Table 6.4: A taxonomy of polynomial individual change trajectories

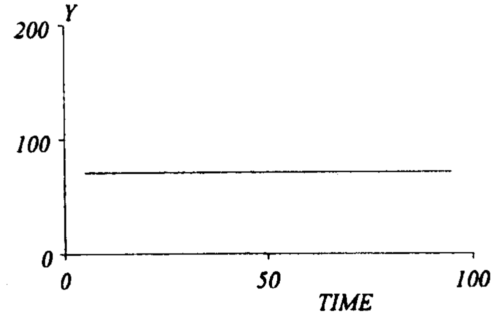
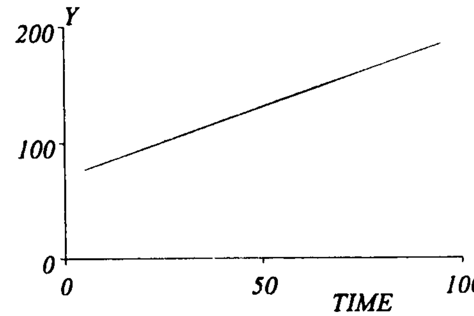
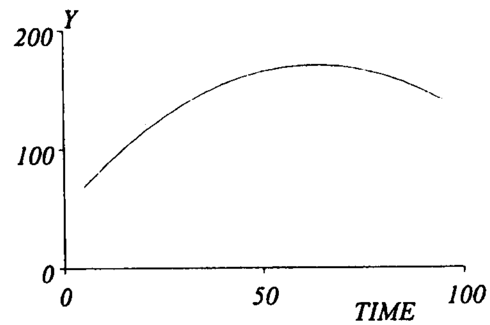
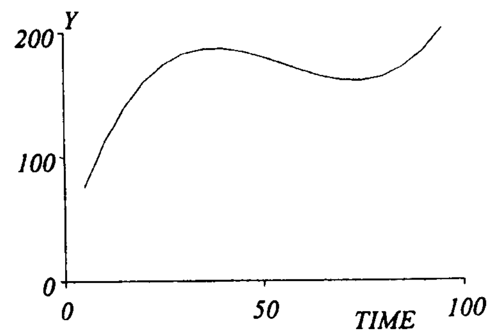
		Illustrative example	
Shape	Level-1 model	Parameter values	Plot of the true change trajectory
No change	$Y_{ij} = \pi_{0i} + \varepsilon_{ij}$	$\pi_{0i} = 71$	
Linear change	$Y_{ij} = \pi_{0i} + \pi_{1i}TIME_{ij} + \varepsilon_{ij}$	$\pi_{0i} = 71$ $\pi_{1i} = 1.2$	
Quadratic change	$Y_{ij} = \pi_{0i} + \pi_{1i}TIME_{ij} + \pi_{2i}TIME_{ij}^2 + \varepsilon_{ij}$	$\pi_{0i} = 50$ $\pi_{1i} = 3.8$ $\pi_{2i} = -0.03$	
Cubic change	$Y_{ij} = \pi_{0i} + \pi_{1i}TIME_{ij} + \pi_{2i}TIME_{ij}^2 + \pi_{3i}TIME_{ij}^3 + \varepsilon_{ij}$	$\pi_{0i} = 30$ $\pi_{1i} = 10$ $\pi_{2i} = -.2$ $\pi_{3i} = .0012$	
⋮	⋮	⋮	⋮

Table 6.7: Selected curvilinear trajectories, and level-1 models, used for truly nonlinear change over time

Family	Specific curve	Level-1 model
Hyperbolic	Rectangular hyperbola	$Y_{ij} = \alpha_i - \frac{1}{\pi_{1i} TIME_{ij}} + \varepsilon_{ij}$
Inverse polynomial	Inverse quadratic	$Y_{ij} = \alpha_i - \frac{1}{(\pi_{1i} TIME_{ij} + \pi_{2i} TIME_{ij}^2)} + \varepsilon_{ij}$
Exponential	Simple exponential	$Y_{ij} = \pi_{0i} e^{(\pi_{1i} TIME_{ij})} + \varepsilon_{ij}$
	Negative exponential	$Y_{ij} = \alpha_i - (\alpha_i - \pi_{0i}) e^{-\pi_{1i} TIME_{ij}} + \varepsilon_{ij}$
	Logistic	$Y_{ij} = \alpha_{1i} + \frac{(\alpha_{2i} - \alpha_{1i})}{(1 + \pi_{0i} e^{-\pi_{1i} TIME_{ij}})} + \varepsilon_{ij}$

