# Examples of Chi-square Difference Tests with Nonnormal and Categorical Variables

The simple chi-square difference test (or likelihood ratio test), which subtracts the chi-square of a less constrained model (more parameters or *M1*) from the chi-square of a more constrained model (fewer parameters or *M0*).

$$\Delta\chi^2 = \chi^2_{Fewer} - \chi^2_{More} = \chi^2_{M0} - \chi^2_{M1}$$
$$\Delta df = df_{Fewer} - df_{More} = df_{M0} - df_{M1}$$

This comparison needs to be adjusted or weighted when data are continuous nonnormal estimation with Satorra-Bentler scaled chi-square when there are no missing values (Satorra & Bentler,1988, 1994), or continuous nonnormal estimation when there are missing values (Yuan-Bentler scale chi-square; Yuan & Bentler, 2000), or when categorical estimation with WLSMV is used.

## Continuous Nonnormal Example

The scaled chi-square difference test (Satorra, 2000; Satorra & Bentler, 2001; 2010) weights the two scaled chi-square values for any nested model test, such as the comparison of a one-factor and two-factor CFA or a model with an equal constraint imposed vs. one without, when estimation is for continuous and nonnormal data (MLM for no missing values or MLR for missing values in Mplus and `lavaan`).

To Illustrate, I used the results from three-factor model illustrated in the handout "Examples of Estimates with Nonnormal Data with Missing Values" from the Mplus output as the less restricted model. The results from that model were $\chi^2_{SB}(51) = 106.973$ with an *scf* value of 1.4504.

The second step is to test the more restricted model. I'll use a two-factor model.

### *Mplus*

The Mplus specification is all the same, except that I now specify two factors.

```
analysis:  type=general; estimator=mlr;
    !MLR is used to request Satorra & Bentler (1988; 1994)
    !robust standard errors and scaled chi-square with missing data;

 model: !two factor, more constrained model;
        cruel by neg6-neg35 neg11-neg14;
        demands by neg16-neg20;

!original three factor, less constrained model;
     !   hostile by neg6-neg35;
      !  badadv by neg11-neg14;
       ! demands by neg16-neg20;
```

Results (excerpt)

```
       Chi-Square Test of Model Fit

                Value                            189.340*
                Degrees of Freedom                    53
                P-Value                           0.0000
                Scaling Correction Factor         1.4664
                  for MLR
```

The next step is to conduct the difference test using the proper approach for the scaled chi-square values described by Bryant and Satorra (2013; see Bryant & Satorra, 2012). The following Excel spreadsheet can be downloaded from http://web.pdx.edu/~newsomj/semclass/sbcorrection.xls to conduct the

weighted difference test.[1] Model 0 is the more constrained (two-factor) model and Model 1 is the less constrained (three-factor) model. "Correction" is the scaling correction factor, *scf*.

The fit of the three-factor model from the example in the earlier nonnormal missing example handout was $\chi^2_{YB}(51) = 106.97$. The fit of the two-factor model, shown above was $\chi^2_{YB}(53) = 189.34$.

| | | | | Chi-square Difference Test for the Satorra-Bentler or Yuan-Bentler Scaled Chi-square | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | MODEL 0 | | | | | MODEL 1 | | | | | | | | |
| SB/YB $\chi$2 | df | correction | $\chi$2 | | SB/YB $\chi$2 | df | correction | $\chi$2 | | $\chi$2 diff | c-hat-d | T-bar-d | df diff | p-value |
| 189.34 | 53 | 1.4664 | 277.6482 | | 106.973 | 51 | 1.4504 | 155.1536 | | 122.4945 | 1.8744 | 65.35133 | 2 | 0.000 |

The difference in the two scaled chi-square values, $\Delta\chi^2_{SB}(2) = 65.35, p < .001$, which appears under the T-bar-d column. If magnitude of effect is desired, the Cohen's $w$ and $\Delta$NCI would be added using the "Spreadsheet for Computing w and $\Delta$NCI" provided on the website and illustrated in the handout "Two-Factor CFA Example in Mplus".

