2 X 2 Contingency Chi-square

The 2 X 2 contingency chi-square is used for the comparison of two groups with a dichotomous dependent variable. We might compare males and females on a yes/no response scale, for instance.

The contingency chi-square is based on the same principles as the simple chi-square analysis in which we examine the expected vs. the observed frequencies. The computation is quite similar, except that the estimate of the expected frequency is a little harder to determine.

Let's use data from the YouGov survey to examine whether voters in the youngest age group (18-24) are more or less likely to have voted in the previous election than older voters.[1] Here are the frequencies:

	Did not vote	Voted	
All other ages	183	824	1007
Youngest	35	50	85
	218	874	1092

To answer the question whether younger or older voters were more likely to vote, we are making a comparison of the proportion of younger responents who voted, $50/85 = .588$, or 58.8%, to the proportion of older respondents who voted, $824/1007 = .818$, or 81.8%. So, the table appears to suggest that older people are much more likely to vote than younger people. Notice that this is a comparison of the conditional proportions, which correspond to column percentages in cross-tabulation output.[2]

First, we need to compute the expected frequencies for each cell. R_1 is the frequency for row 1, C_1 is the frequency for row 2, and N is the total sample size. The first cell is:

$$E_{11} = \frac{R_1 C_1}{N} = \frac{(1007)(218)}{1092} = 201.03$$

Filling in the rest of the cells in the same way for each expected value, E_{ij}, using the same equation but by using frequencies from the corresponding row R_i and column C_j for each cell, I obtained the following expected values:

$E_{11} = \dfrac{(1007)(218)}{1092} = 201.03$	$E_{12} = \dfrac{(1007)(874)}{1092} = 805.97$
$E_{21} = \dfrac{(85)(218)}{1092} = 16.97$	$E_{22} = \dfrac{(85)(874)}{1092} = 68.03$

$$\chi^2 = \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}} = \frac{(183-201.03)^2}{201.03} + \frac{(824-805.97)^2}{805.97} + \frac{(35-16.97)^2}{16.97} + \frac{(50-68.03)^2}{68.03}$$

$$= 1.62 + 0.40 + 19.16 + 4.78$$

$$= 25.96$$

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[1] These results use a random sample taken from a YouGov/Harvard/MIT survey called the Cooperation Election Study (CES) conducted by Stephen Ansolabehere & Brian Schaffner, 2022, "CES Common Content, 2021", https://doi.org/10.7910/DVN/OPQOCU

[2] There are other questions we might ask, of course. Asking whether the proportion of nonvoters who are older is higher than the proportion of voters who are older is an equivalent question to the one above (comparison of conditional row proportions rather than conditional column proportions). We also might ask whether younger respondents are more likely to vote than not vote, which is a simple two-cell comparison among younger respondents, which would be made by simply selecting out younger voters and using the z-proportions or chi-square test previously discussed.

The result of the chi-square is compared to the tabled critical value based on $df = (R - 1)(C - 1)$, where R and C represent the number of rows and the number of columns, respectively.[3] So, with $df = (R - 1)(C - 1) = 1$, the critical value is 3.84, and the computed value is significant.

Minimum Expected Frequencies and Fisher's Exact Test
Fisher's exact test, proposed by R.A. Fisher (Fisher, 1935) and sometimes called the "Fisher-Irwin" test (or just "exact test" or "exact p-value" in computer outputs), is often printed along with the Pearson χ^2. It is not so much a modification of the chi-square test as an alternative approach to testing the association between two binary variables for significance. The test has been suggested for use with small samples in which the expected frequencies in some cells are low. The concept is to use the hypergeometric distribution to compute the exact probability of the particular configuration of obtained frequencies. The problem with Fisher' exact test is that it can be overly conservative and its use is often recommended when not necessary. Some software packages print a warning when 20% of the cells have an expected frequency below 5 (known as Cochran's rule). First thing to notice, however, is that it is the expected frequency that is of concern and not the observed frequency. Secondly, simulation studies (e.g., Camilli & Hopkins, 1978) suggest that Pearson's χ^2 has nominal alpha values with expected values as low as 1 as long as the total sample size is 20 or larger. So, the upshot is that Fisher's exact test is not needed in very many circumstances.

