The Importance of Reliability

I. Attenuation

- Correlation coefficient
- Statistical control

II. Means

- Single mean
- Group comparison
- Over time: regression toward the mean

I. Attenuation

Measurement error *attenuates* correlations

- Imagine if a score was only random error
- If observed scores are a function of true scores and measurement error, degree of error will cloud estimation of the relationship between two variables
- Example: child’s age and reading ability

I. Attenuation

Remember that measurement error will increase the variance of the observed score, so the denominator in the correlation coefficient will be larger

This makes the estimate of the correlation smaller in magnitude

$$r_{xy} = \frac{\sum (X - \bar{X})(Y - \bar{Y})}{\sqrt{\sum (X - \bar{X})^2 \sum (Y - \bar{Y})^2}} = \frac{c_{xy}}{s_x s_y}$$

I. Attenuation

$$r_{x_o y_o} = r_{x_t y_t} \sqrt{R_{xx} R_{yy}}$$

$r_{x_o y_o}$ is the correlation estimated from the data (between observed scores), $r_{x_t y_t}$ is the correlation between the true scores (if we could know them), R_{xx} and R_{yy} are the reliabilities of the two measures

I. Attenuation

Example 1: say the reliability for my guess at the age is .6 and the measurement of reading ability is .5 and that the true score correlation is .4

$$r_{x_o y_o} = r_{x_t y_t}\sqrt{R_{xx}R_{yy}}$$
$$= .4\sqrt{(.5)(.6)} = .4\sqrt{.3} = .4(.548) = .21$$

When the true score correlation is .4, the estimated correlation is .21—a substantial underestimate—almost half!

I. Attenuation

Example 2: say the reliability for my guess at the age is .9 and the measurement of reading ability is .9 and that the true score correlation is .4

$$r_{x_o y_o} = r_{x_t y_t}\sqrt{R_{xx}R_{yy}}$$
$$= .4\sqrt{(.9)(.9)} = .4\sqrt{.81} = .4(.9) = .36$$

When the true score correlation is .4, the estimated correlation is .36—not nearly as bad

I. Attenuation

Some relationships might be missed entirely

Imagine a lower true score correlation between age and reading of .15, with the low reliabilities in the first example, the observed correlation would be only .08, which is getting pretty close to zero and might be nonsignificant

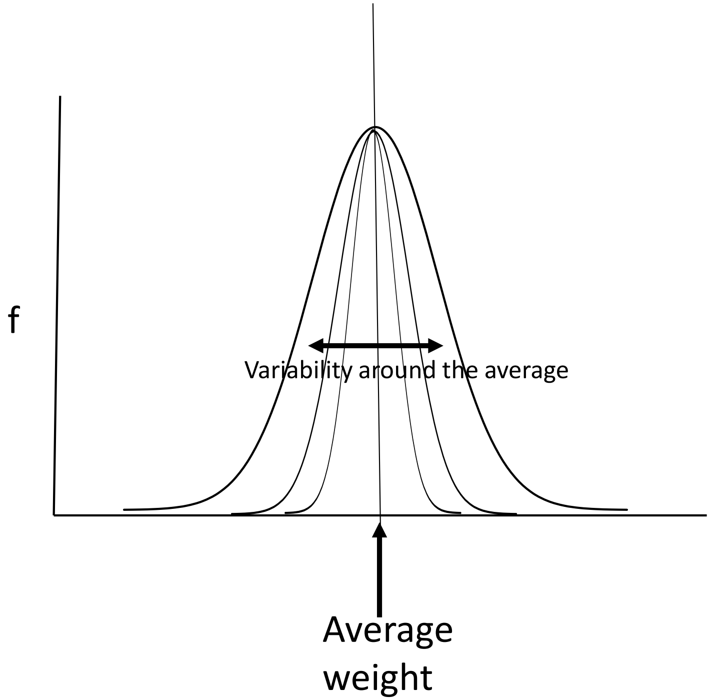
I. Attenuation

Simple relationships are attenuated, but when we wish to "statistically control" for other factors (e.g., race differences in test scores *controlling for* SES), measurement error in the control variable means we ***do not adequately control*** for that variable

II. Means

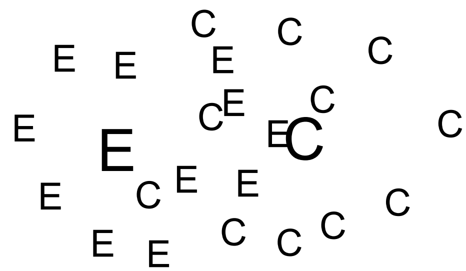
Remember that random measurement error sometimes leads to overestimates and sometimes leads to underestimates