*lavaan*

Here is the code for the two-factor, more constrained, model in `lavaan`.

```
> model = '
+         cruel =~ neg6 + neg26 + neg30 + neg35 + neg11 + neg12 + neg13 + neg14
+            demands =~ neg16 + neg17 + neg19 + neg20
+ '
> #Yuan-Bentler robust estimates for nonnormal missing data
> fit = sem(model, data = missdat, missing = "fiml", estimator="mlr")
> summary(fit,fit.measures=TRUE, rsquare=TRUE, standardized=TRUE)

lavaan 0.6-18 ended normally after 42 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        37

  Number of observations                           275
  Number of missing patterns                         7

Model Test User Model:
                                              Standard      Scaled
  Test Statistic                               277.652     189.340
  Degrees of freedom                                53          53
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  1.466
```

**For the MLM with nonmissing data only**, the R `lavTestLRT` function can be used to conducted the scaled chi-square test. Note that there is a special `method="satorra.bentler.2010"` statement that obtains the correction to the Satorra-Bentler scaled chi-square that avoids negative values (Satorra & Bentler, 2010; automatically implemented in Mplus). The scaled chi-square value is close to the Mplus result but not equal. The fit model is the original model the fit2 model is the more constrained model.

```
> #compute the correct weighted chi-square test for mlm
> #add method = "satorra.bentler.2010", in the parentheses for correction of any negative values
> lavTestLRT(fit,fit2, method = "satorra.bentler.2010")
```

### Categorical Example

For models with categorical variables using the WLSMV estimator, a different adjust is needed. A second-order test has been proposed by Asparouhov & Muthén (2010) and is implemented in Mplus using

[1] The original sheet from Byrant can be found here http://web.pdx.edu/~newsomj/semclass/BryantSatorraScaledDifferenceTestsForLISREL8LISREL9EQSandMplus.xls, and was originally posted by Satorra (but no longer available) at http://www.econ.upf.edu/~satorra/dades/BryantSatorraScaledDifferenceTestsForLISREL8LISREL9EQSandMplus.xls

the DIFFTEST procedure.[2] This test is not the same as the Satorra-Bentler test described above, but instead uses an estimated degrees of freedom in order to better approximate the chi-square distribution's mean and variance under these conditions. The file name `deriv.dat` used below is arbitrary (but has to match in the two steps) and is stored in the same folder as the input file by default.

**Step 1**

```
INPUT INSTRUCTIONS


  title:  CESD using categorical indicators (4 ordinal values);

  data:  file=c:\jason\mplus\negex\wave1\pncesdw1.dat;
               format=35f1.0;
          listwise=on;

  variable:  names = w1adv1 w1adv2 w1adv3 w1hlp1 w1hlp2
                     w1hlp3 w1vst1 w1vst2 w1vst3 w1emo1
                     w1emo2 w1emo3 w1unw1 w1unw2 w1unw3
                     w1dwn1 w1dwn2 w1dwn3 w1out1 w1out2
                     w1out3 w1fai1 w1fai2 w1fai3 w1dboth
                     w1dblues w1dmind w1ddep w1defrt w1dsleep
                     w1dhappy w1dlone w1dnogo w1dsad w1denjoy;


             usevariable= w1dboth w1dblues w1dmind w1ddep w1defrt
                w1dsleep w1dhappy w1dsad w1denjoy;

             categorical=w1dboth w1dblues w1dmind w1ddep w1defrt
                       w1dsleep w1dhappy w1dsad w1denjoy;

  analysis:  type=general;  estimator=wlsmv; parameterization=theta;
      !WLSMV is the default and estimator= is not needed here;
      !parameterization=theta changes the default delta parameterization to theta;
      !WLSMV gives probit estimates;

  model: negaff by w1dboth w1dblues w1ddep w1dsad;
         somatic by w1dmind w1defrt w1dsleep;
         positive by w1dhappy  w1denjoy;

  output: stdyx;

  savedata: difftest = deriv.dat;
  !this is the model and first step in the chi-square difference test
  !savedata command with difftest statement saves results from H1 (less restricted model);


INPUT READING TERMINATED NORMALLY

CESD using categorical indicators (4 ordinal values);

SUMMARY OF ANALYSIS

Number of groups                                                 1
Number of observations                                         720

Number of dependent variables                                    9
Number of independent variables                                  0
Number of continuous latent variables                            3

Observed dependent variables

  Binary and ordered categorical (ordinal)
   W1DBOTH     W1DBLUES    W1DMIND     W1DDEP      W1DEFRT     W1DSLEEP
   W1DHAPPY    W1DSAD      W1DENJOY

Continuous latent variables
   NEGAFF      SOMATIC     POSITIVE
```