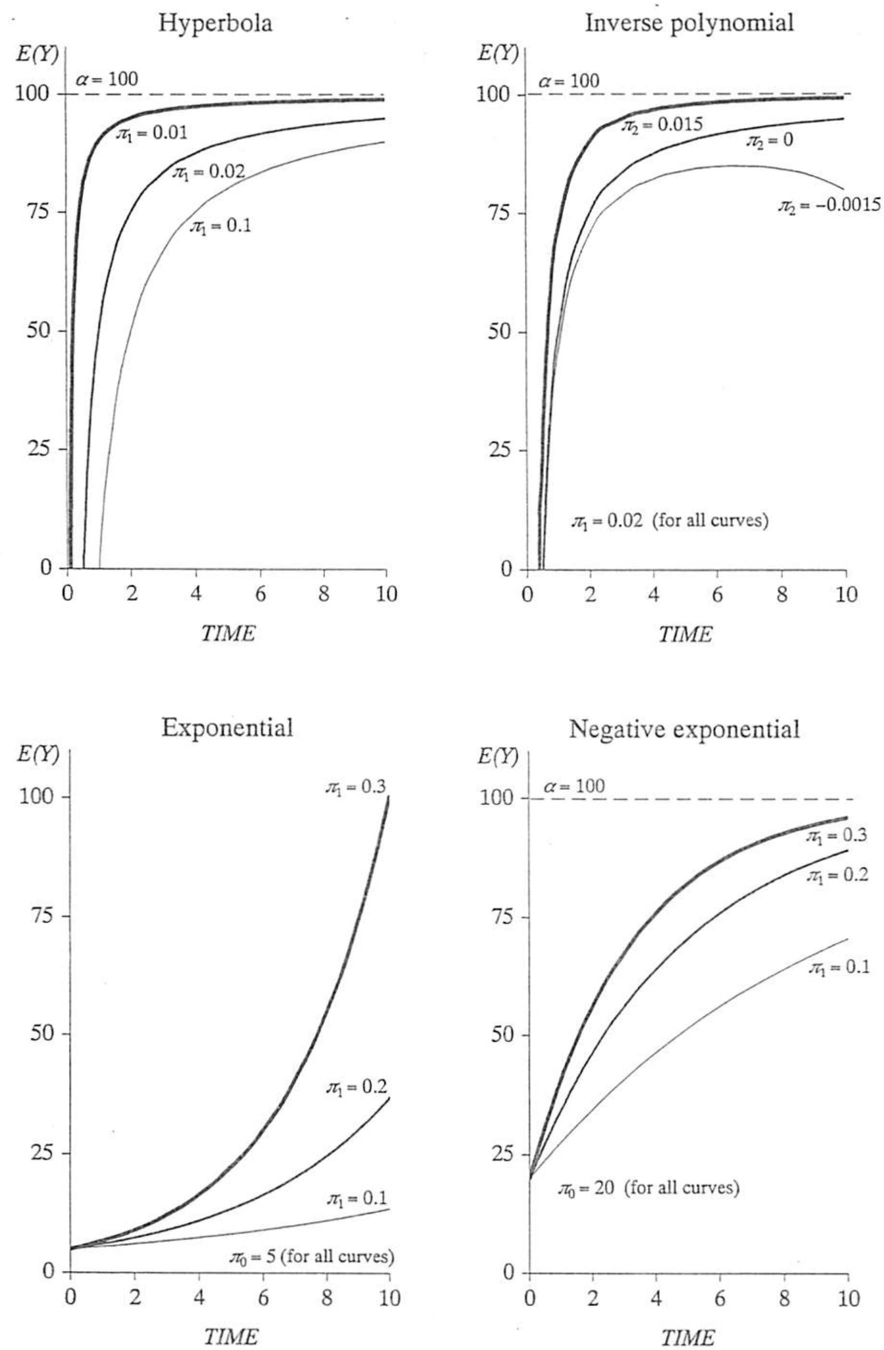


Figure 6.11. Understanding how alternative nonlinear change trajectories represent different patterns of change over time.

nate *E(Y)*, the "population expectation of *Y*." For clarity, we also drop the subscript *i*.

