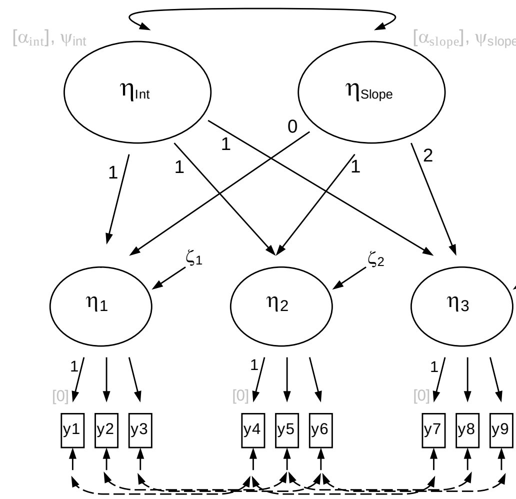
Second-Order Latent Growth Curve Models

Overview

Second-order growth curve models include multiple indicators of a latent variable at each time point (McArdle, 1988; Tisak & Meredith, 1990). They are sometimes referred to as *multiple indicator growth curve* models, *curve-of-factors* models, or *latent variable growth curve* models.

The second-order intercept and slope factors then model change in these variables and are specified much like they are with the more common first-order growth curve models, with intercept and slope factor means and variances estimated.



The second-order latent growth curve models require attention to several specification details (see below and Sayer & Cumsille, 2001 for more detail). Scaling constraints are needed for measurement intercepts of each first order latent variable (i.e., ν are set to zero for one loading of η_1, η_2, and η_3).

Constant variance across time points can be assumed by constraining the residual disturbances (ζ_1, ζ_2, ζ_3) to be equal. This assumption can be tested using a nested chi-square test. Autocorrelations of measurement errors over time may also be desirable.

The estimation of latent variables at each time point, which separately estimates measurement error (specific variance), will not impact mean slope estimates, because means are unbiased by measurement error. The second-order growth curve model should increase growth curve reliability and therefore increase statistical precision and increase power, however (von Oertzen, Hertzog, Lindenberger, & Ghisletta, 2010; Wänström, 2009).

Model Equations

An intercept and slope factor variance are specified for the second-order factors, with loadings set to 1.0 for the intercept factor, η_{int}, and slope factor, η_{slope}, loadings to unit increments (frequently, 0, 1, 2, 3…).

$$\eta_{ti} = \eta_{int} + \lambda_{1i}\eta_{slope} + \zeta_{ti}$$

Each factor estimates a mean, α, and deviation, ζ. α_{int} represents the average baseline value if time codes of 0, 1, 2, 3… are used for growth factor loadings, and α_{slope} represents the average slope.

$$\eta_{int} = \alpha_{int} + \zeta_{int}$$
$$\eta_{slope} = \alpha_{slope} + \zeta_{slope}$$

The first-order factors at each time point, *t*, are defined by measurement intercept values for each indicator, ν_{jt}, factor loadings, Λ_{jt}, a latent variable, η_{ti}, and indicator-specific variance, ε_{jti} where the subscript *j* refers to particular indicator of the first-order factor.

$$Y_{jti} = \nu_{jt} + \Lambda_{jt}\eta_{ti} + \varepsilon_{jti}$$

Model Specification Details

Identification. The most common method of identifying latent variables at each time point is to use a referent indicator for the first-order latent variable at each time point. Often the default in software programs, one loading, λ_{jti} = 1.0 and one measurement intercept, ν_{jti} = 0 for the first order factor are set (Bollen & Curran, 2006; Sayer & Cumsille, 2001), but the remaining intercepts can be freely estimated. The referent for the loading and the intercept should be the same and should be consistent across time points. The scaling of the factor mean at each time point is based on the observed mean of the referent measurement intercept, so the mean and variance of growth parameters will be a function of the observed mean of the chosen referent indicator variable.

An alternative identification approach that has been suggested is to standardize the factor mean and variance ($\alpha_1 = 0$ and $\psi_{11} = 1$) at the first time point (Chan, 1988; Widaman & Reise, 1997). The advantage of this approach is that a single indicator does not have to be chosen as a referent arbitrarily. Measurement invariance can be used with this method.

The effects coding approach to identification of Little, Slegers, and Card (2006) has several advantages over the aforementioned approaches. This approach places constraints on the set of loadings and measurement intercepts for each factor, avoiding the arbitrary choice of one referent over another. The approach can be used in conjunction with longitudinal measurement constraints as well. An appealing advantage is that the means at each time point are defined as a weighted average of the indicators at each time point rather than based on a single referent indicator mean. It also allows the researcher to retain the original metric of the measured variables rather than scaling each time point as difference values from the first time point, as with the single occasion identification approach. Example code for a second-order growth model using Mplus and lavaan are posted online at www.longitudinalsem.com (ex7-6e.inp, ex7-6e.R).

Longitudinal measurement invariance. The second-order growth curve framework allows the researcher to examine factorial invariance over time (Bontempo & Hofer, 2007; Newsom, 2015), which is a key advantage To the extent that measurement properties change over time, growth estimates are likely to be biased. As with between group factorial tests, one can distinguish between less and more strict invariance assumptions that include measurement intercepts, loadings, and measurement errors. Because mean estimates of latent slopes and intercepts are of interest and depend on values of measurement intercepts, invariance tests of measurement intercepts become more important than in models without interest in means (strict invariance). If invariance is established, loadings and intercepts can be set equal. Invariance tests may be affected by the choice of referent variable, so careful strategies for testing are needed when the referent identification approach is used (Ferrer, Balluerka, & Widaman, 2008).

Homogeneneous variance across time points. The researcher can set the disturbances associated with the first order factors to be equal over time to impose homogeneity of error variance, an assumption that can be tested for significance.

Correlated measurement residuals. Correlated errors of same items over time are a reasonable addition to these models and should improve estimates of the variance and covariances of the growth factors.

References and Further Reading

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