Correlation Example[1]

Time since Ph.D.			**Publications**			
X	$X-\bar{X}$	$(X-\bar{X})^2$	Y	$Y-\bar{Y}$	$(Y-\bar{Y})^2$	$(X-\bar{X})(Y-\bar{Y})$
3	-4.67	21.81	**18**	-1.93	3.74	9.02
6	-1.67	2.79	**3**	-16.93	286.74	28.22
3	-4.67	21.81	**2**	-17.93	321.60	83.69
8	0.33	0.11	**17**	-2.93	8.60	-0.98
9	1.33	1.77	**11**	-8.93	79.80	-11.91
6	-1.67	2.79	**6**	-13.93	194.14	23.22
16	8.33	69.39	**38**	18.07	326.40	150.56
10	2.33	5.43	**48**	28.07	787.74	65.49
2	-5.67	32.15	**9**	-10.93	119.54	61.96
5	-2.67	7.13	**22**	2.07	4.27	-5.51
5	-2.67	7.13	**30**	10.07	101.34	-26.84
6	-1.67	2.79	**21**	1.07	1.14	-1.78
7	-0.67	0.45	**10**	-9.93	98.67	6.62
11	3.33	11.09	**27**	7.07	49.94	23.56
18	10.33	106.71	**37**	17.07	291.27	176.36
$\bar{X}=7.67$		$\sum(X-\bar{X})^2=293.33$	$\bar{Y}=19.93$		$\sum(Y-\bar{Y})^2 = 2674.93$	$\sum(X-\bar{X})(Y-\bar{Y}) = 581.67$

$$r=\frac{\sum(X-\bar{X})(Y-\bar{Y})}{\sqrt{\sum(X-\bar{X})^2\sum(Y-\bar{Y})^2}} = \frac{581.67}{\sqrt{(293.33)(2674.93)}} = .66$$

$$t=\frac{r\sqrt{N-2}}{\sqrt{1-r^2}} = \frac{.66\sqrt{13}}{\sqrt{1-.66^2}} = \frac{2.38}{.75} = 3.17$$

$$df = N-2 = 15-2 = 13,\ t_{crit,\alpha=.05} = 2.160$$

Confidence limits seem to be rarely reported, perhaps because software packages have often not computed them in the past and perhaps because they are a bit complicated to calculate than other types of confidence intervals.[2] They are useful for having a sense of sampling variability of the sample correlation estimate, however, and should probably be used more often. To compute them, the r value must be converted to a z value first, then the intervals are calculated, and then the interval values are converted back to the r scale. You will not need to compute these by hand, but just for illustrating, I have included the computation ($\ln$ is the natural log function and e is Euler's mathematical constant).

$$z=.5\left[\ln\left(\frac{1+r}{1-r}\right)\right]=.5\left[\ln\left(\frac{1+.66}{1-.66}\right)\right]=.787$$

$$SE_z=\sqrt{\frac{1}{n-3}}=\sqrt{\frac{1}{15-3}}=.289$$

$$z\pm(1.96)(SE_z)=.787\pm(1.96)(.289)=.222,1.353$$

$$r=\frac{e^{2z}-1}{e^{2z}+1}=\frac{e^{2\times.222}-1}{e^{2\times.222}+1}=\frac{.559}{2.559}=.218$$

$$r=\frac{e^{2z}-1}{e^{2z}+1}=\frac{e^{2\times1.353}-1}{e^{2\times1.353}+1}=\frac{13.969}{15.969}=.875$$

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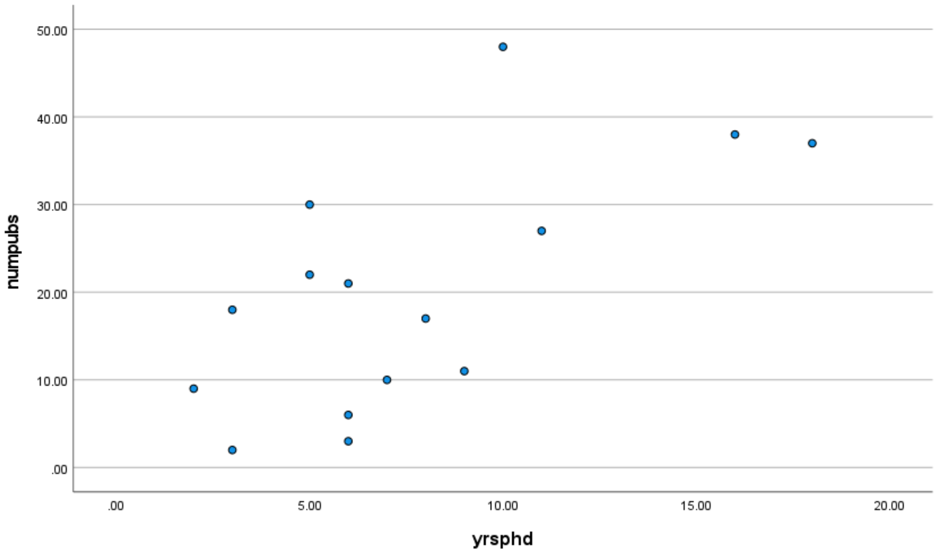
[1] Numerical example from Cohen, Cohen, West, and Aiken (2003) Table 2.2.2. Note that standard error in this text is SE rather than s, which was used in Myers et al. text, and the current book often uses r_{XY} in place of r in many places.

[2] Until the most recent version of SPSS, confidence intervals for correlations were not available through the CORRELATIONS procedure. They are available now, so I don't anticipate that you will ever need to compute confidence intervals manually. But just in case the occasion arises, I have created an Excel spreadsheet that will do these computations automatically here: http://web.pdx.edu/~newsomj/uvclass/correlation confidence intervals.xlsx.