All three hyperbolae decelerate smoothly toward a ceiling, regardless of the parameters' specific values. What role does each parameter play in determining the trajectory's shape? To highlight these roles, we have selected the hyperbolic trajectories in figure 6.11 so that α_i is constant at

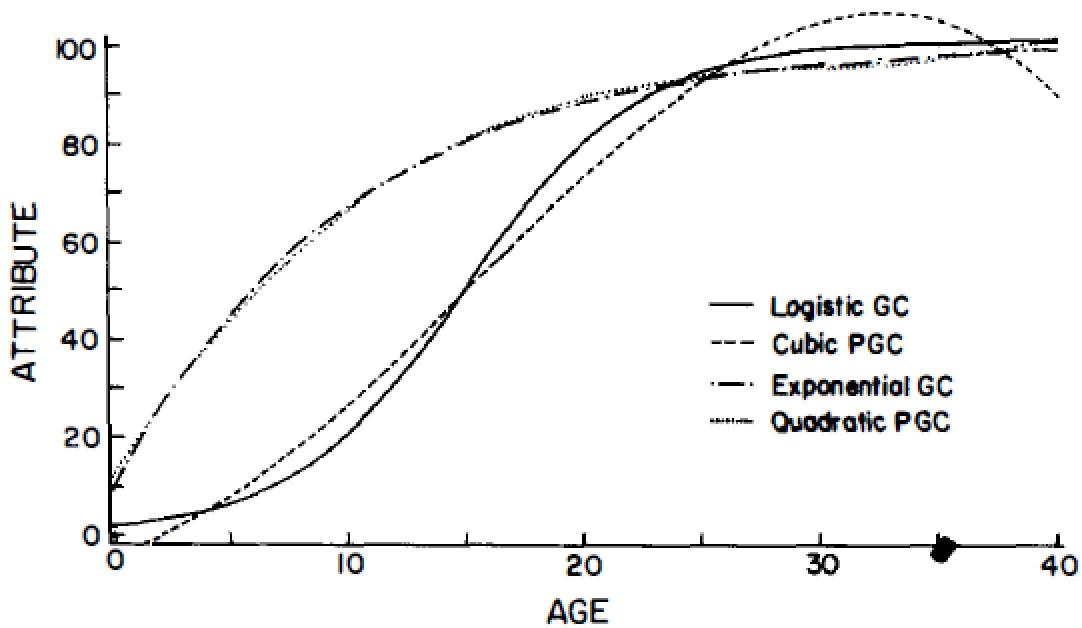


FIG. 1.—Plots of nonlinear growth curves and their polynomial approximations

from Burchinal, M., & Appelbaum, M. I. (1991). Estimating individual developmental functions: Methods and their assumptions. *Child development, 62*, 23-43.