[2] The second-order test replaces a first-order Satterthwaite test (Muthén, du Toit, & Spisic, 1997) originally implemented before Version 6 of Mplus. For implementation details in Mplus, see https://www.statmodel.com/chidiff.shtml.

```
Estimator                                                      WLSMV
Maximum number of iterations                                    1000
Convergence criterion                                     0.500D-04
Maximum number of steepest descent iterations                     20
Parameterization                                              THETA
Link                                                          PROBIT


MODEL FIT INFORMATION

Number of Free Parameters                       39

Chi-Square Test of Model Fit

          Value                             39.254*
          Degrees of Freedom                    24
          P-Value                           0.0257
```

**Step 2**

```
INPUT INSTRUCTIONS

  title:  CESD using categorical indicators (4 ordinal values);

  data:  file=c:\jason\mplus\negex\wave1\pncesdw1.dat;
              format=35f1.0;
           listwise=on;

  variable:  names = w1adv1 w1adv2 w1adv3 w1hlp1 w1hlp2
                     w1hlp3 w1vst1 w1vst2 w1vst3 w1emo1
                     w1emo2 w1emo3 w1unw1 w1unw2 w1unw3
                     w1dwn1 w1dwn2 w1dwn3 w1out1 w1out2
                     w1out3 w1fai1 w1fai2 w1fai3 w1dboth
                     w1dblues w1dmind w1ddep w1defrt w1dsleep
                     w1dhappy w1dlone w1dnogo w1dsad w1denjoy;

             usevariable= w1dboth w1dblues w1dmind w1ddep w1defrt
                w1dsleep w1dhappy w1dsad w1denjoy;

             categorical=w1dboth w1dblues w1dmind w1ddep w1defrt
                       w1dsleep w1dhappy w1dsad w1denjoy;

  analysis:  type=general;  estimator=wlsmv; parameterization=theta;
      !WLSMV is the default and estimator= is not needed here;
      !parameterization=theta changes the default delta parameterization to theta;
      !WLSMV gives probit estimates;
      difftest = deriv.dat;
      !the above line is the second model and step that requests the comparison of
      !this more restricted model to the prior less restricted model
      !(need to comment out or delete prior savedata command);

  model: cesd by w1dboth w1dblues w1ddep w1dsad
        w1dmind w1defrt w1dsleep w1dhappy  w1denjoy;

  !negaff by w1dboth w1dblues w1ddep w1dsad;
  !       somatic by w1dmind w1defrt w1dsleep;
  !       positive by w1dhappy  w1denjoy;

  output: stdyx;

MODEL FIT INFORMATION

Number of Free Parameters                       36

Chi-Square Test of Model Fit

          Value                            301.292*
          Degrees of Freedom                    27
          P-Value                           0.0000

Chi-Square Test for Difference Testing
```

```
          Value                            148.080
          Degrees of Freedom                     3
          P-Value                          0.0000

*   The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used
    for chi-square difference testing in the regular way.  MLM, MLR and WLSM
    chi-square difference testing is described on the Mplus website.  MLMV, WLSMV,
    and ULSMV difference testing is done using the DIFFTEST option.
```

## R

There is currently no procedure in `lavaan` to conduct the same test with the second-order adjustments used in DIFFTEST in Mplus, nor an equivalent procedure that seems to be available in any R package at the moment. See Kite, Johnson, and Xing which appear to be developing a function for the MplusAutomation package, however, https://pj.freefaculty.org/guides/crmda_guides/44.difftest/44.difftest.html#:~:text=The%20crmda_difftest%20function%20imports%20the,Asparouhov%20and%20Muth%C3%A9n%2C%202010.

## References

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