SPSS Syntax

```
graph
  /scatterplot(bivar)=numpubs with yrsphd.

 correlations vars=yrsphd numpubs
        /ci.
```

Correlations



Correlations

		yrsphd	numpubs
yrsphd	Pearson Correlation	1	.657
	Sig. (2-tailed)		.008
	N	15	15
numpubs	Pearson Correlation	.657	1
	Sig. (2-tailed)	.008	
	N	15	15

Confidence Intervals

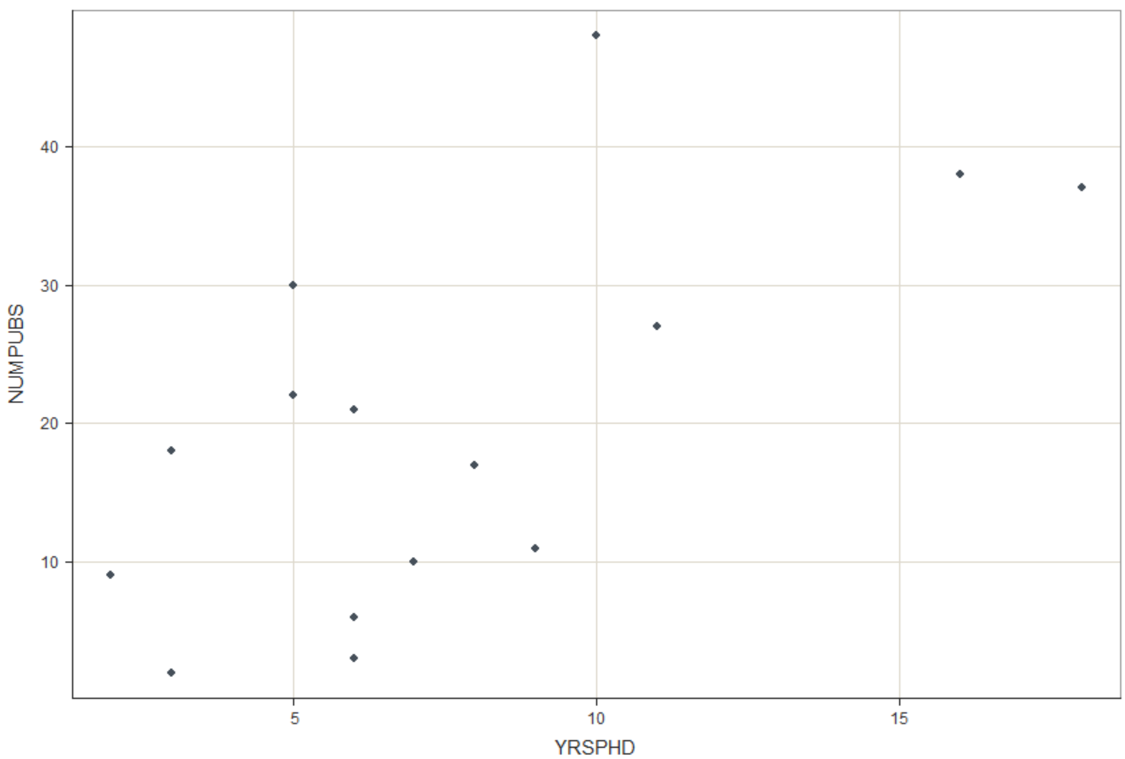
	Pearson Correlation	Sig. (2-tailed)	95% Confidence Intervals (2-tailed)[a]	
			Lower	Upper
yrsphd - numpubs	.657	.008	.218	.875

a. Estimation is based on Fisher's r-to-z transformation.

R

```
> #clear active frame from previous analyses
> rm(d)
> #clear console
> cat("\014")
> library(haven)
> d = read_sav("c:/jason/spsswin/uvclass/ccwa2_2_2.sav")
>
>
> #add digits to get more digits for CIs
> Correlation(YRSPHD, NUMPUBS)

Correlation Analysis for Variables YRSPHD and NUMPUBS
----------------------------------------------------
>>> Pearson's product-moment correlation
```

```
Number of paired values with neither missing, n = 15
Number of cases (rows of data) deleted: 0


Sample Covariance: s = 41.548

Sample Correlation: r = 0.657


Hypothesis Test of 0 Correlation:  t = 3.139,  df = 13,  p-value = 0.008
95% Confidence Interval for Correlation:  0.218 to 0.875

> #an alternative method[3]
> #base R function
> #cor.test(d$YRSPHD,d$NUMPUBS)


> #scatterplot (this function also reports the correlation)
> ScatterPlot(YRSPHD, NUMPUBS)

>>> Pearson's product-moment correlation

Number of paired values with neither missing, n = 15


Sample Correlation of YRSPHD and NUMPUBS: r = 0.657


Hypothesis Test of 0 Correlation:  t = 3.139,  df = 13,  p-value = 0.008
95% Confidence Interval for Correlation:  0.2 to 0.9
```



Example Write-up
A Pearson correlation coefficient was computed to examine the relationship between the amount of time since a faculty member received a PhD and the number of peer-reviewed publications. There was a significant positive correlation between the time since receiving the PhD and the number of publications, r = .66, p = .008, 95% [CI = .22, .88]. Approximately 44% of the variance was shared between the two variables, r^2 = .44.

[3] `lessR` will also generate a correlation matrix, but the significance tests of the correlations are not provided.
```
Correlation(c(x1,x2,x3))
```