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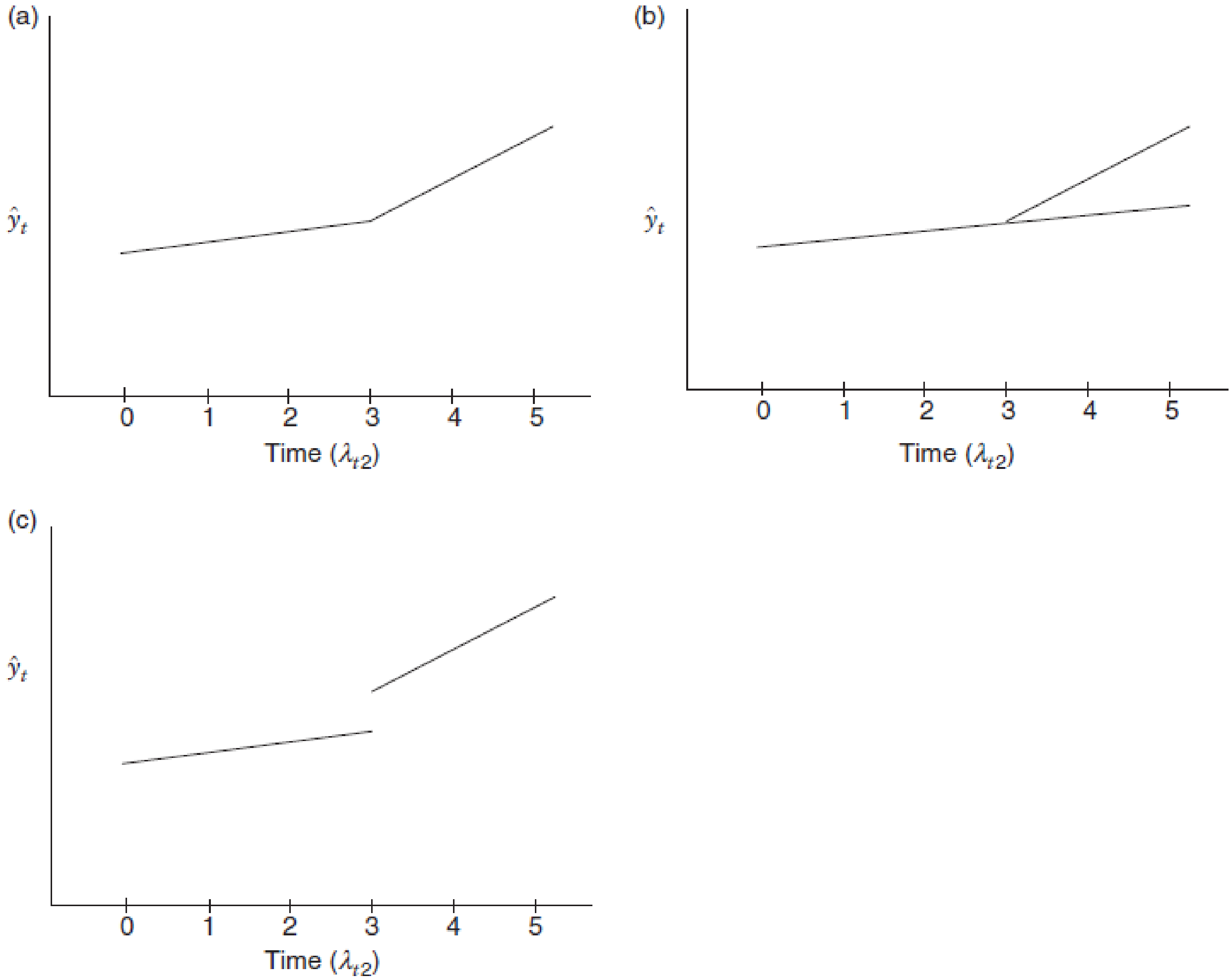

Figure 8.2 Plots Illustrating Piecewise Slope Patterns: (a) differing slopes; (b) increment (decrement) model; (c) differing slopes and intercepts.

From Newsom, J.T. (2015). *Longitudinal Structural Equation Modeling: A Comprehensive Introduction*. New York: Routledge.

