From Shapes to Systems: Nonlinear Modeling of Longitudinal Change

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Brief Intro to Nonlinear Models

— The Shape of Change —
  Conceptual definition

  Gompertz Growth
  Methods, Findings & Implications

— Systems of Change —
  Conceptual definition

  Latent Differential Equations
  Methods, Findings & Implications

Theoretical Implications
Nonlinear Models – Brief Introduction

The question is... How to model longitudinal data effectively?
Nonlinear Models – Brief Introduction

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Lines are fine. They describe most phenomena well and are “straight” forward regarding interpretation.
The question is... How to model longitudinal data effectively?

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However, not all models are created equal. The key component is the utility. That is, what is gained from moving away from strictly rectilinear models?
The question is... How to model longitudinal data effectively?

Lines are fine. They describe most phenomena well and are “straight” forward regarding interpretation.

However, not all models are created equal. The key component is the utility. That is, what is gained from moving away from strictly rectilinear models?

Nonlinearity can be used to describe a few different things.

(1) Nonlinear mean trajectories
(2) Nonlinear equation parameters
The Shape of Change
Shape of Change – Conceptual Definition

(1) Nonlinear mean trajectories

Simply that the expected trajectory is not a straight line.

In order to account for this lack of “rectilinearity”, polynomial terms can be included in the equation.

\[ y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + e_t \]
Shape of Change – Conceptual Definition

\[ y_t = \beta_0 + \beta_1 t + \beta_2 t^2 \]

- \( \beta_1 > 0 \)
- \( \beta_2 > 0 \)

- \( \beta_1 > 0 \)
- \( \beta_2 < 0 \)

- \( \beta_1 < 0 \)
- \( \beta_2 > 0 \)

- \( \beta_1 < 0 \)
- \( \beta_2 < 0 \)
Shape of Change – Conceptual Definition

(1) Nonlinear mean trajectories
Simply that the expected trajectory is not a straight line.
In order to account for this lack of “rectilinearity”, polynomial terms can be included in the equation.

\[ y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + e_t \]

(2) Nonlinear equation parameters
Here the equation parameters enter nonlinearly (not additively).
This in-turn can give rise to nonlinear trajectories.

\[ y_t = \beta_0 e^{\beta_1 t} + e_t \]
Shape of Change – Conceptual Definition

\[ y_t = \beta_0 e^{\beta_1 t} \]
Shape of Change – Academic Achievement

Non-linear growth achievement data from the ECLS-K8 and the NLSY
Shape of Change – Methods

Gompertz $y_t = \alpha e^{\beta e^y}$
Shape of Change – Methods

\[ f(t) = \alpha e^{\beta y_t} + e_t \]
Shape of Change – Methods

\[ f(t) = \alpha e^{\beta y'} + e_t \]

\[ \frac{\partial f}{\partial \alpha} = e^{\beta y'} \]
Shape of Change – Methods

\[ f(t) = \alpha e^{\beta y_t} + e_t \]

\[ \frac{\partial f}{\partial \alpha} = e^{\beta y_t} \]

\[ \frac{\partial f}{\partial \beta} = \alpha e^{y_t} e^{\beta y_t} \]
Shape of Change – Methods

\[ f(t) = \alpha e^{\beta e^{\gamma t}} + e_t \]

\[ \frac{\partial f}{\partial \alpha} = e^{\beta e^{\gamma t}} \]

\[ \frac{\partial f}{\partial \beta} = \alpha e^{\gamma t} e^{\beta e^{\gamma t}} \]

\[ \frac{\partial f}{\partial \gamma} = \alpha \beta t e^{\gamma t} e^{\beta e^{\gamma t}} \]
Shape of Change – Methods

\[ f(t) = \alpha e^{\beta e^{y_t}} + e_t \]

<table>
<thead>
<tr>
<th>t</th>
<th>( \frac{\partial f}{\partial \alpha} )</th>
<th>( \frac{\partial f}{\partial \beta} )</th>
<th>( \frac{\partial f}{\partial \gamma} )</th>
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<tr>
<td>1</td>
<td>( e^{\beta e^{y_1}} )</td>
<td>( \alpha e^{y_1} e^{\beta e^{y_1}} )</td>
<td>( 1 \cdot \alpha \beta e^{y_1} e^{\beta e^{y_1}} )</td>
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<tr>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
</tr>
<tr>
<td>( T )</td>
<td>( e^{\beta e^{y_T}} )</td>
<td>( \alpha e^{y_T} e^{\beta e^{y_T}} )</td>
<td>( T \cdot \alpha \beta e^{y_T} e^{\beta e^{y_T}} )</td>
</tr>
</tbody>
</table>