Yates' Continuity Correction
Yates suggested a correction to the Pearson's χ^2 based on the notion that a test of discrete variables should follow a discrete distribution are tested using a normal approximation, the chi-squared distribution. The Yates' correction for continuity (sometimes called just "continuity correction" in computer output) is a simple modification of the chi-squared test formula by subtracting ½ or .5 from the frequency difference.

$$\chi^2 = \sum \frac{\left(\left| O_i - E_{ij} \right| - .5 \right)^2}{E_{ij}}$$

There is good evidence and fairly wide consensus that the results with the Yates correction are too conservative (e.g., Grizzle, 1967; Camilli & Hopkins, 1978).

Magnitude of Effect
The most commonly used effect size measure associated with the 2 × 2 chi-square test is phi, ϕ (the Greek lower case "fee", as pronounced by statisticians). Phi is a simple computation, based on chi-square.

$$\phi = \sqrt{\frac{\chi^2}{N}}$$

According to Cohen's (1992) guidelines, .1 is a small effect, .3 is a medium effect, and .5 is a large effect. Cramer's V is used for more than a 2 × 2 chi-square, and it is equivalent to phi for the 2 × 2 design. It is also the case the Cohen's w is equivalent to phi in this circumstance, and if you look back at the two-group chi-square, you will see that the computations are the same. Cohen's w can be used for any chi-square test, whether for a one-or two-dimensional table or other. These are all what Howell (2010) refers to as r-type effect size measures, because, as we will soon see, phi is the same as the Pearson correlation coefficient. Howell also discusses what he calls d-type effect size measures, odds ratios and relative risk, and we will discuss those next term when we discuss logistic regression.

Three or More Dimensions
Although 2 × 2 contingency table looks like a 2 × 2 factorial table (to be discussed later in the term), they are not analogous. The homogeneity conceptualization of chi-squared tests involves a two-group comparison of a binary outcome, which is analogous to a t-test in the continuous case. Because one of the columns (or rows)

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[3] I use Howell's notation, which is understandably confusing in this case, because above R_i and C_j refer to the frequencies (i.e., number of cases in a row or column) and here the R and C without subscript refer to the number of rows and columns.

is for the dependent variable, it is really the three-way table that is analogous to the factorial design in ANOVA, which requires an analysis of a three-way contingency table (2 × 2 × 2) in the binary outcome case (to be discussed later this term and next term in greater depth).

Partitioning

The chi-squared values for the set of all possible *orthogonal* (or independent) chi-squares add up to the chi-square for the whole design. The likelihood ratio test (discussed next term), G^2, however, cannot be partitioned in the same way. Planned follow-up analyses to a significant Pearson χ^2 for contingency tables are simply chi-square analyses based on chi-squared tests for two or more cell comparisons, including smaller contingency tables (e.g., a 2 × 2 from a 5 × 3 design; Delucchi, 1993). Such tests may involve marginal proportions or individual cell proportions as well.

Chi-square Software Examples

SPSS
Syntax

```
crosstabs /tables=youth by voted
  /cells=count row column expected
 /statistics=chisq phi.
```

Menus

1. Analyze→Descriptive statistics→ crosstabs
2. Move variables over
3. Click on "statistics"
4. Check Chi-square box and the Phi and Cramer's V box, click "continue," click "ok"
5. Click on "cells" and choose row and column percentages.

youth * voted voted in 2020 general election Crosstabulation

			voted voted in 2020 general election 0 no	1 yes	Total
youth	.00 older ages	Count	183	824	1007
		Expected Count	201.0	806.0	1007.0
		% within youth	18.2%	81.8%	100.0%
		% within voted voted in 2020 general election	83.9%	94.3%	92.2%
	1.00 youngest age group	Count	35	50	85
		Expected Count	17.0	68.0	85.0
		% within youth	41.2%	58.8%	100.0%
		% within voted voted in 2020 general election	16.1%	5.7%	7.8%
Total		Count	218	874	1092
		Expected Count	218.0	874.0	1092.0
		% within youth	20.0%	80.0%	100.0%
		% within voted voted in 2020 general election	100.0%	100.0%	100.0%

Chi-Square Tests

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	25.960 [a]	1	<.001		
Continuity Correction [b]	24.540	1	<.001		
Likelihood Ratio	21.952	1	<.001		
Fisher's Exact Test				<.001	<.001
Linear-by-Linear Association	25.936	1	<.001		
N of Valid Cases	1092				

a. 0 cells (0.0%) have expected count less than 5. The minimum expected count is 16.97.

b. Computed only for a 2x2 table

Symmetric Measures

		Value	Approximate Significance
Nominal by Nominal	Phi	-.154	<.001
	Cramer's V	.154	<.001
N of Valid Cases		1092	