On average the estimate will be accurate



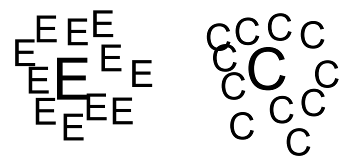
II. Means

Comparing two means – a lot of random variation



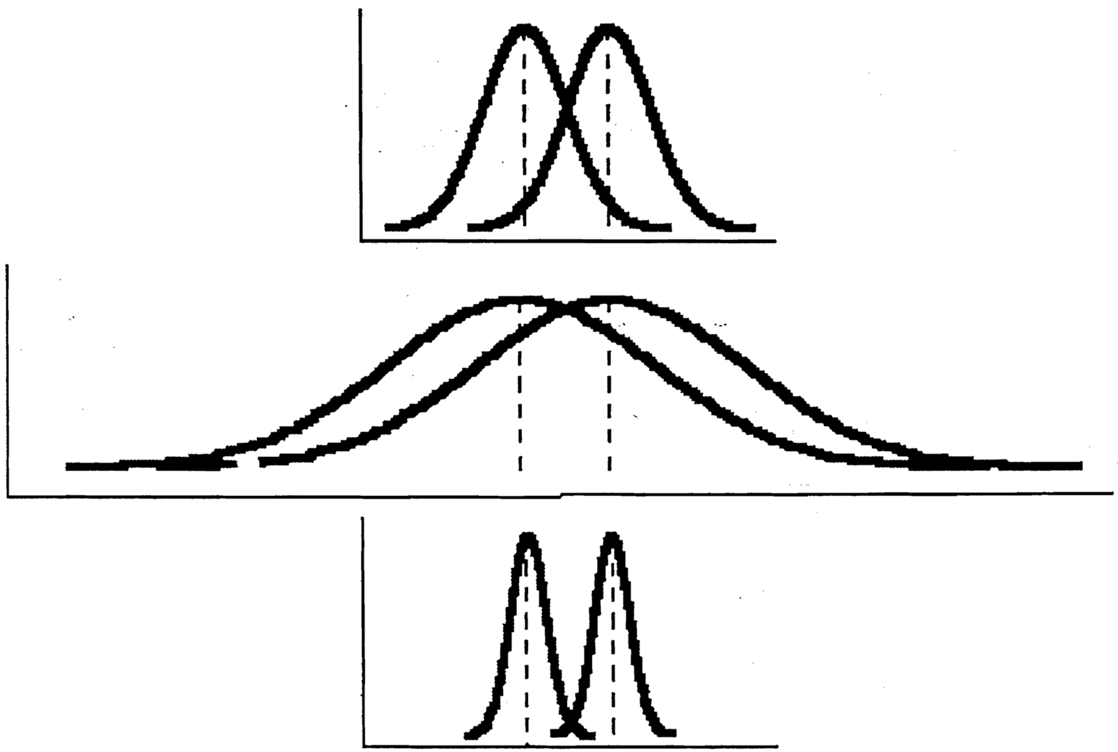
II. Means

Comparing two means – little random variation



II. Means

Comparing two means



II. Means

Comparing means

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

If X_1 and X_2 observed scores have larger variance ($s_1{}^2$ and $s_2{}^2$) than their true score counterparts, then the denominator will be larger and the t will be smaller, so ***less likely to be significant***

II. Means

Comparing means

$$d_{x_o} = \frac{\left|\bar{X}_{o1} - \bar{X}_{o2}\right|}{\sqrt{\dfrac{s_{o1}^2 + s_{o2}^2}{2}}}$$

Also seen in the estimate of the effect size, which gives the magnitude of the group difference (where $o1$ and $o2$ subscripts indicate observed values for group one and two)

Over time: Regression Toward The Mean

Regression toward the mean is a phenomenon in which high initial scores will tend to be lower on retest and

Low initial scores will tend to be higher on retest

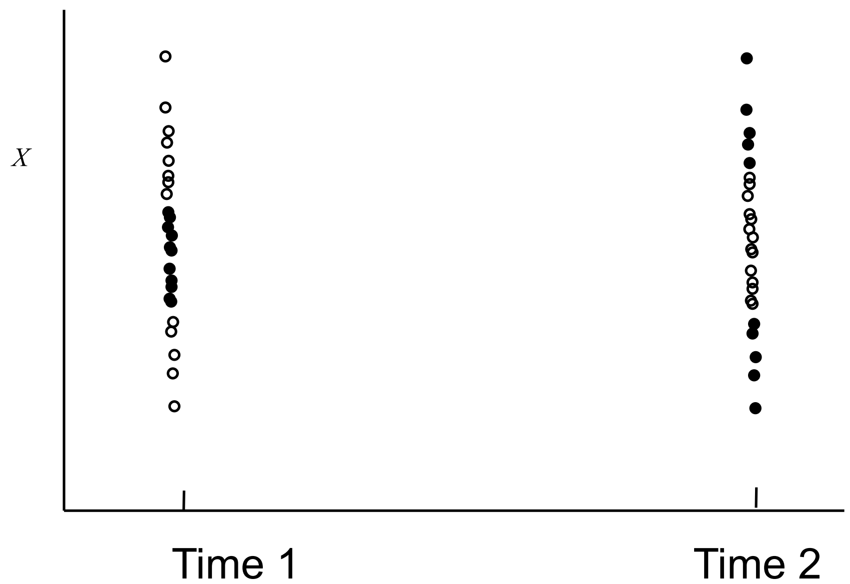
Both extremes move toward the middle

(and the middle scores move toward the extremes)

The more random error due to measurement error, the more likely this is to occur

Over time: Regression Toward The Mean

Figure



Over time: Regression Toward The Mean

Figure