Browne & Du Toit, 1991; Browne, 1993
Shape of Change – Methods

\[ f(t) = t + \alpha e^{\beta e^{y_t}} + e_t \]

**MODEL:**

\[
\begin{align*}
    i & \text{ BY } math05*(I1) \\
     & \text{ math06 (I2)} \\
     & \text{ math07 (I3)} \\
     & \text{ math08 (I4)} \\
     & \text{ math09 (I5)};
\end{align*}
\]

\[
\begin{align*}
    a & \text{ BY } math05*(A1) \\
     & \text{ math06 (A2)} \\
     & \text{ math07 (A3)} \\
     & \text{ math08 (A4)} \\
     & \text{ math09 (A5)};
\end{align*}
\]

\[
\begin{align*}
    b & \text{ BY } math05*(B1) \\
     & \text{ math06 (B2)} \\
     & \text{ math07 (B3)} \\
     & \text{ math08 (B4)} \\
     & \text{ math09 (B5)};
\end{align*}
\]

**MODEL CONSTRAINT:**

\[
\begin{align*}
    g & \text{ BY } math05*(G1) \\
     & \text{ math06 (G2)} \\
     & \text{ math07 (G3)} \\
     & \text{ math08 (G4)} \\
     & \text{ math09 (G5)};
\end{align*}
\]

\[
\begin{align*}
    \text{new}(g*0.451 & \text{ b*3.119}); \\
    I1=1; & I2=1; I3=1; I4=1; I5=1; \\
    A1 = \exp(-b*(\exp((1-1)*g))); & A2 = \exp(-b*(\exp((1-2)*g))); \\
    A3 = \exp(-b*(\exp((1-3)*g))); & A4 = \exp(-b*(\exp((1-4)*g))); \\
    A5 = \exp(-b*(\exp((1-5)*g))); \\
    B1 = (-a)*\exp(-\exp(-1*g)*((1-1)*g*\exp(1*g)+b*\exp(g)))); & B2 = (-a)*\exp(-\exp(-2*g)*((2-1)*g*\exp(2*g)+b*\exp(g)))); \\
    B3 = (-a)*\exp(-\exp(-3*g)*((3-1)*g*\exp(3*g)+b*\exp(g)))); & B4 = (-a)*\exp(-\exp(-4*g)*((4-1)*g*\exp(4*g)+b*\exp(g)))); \\
    B5 = (-a)*\exp(-\exp(-5*g)*((5-1)*g*\exp(5*g)+b*\exp(g)))); \\
    G1 = (1-1)*a*b*\exp(-\exp(-1*g)*((1-1)*g*\exp(1*g)+b*\exp(g)))); & G2 = (2-1)*a*b*\exp(-\exp(-2*g)*((2-1)*g*\exp(2*g)+b*\exp(g)))); \\
    G3 = (3-1)*a*b*\exp(-\exp(-3*g)*((3-1)*g*\exp(3*g)+b*\exp(g)))); & G4 = (4-1)*a*b*\exp(-\exp(-4*g)*((4-1)*g*\exp(4*g)+b*\exp(g)))); \\
    G5 = (5-1)*a*b*\exp(-\exp(-5*g)*((5-1)*g*\exp(5*g)+b*\exp(g))));
\end{align*}
\]

Grimm & Ram, 2009; Grimm, Ram & Hamagami, *in press*
Shape of Change – Methods

\[ f(t) = i + \alpha e^{\beta v_t} + e_t \]