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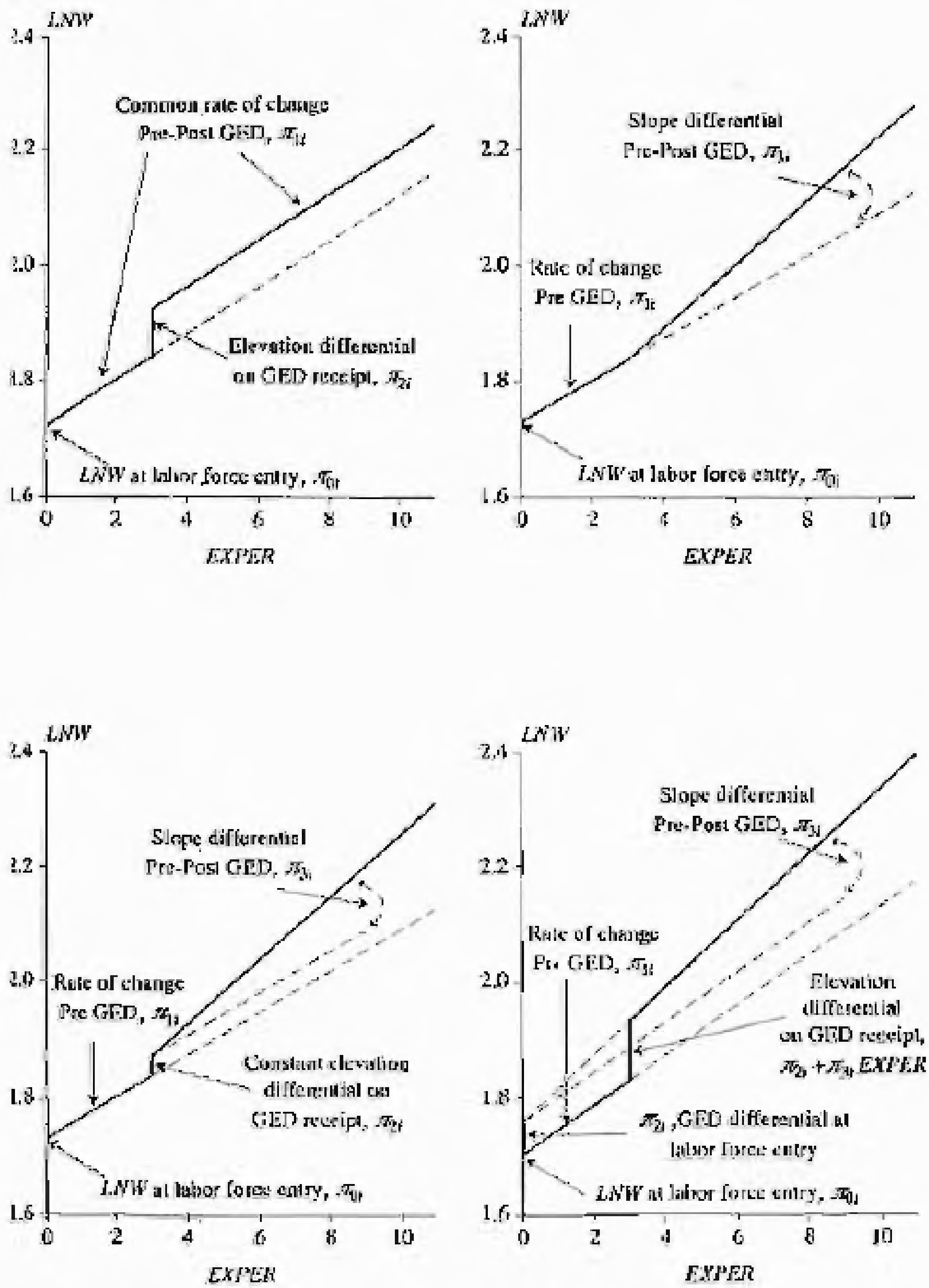

gure 6.2. Alternative discontinuous change trajectories for the high school dropout
ge data.

From Singer & Willett, (2003). *Applied longitudinal data analysis: Modeling change and event occurrence.* New York: Oxford University Press.

TABLE 6.9 Some Possible Coding Schemes for a Two-Piece Linear Model (Reading Achievement Example)

(a) Two-Rate Model

	Grades 1	2	3	4	5	6	Interpretation of πs:
a_{1t}	0	1	2	2	2	2	π_1 growth rate period 1
a_{2t}	0	0	0	1	2	3	π_2 growth rate period 2
							π_0 status Grade 1
a_{1t}	−2	−1	0	0	0	0	π_1 growth rate period 1
a_{2t}	0	0	0	1	2	3	π_2 growth rate period 2
							π_0 status Grade 3

(b) Increment (Decrement) Model

	Grades 1	2	3	4	5	6	Interpretation of πs:
a_{1t}	0	1	2	3	4	5	π_1 base growth rate
a_{2t}	0	0	0	1	2	3	π_2 increment (decrement) to growth in period 2
							π_0 status Grade 1
a_{1t}	−2	−1	0	1	2	3	π_1 base growth rate
a_{2t}	0	0	0	1	2	3	π_2 increment (decrement) to growth in period 2
							π_0 status Grade 3

From Raudenbush, S.W., & Bryk, A.S., (2002) *Hierarchical linear models: Applications and data analysis methods.* Thousand Oaks, CA: Sage