimates:
  Standard errors                             Standard
  Information                                 Expected
  Information saturated (h1) model          Structured

Group 1 [0]:

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  posaff =~
    panas1   (ly1)    0.960    0.027   35.266    0.000    0.786    0.884
    panas2   (ly2)    1.100    0.040   27.696    0.000    0.901    0.818
    panas3   (ly3)    0.959    0.030   31.654    0.000    0.786    0.896
    panas4   (ly4)    1.018    0.031   33.315    0.000    0.834    0.909
    panas5   (ly5)    0.963    0.038   25.268    0.000    0.789    0.840

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .panas1            0.172    0.024    7.051    0.000    0.172    0.218
   .panas2            0.402    0.051    7.900    0.000    0.402    0.331
   .panas3            0.152    0.022    6.770    0.000    0.152    0.198
   .panas4            0.147    0.023    6.393    0.000    0.147    0.174
   .panas5            0.260    0.034    7.686    0.000    0.260    0.295
    posaff            0.671    0.080    8.426    0.000    1.000    1.000

R-Square:
                   Estimate
    panas1            0.782
    panas2            0.669
    panas3            0.802
    panas4            0.826
    panas5            0.705

Group 2 [1]:

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  posaff =~
    panas1   (ly1)    0.960    0.027   35.266    0.000    1.041    0.977
    panas2   (ly2)    1.100    0.040   27.696    0.000    1.193    0.928
    panas3   (ly3)    0.959    0.030   31.654    0.000    1.040    0.935
    panas4   (ly4)    1.018    0.031   33.315    0.000    1.105    0.938
    panas5   (ly5)    0.963    0.038   25.268    0.000    1.044    0.873
```

```
Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .panas1            0.052    0.022    2.353    0.019    0.052    0.046
   .panas2            0.229    0.059    3.865    0.000    0.229    0.139
   .panas3            0.155    0.041    3.778    0.000    0.155    0.125
   .panas4            0.166    0.044    3.737    0.000    0.166    0.120
   .panas5            0.340    0.081    4.197    0.000    0.340    0.238
    posaff            1.177    0.267    4.410    0.000    1.000    1.000

R-Square:
                   Estimate
    panas1            0.954
    panas2            0.861
    panas3            0.875
    panas4            0.880
    panas5            0.762

Constraints:
                                               |Slack|
    ly1 - (5-ly2-ly3-ly4-ly5)                    0.000
```

Conduct Chi-square Difference Tests

```
> library(psych)
>
> # Chi-square difference test (compare model1 and model2 -- compare fit with single constraint)
> fitMeasures(fit1, c("chisq", "df"))
 chisq     df
40.855 10.000
> fitMeasures(fit2, c("chisq", "df"))
 chisq     df
49.397 14.000
> diffchisq=fitMeasures(fit2, c("chisq", "df"))-fitMeasures(fit1, c("chisq", "df"))
> diffchisq
chisq    df
8.542 4.000
> 1-pchisq(diffchisq[2], diffchisq[1])
       df
0.8891296
> rm(diffchisq)
```

Chi-square comparisons

			Comparison to all-free model			
	χ^2	*df*	$\Delta\chi^2$	Δdf	*w*	ΔNCI
All parameters free	40.855	10				
Loadings only equal	49.397	14	8.542***	4	.061	.004

* p < .05, ** p < .01, *** p < .001. If robust estimates had beed used, this test would have used a scaled chi-square comparison that weights by the scaling correction factor instead of the simple subtraction method.

Sample Write-up

A multigroup structural equation modeling approach was used to compare widows and nonwidows on the factor loadings of the positive affect scale. To test for weak factorial invariance (Meredith, 1993) across groups, the chi-square from a model with all parameters allowed to be unequal across groups was compared to the chi-square from a model with only the loadings constrained to be equal across groups. No means or intercepts were estimated in these models. The model with all parameters freely estimated in the two groups, fit the data well, CFI = .969, SRMR = .028, according to fit criteria suggested by Hu and Bentler (1999), although the overall chi-square was significant, $\chi^2(68) = 40.855$ $p < .001$. The weak invariance model with loadings constrained to be equal across groups had fit that was significantly poorer, $\chi^2(14) = 49.397$, $p < .001$, $\Delta\chi^2(4) = 8.542$, $p < .001$, suggesting that the loadings differed significantly across groups. This difference was small in magnitude, however, *w* = .061, ΔNCI = .004. Further analyses are required to determine which loadings may differ across groups. The findings suggest that the measurement of the one-factor positive affect scale differs for widows and non-widows, and, thus, caution may be warranted in comparing these groups.[2]

[2] In an actual analysis, the researcher might stop here if it was concluded that the degree of invariance was significant but trivial. Otherwise, the next step would be identifying the source of the difference (see Cheung and Lau, 2012, for one method of exploring particular loading differences) and consider possible modifications to the scale so that it might have comparable measurement properties.