MODEL CONSTRAINT:

\[ \text{new}(g*0.451 \ b*3.119); \]

\[ I1=1; \ I2=1; \ I3=1; \ I4=1; \ I5=1; \]

\[ A1 = \exp(-b*(\exp((1-1)*g))); \]
\[ A2 = \exp(-b*(\exp((1-2)*g))); \]
\[ A3 = \exp(-b*(\exp((1-3)*g))); \]
\[ A4 = \exp(-b*(\exp((1-4)*g))); \]
\[ A5 = \exp(-b*(\exp((1-5)*g))); \]

\[ B1 = (-a)*\exp(-\exp(-1*g)*((1-1)*g*\exp(1*g)+b*\exp(g)))); \]
\[ B2 = (-a)*\exp(-\exp(-2*g)*((2-1)*g*\exp(2*g)+b*\exp(g)))); \]
\[ B3 = (-a)*\exp(-\exp(-3*g)*((3-1)*g*\exp(3*g)+b*\exp(g)))); \]
\[ B4 = (-a)*\exp(-\exp(-4*g)*((4-1)*g*\exp(4*g)+b*\exp(g)))); \]
\[ B5 = (-a)*\exp(-\exp(-5*g)*((5-1)*g*\exp(5*g)+b*\exp(g)))); \]

\[ G1 = (1-1)*a*b*\exp(-\exp(-1*g)*((1-1)*g*\exp(1*g)+b*\exp(g)))); \]
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\[ G3 = (3-1)*a*b*\exp(-\exp(-3*g)*((3-1)*g*\exp(3*g)+b*\exp(g)))); \]
\[ G4 = (4-1)*a*b*\exp(-\exp(-4*g)*((4-1)*g*\exp(4*g)+b*\exp(g)))); \]
\[ G5 = (5-1)*a*b*\exp(-\exp(-5*g)*((5-1)*g*\exp(5*g)+b*\exp(g)))); \]

\[ f(t) = i + \alpha e^{\beta v_t} + e_t \]

Grimm & Ram, 2009; Grimm, Ram & Hamagami, in press
Shape of Change – Findings

ECLS-K8

Reading (CFI=.896; TLI=.879; RMSEA=.147)
Math (CFI=.955; TLI=.947; RMSEA=.103)

<table>
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<tr>
<th>Covariate</th>
<th>Reading</th>
<th>Math</th>
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<td>-.005</td>
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<td>Hispanic</td>
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<td>High School</td>
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<td>College</td>
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<td>Father Education</td>
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<td>High School</td>
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<td>College</td>
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<td>Poverty Status</td>
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N = 21,260
Shape of Change – Findings

NLSY

Reading (CFI=.956; TLI=.956; RMSEA=.045)
Math     (CFI=.975; TLI=.975; RMSEA=.029)

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<th>Math</th>
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<td>High School</td>
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<td>Father Education</td>
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<td>High School</td>
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<td>.024</td>
<td>.045</td>
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N = 9,032
Shape of Change – Findings

**ECLS-K**

*Reading*

Most rapid growth during Spring of 1st grade.
\[ \rho_{\alpha \gamma} = .462, \rho_{\alpha \beta} = -.475, \rho_{\beta \gamma} = -.336 \]

*Math*

Most rapid growth during Fall of 1st grade.
\[ \rho_{\alpha \gamma} = .314, \rho_{\alpha \beta} = -.215, \rho_{\beta \gamma} = -.726 \]

**NLSY**

*Reading*

Most rapid growth at 7.36 years.
\[ \rho_{\alpha \gamma} = -.150, \rho_{\alpha \beta} = .125, \rho_{\beta \gamma} = -.653 \]

*Math*

Most rapid growth at 7.5 years.
\[ \rho_{\alpha \gamma} = -.300, \rho_{\alpha \beta} = -.563, \rho_{\beta \gamma} = .085 \]
Of the curves fit, the Gompertz curve provided the best overall fit to the data.

The modeling of the parameters as latent variables allowed for a more flexible investigation of the characteristics of change.

The parameters were related elucidating a general trend of greater overall growth being related to earlier and more rapid change.

Salient effects of Gender, Ethnicity, Parental Education and Socio-economic Status were demonstrated for each latent parameter. These effects could serve to inform policy makers and educators in the future.
Systems of Change
Systems of Change – Conceptual Definition

The question is... How to model interacting systems over time?
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With these models we are less concerned with characteristics of the shape (though shape is still important), but more so with reciprocal effects.
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How do these interactions lead to change? This view allows for specification of coupled influences between processes with change as the outcome.
Systems of Change – Conceptual Definition

The question is... How to model interacting systems over time?