R

```
#this lessR BarChart function produces a chi-square test by default
BarChart (voted, by=youth, horiz = FALSE, stat = "proportion", beside = TRUE)


Joint and Marginal Frequencies
-----------------------------
      voted
youth      0    1   Sum
    0    183  824  1007
    1     35   50    85
  Sum    218  874  1092
```

```
Cramer's V (phi): 0.154


Chi-square Test of Independence:
     Chisq = 25.960, df = 1, p-value = 0.000


Cell Proportions within Each Column
-----------------------------------


      voted
youth         0       1
  0       0.839   0.943
  1       0.161   0.057
  Sum     1.000   1.000
```
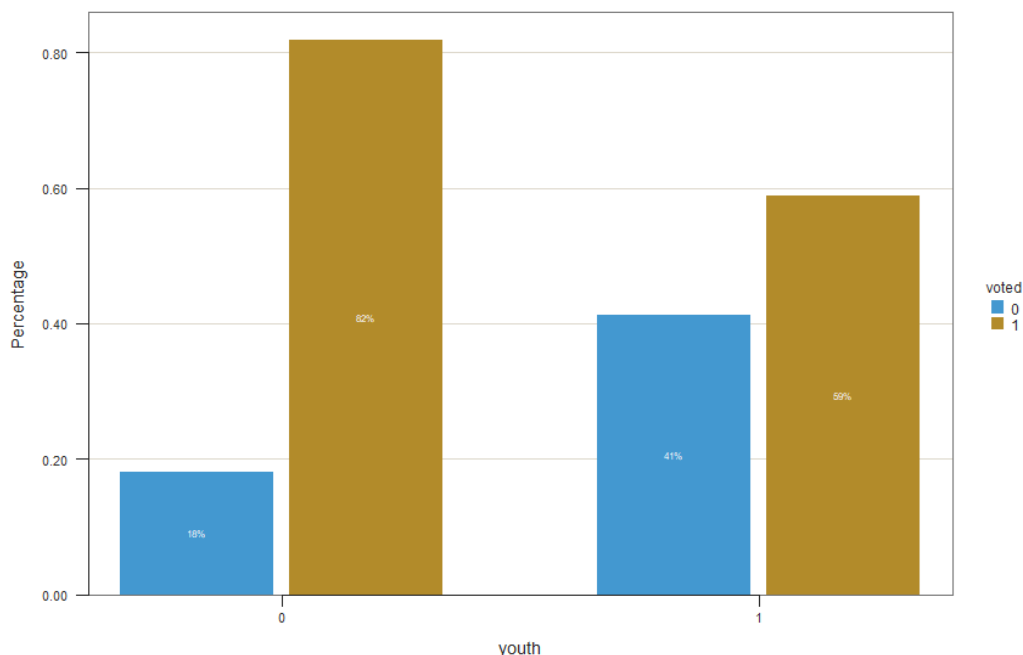
```
#I switched the voted and youth variables to the youth variable on the x-axis
BarChart (youth, by=voted, horiz = FALSE, stat = "proportion", beside = TRUE)
```


To get marginal frequencies and proportions in R

```
> tbl = table(d$youth, d$voted)
> tbl

     0   1
  0 183 824
  1  35  50
> #and get marginal and cell proportions
> #margin.table(tbl, 1) # Frequencies summed over voted
> margin.table(tbl, 2) # Frequencies summed over youth

  0   1
218 874

> #prop.table(tbl) # cell proportions
> #note that rows are 0 for older and 1 for younger
> #and columns are 0 for not voted and 1 for voted
> prop.table(tbl, 1) # row proportions (within each level of youth)

          0         1
  0 0.1817279 0.8182721
  1 0.4117647 0.5882353

> #alternative base R method of getting the chi-square
> chisq.test(tbl,correct = FALSE)  #correct = FALSE turns off Yates continuity correction

Pearson's Chi-squared test
```

```
data:  tbl
X-squared = 25.96, df = 1, p-value = 0.0000003486
```

Example write-up. A chi-square test was used to determine whether there was a significant difference between the proportion of younger and older respondents who voted. Results indicated that 58.8% of younger respondents voted, whereas 81.8% of older respondents voted. This difference was significant, $\chi^2(1) = 25.96$, $p < .001$ The phi coefficient, $\phi = -.15$, suggested a small effect.

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