With these models we are less concerned with characteristics of the shape (though shape is still important), but more so with reciprocal effects. How do these interactions lead to change? This view allows for specification of coupled influences between processes with change as the outcome.

This can describe interacting dimensions within the individual, or interactions between individuals such as Mother-Child dyads, Teacher-Student dyads, Counselor-Client dyads, or, as in the present case, Romantic dyads.
Systems of Change – DDIP

A project focused on developing mathematical and statistical models for analyzing the dynamics of dyadic interactions and applying these models to data collected from couples at multiple time points to study dyadic affective processes, emotion regulation, and stability of relationships over time.

Measures include:

- Behavioral and psychometric scales (Affect ratings, BFI, Attachment, etc.)
- Physiological signals
- One year follow-up (relationship status, etc.)
Individuals provided daily ratings of 18 items intended to measure relationship specific affect (9 positive and 9 negative). Items rated on a Likert-type scale from 1 (not at all) to 5 (extremely). Daily averages were computed from the 9 positive and 9 negative items.

**positive items**: emotionally intimate, physically intimate, trusted, committed, free, loved, happy, loving, and socially supported.

**negative items**: sad, blue, trapped, argumentative, discouraged, doubtful, lonely, angry, and deceived.
The resulting reliability coefficients of change ranged from .818 for male positive affect to .869 for female negative affect.

The first 30 days of observations from 178 heterosexual couples.

Mean age of 23.31 years ($SD = 8.38$)
Systems of Change – Methods

Brief introduction to (latent) differential equations

Newton vs Leibniz

\[
\dot{x} = \frac{dx}{dt} \\
\ddot{x} = \frac{d^2 x}{dt^2}
\]
Systems of Change – Methods

Brief introduction to (latent) differential equations

The goals:
(1) to investigate fluctuations in daily affect
(2) to determine if these fluctuations damped with time
(3) to determine the nature and strength of coupled influences

The *Damped Linear Oscillator* model.

\[ \ddot{x}_t = \eta x_t + \zeta \dot{x}_t + e_t \]

Where:
- \( x \) Position
- \( \dot{x} \) First derivative (speed)
- \( \ddot{x} \) Second derivative (acceleration)
- \( \eta \) Related to the frequency (\( \omega \)) of oscillation
  such that \( \eta = -(2\pi\omega)^2 \)
- \( \zeta \) Damping parameter

Figure 1. A linear oscillator model with parameters (a) damping, (b) 1/frequency, and (c) amplitude.
Systems of Change – Methods

\[
L = \begin{bmatrix}
1 & -2\tau \Delta t & (\frac{-2\tau \Delta t}{2})^2/2 \\
1 & -1\tau \Delta t & (\frac{1\tau \Delta t}{2})^2/2 \\
1 & 0 & 0 \\
1 & 1\tau \Delta t & (\frac{-1\tau \Delta t}{2})^2/2 \\
1 & 2\tau \Delta t & (\frac{2\tau \Delta t}{2})^2/2 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
v_1 \\
v_2 \\
v_3 \\
v_4 \\
v_5 \\
\vdots \\
v_{n-4} \\
v_{n-3} \\
v_{n-2} \\
v_{n-1} \\
v_n
\end{bmatrix}
\]
Systems of Change – Methods

\[ R = L (I-A)^{-1} S (I-A)^{-1}' L' + E \]
Systems of Change – Methods

Assume the influence of each partner on the other is the same.

\[
\begin{align*}
\ddot{x}_t &= \eta_x x_t + \zeta_x \dot{x}_t + \gamma (\eta_y y_t + \zeta_y \dot{y}_t) + e_{\ddot{x}_t} \\
\ddot{y}_t &= \eta_y y_t + \zeta_y \dot{y}_t + \gamma (\eta_x x_t + \zeta_x \dot{x}_t) + e_{\ddot{y}_t}
\end{align*}
\]

Assume a single but different influence of each partner on the other.

\[
\begin{align*}
\ddot{x}_t &= \eta_x x_t + \zeta_x \dot{x}_t + \gamma_y (\eta_y y_t + \zeta_y \dot{y}_t) + e_{\ddot{x}_t} \\
\ddot{y}_t &= \eta_y y_t + \zeta_y \dot{y}_t + \gamma_x (\eta_x x_t + \zeta_x \dot{x}_t) + e_{\ddot{y}_t}
\end{align*}
\]

Simply assume that there are influences...

\[
\begin{align*}
\ddot{x}_t &= \eta_x x_t + \zeta_x \dot{x}_t + \eta_{xy} y_t + \zeta_{xy} \dot{y}_t + e_{\ddot{x}_t} \\
\ddot{y}_t &= \eta_y y_t + \zeta_y \dot{y}_t + \eta_{yx} x_t + \zeta_{yx} \dot{x}_t + e_{\ddot{y}_t}
\end{align*}
\]

\[ \ddot{x}_t = \eta_x x_t + \zeta_x \dot{x}_t + \eta_{xy} y_t + \zeta_{xy} \dot{y}_t + e_{\ddot{x}_t} \]
\[ \ddot{y}_t = \eta_y y_t + \zeta_y \dot{y}_t + \eta_{yx} x_t + \zeta_{yx} \dot{x}_t + e_{\ddot{y}_t} \]

Source: Steele, J.S. & Ferrer, E. Systems of Change – Methods
### Parameter Estimates from the Dyadic Coupled Model (Positive Affect)

<table>
<thead>
<tr>
<th></th>
<th>Female Estimate (S.E.)</th>
<th>Est/S.E.</th>
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<td>$\zeta$</td>
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<td>-.034 (.074)</td>
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<td>$Cov_{pp}$</td>
<td>.000 (.001)</td>
<td>.02</td>
<td>.000 (.001)</td>
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### Coupled Parameters – partner’s influence

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<th>Male</th>
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<td>$\eta_{cp}$</td>
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<td>$\zeta_{cp}$</td>
<td>-.113 (.046)</td>
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<td>1.48</td>
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### Latent Variable Variances

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<th>Male</th>
<th>Est/S.E.</th>
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<tr>
<td>$S_p^2$</td>
<td>.039 (.004)</td>
<td>9.61</td>
<td>.033 (.003)</td>
<td>9.61</td>
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<tr>
<td>$S_{p0}^2$</td>
<td>.013 (.001)</td>
<td>10.88</td>
<td>.011 (.001)</td>
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### Residual Variances

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<th>Est/S.E.</th>
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<tr>
<td>$P_0$</td>
<td>.209 (.012)</td>
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<td>$P_i$</td>
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<td>35.35</td>
<td>.205 (.006)</td>
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<td>$P_2$</td>
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<td>.199 (.006)</td>
<td>33.13</td>
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<td>$P_3$</td>
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<td>.202 (.006)</td>
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<td>$P_4$</td>
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<td>4.81</td>
<td>.007 (.001)</td>
<td>6.45</td>
</tr>
</tbody>
</table>

### Residual Covariances

<table>
<thead>
<tr>
<th></th>
<th>Female</th>
<th>Est/S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_8 \leftrightarrow u_8$</td>
<td>.094 (.003)</td>
<td>31.43</td>
</tr>
</tbody>
</table>

Source: Steele, J.S. & Ferrer, E.
### Parameter Estimates from the Dyadic Coupled Model (Negative Affect)

<table>
<thead>
<tr>
<th></th>
<th>Female</th>
<th>Male</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate (S.E.)</td>
<td>Est/S.E.</td>
</tr>
<tr>
<td>$\eta$</td>
<td>-.544 (.057)</td>
<td>-9.55</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>.012 (.123)</td>
<td>0.09</td>
</tr>
<tr>
<td>$\text{Cov}_{n\bar{n}}$</td>
<td>.000 (.001)</td>
<td>0.35</td>
</tr>
</tbody>
</table>

**Coupled Parameters – partner’s influence**

<table>
<thead>
<tr>
<th></th>
<th>Female</th>
<th>Male</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate (S.E.)</td>
<td>Est/S.E.</td>
</tr>
<tr>
<td>$\eta_{cp}$</td>
<td>.235 (.092)</td>
<td>2.55</td>
</tr>
<tr>
<td>$\zeta_{cp}$</td>
<td>-.098 (.077)</td>
<td>-1.27</td>
</tr>
</tbody>
</table>

**Latent Variable Variances**

<table>
<thead>
<tr>
<th></th>
<th>Female</th>
<th>Male</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate (S.E.)</td>
<td></td>
</tr>
<tr>
<td>$S_{n}^2$</td>
<td>.018 (.004)</td>
<td>5.02</td>
</tr>
<tr>
<td>$S_{\bar{n}}^2$</td>
<td>.007 (.001)</td>
<td>7.28</td>
</tr>
</tbody>
</table>

**Residual Variances**

<table>
<thead>
<tr>
<th></th>
<th>Female</th>
<th>Male</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate (S.E.)</td>
<td></td>
</tr>
<tr>
<td>$N_0$</td>
<td>.172 (.012)</td>
<td>14.40</td>
</tr>
<tr>
<td>$N_1$</td>
<td>.195 (.008)</td>
<td>24.91</td>
</tr>
<tr>
<td>$N_2$</td>
<td>.190 (.008)</td>
<td>22.88</td>
</tr>
<tr>
<td>$N_3$</td>
<td>.189 (.008)</td>
<td>24.09</td>
</tr>
<tr>
<td>$N_4$</td>
<td>.157 (.013)</td>
<td>12.47</td>
</tr>
<tr>
<td>$e_{\bar{n}}$</td>
<td>.002 (.001)</td>
<td>2.07</td>
</tr>
</tbody>
</table>

**Residual Covariances**

<table>
<thead>
<tr>
<th></th>
<th>Female</th>
<th>Male</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate (S.E.)</td>
<td></td>
</tr>
<tr>
<td>$\Phi_{u\bar{u}}$</td>
<td>.075 (.003)</td>
<td>23.68</td>
</tr>
</tbody>
</table>

Source: Steele, J.S. & Ferrer, E.
Couples fluctuate in their assessments of the relationship.

How much?

<table>
<thead>
<tr>
<th></th>
<th>Positive</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Females:</td>
<td>~9.8 days</td>
<td>~8.5 days</td>
</tr>
<tr>
<td>Males:</td>
<td>~9.4 days</td>
<td>~8.0 days</td>
</tr>
</tbody>
</table>

There was no salient damping present in the sample. Perhaps this is too short of a window, or theoretically reaching a point of no fluctuation is not reasonable.
Systems of Change – Findings

Interpreting the coupled influences...

Coupled Positive Affect

<table>
<thead>
<tr>
<th></th>
<th>Female</th>
<th>Male</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>0.168 [5.49]</td>
<td>0.135 [5.19]</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>-0.113 [-2.49]</td>
<td>0.058 [1.48]</td>
</tr>
</tbody>
</table>

Coupled Negative Affect

<table>
<thead>
<tr>
<th></th>
<th>Female</th>
<th>Male</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>0.235 [2.55]</td>
<td>0.165 [2.08]</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>-0.098 [-1.27]</td>
<td>-0.019 [-0.29]</td>
</tr>
</tbody>
</table>

Females are attuned not only to how far off from average their partners are but also how much they are changing. This is only true for positive affect.
Systems of Change – Implications

LDEM allows for dynamical modeling in SEM.

The use of LLA of derivatives mirrors polynomial bases.

The coupled model allows for investigation of the characteristics of the univariate system as well as reciprocal effects of other systems.

Change is the outcome, which may be more germane to issues of intervention than absolute levels of outcome.
Theoretical Implications
Theoretical Implications – Nonlinear Models

The use of nonlinear models can prove more informative than traditional linear approaches, without a great deal more overhead.
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Nonlinearity arises in varied form, from means structure to equation parameters. In each instance the same tool, SEM, can be used.
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Nonlinearity arises in varied form, from means structure to equation parameters. In each instance the same tool, SEM, can be used.

Most processes in life are nonlinear, this trend is well established in other fields precisely because of the heightened information it provides.
Theoretical Implications – Dynamical Systems

Here again SEM can be used to model complex, interacting systems.
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The focus on change as an outcome is not new – Newton got things rolling in 1667 – and has been echoed throughout the years.

The modeling of system dynamics can serve to divorce analyses from a dependence on the ostensible shape of change, and return focus to the underlying causes.
Thank You