

Simple Slope Tests of Cross-level Interactions

Defining Simple Slopes

Simple slopes involve the regression equation for one predictor at specific levels of a second predictor, usually termed a "moderator." Finding simple slopes is not a difficult matter, but testing them for significance is more complicated. In this handout, I illustrate simple slopes for a cross-level interaction.

Curran and Bauer (2006) illustrated how to test simple slopes with multilevel models. To be consistent with notation in their paper and with Kris Preacher's online calculator for simple slope tests (see link below), I use a slightly different notation (Bauer & Curran 2005, Preacher, Curran, & Bauer, 2006) with w instead of z as the level-2 variable.

The two-level equations for a cross-level interaction with one predictor at each level are below:

$$\begin{aligned}y_{ij} &= \beta_{0j} + \beta_{1j}x_{ij} + r_{ij} \\ \beta_{0j} &= \gamma_{00} + \gamma_{01}w_j + u_{0j} \\ \beta_{1j} &= \gamma_{10} + \gamma_{11}w_j + u_{1j}\end{aligned}$$

Substituting the second level-equations into the first-level equation gives the single multilevel equation, with $x_{ij}w_j$ as the cross-level interaction term and γ_{11} as the slope for the interaction.

$$y_{ij} = (\gamma_{00} + \gamma_{10}x_{ij} + \gamma_{01}w_j + \gamma_{11}x_{ij}w_j) + (u_{0j} + u_{1j}x_{ij} + r_{ij})$$

If we revert to the expected or predicted regression equation by removing the error terms and doing a little rearranging of terms, we have:

$$\hat{y}_{ij} = (\gamma_{00} + \gamma_{01}w_j) + (\gamma_{10} + \gamma_{11}w_j)x_{ij}$$

The first parenthetical term can be thought of as an intercept (which Curran and Bauer call ω_0) and the second parenthetical term can be thought of as the simple slope (called ω_1). A particular value for w_j , the level-2 moderator, can be substituted into the equation to estimate a certain simple slope for y regressed on x at that particular value of w . Usually more than one simple slope is calculated. If w is dichotomous, 0 and 1 might be used. If w is continuous, two or three values of w might be used, such as -1 S.D. below the mean of w , the mean of w , and +1 S.D. above the mean of w .

To test a simple slope for significance, values from the asymptotic covariance matrix (sometimes simply called "acov" or "covb") of fixed regression estimates is needed (a quantity not usually printed by default). The diagonal elements of this matrix are equal to the square of the standard error estimates (i.e., the variance error) for the intercept and slope fixed effects. The off-diagonal estimates represent the sample estimate of the covariation between fixed effects across the sampling distribution.

I've chosen the level-2 variable as a moderator, but the level-1 variable might possibly be thought of as the "moderator" instead. One can also test interactions between two level-1 variables or two level-2 variables.

Example of Simple Slope Significance Tests and Plots

Below I show how to obtain the asymptotic covariance elements (i.e., the estimated variances and covariance of coefficients in the sampling distribution) needed for simple slope significance tests and plots using SPSS, R, and HLM. For the significance tests, I use an online calculator, which requires the asymptotic covariance elements. As an example, I used the same model as the one illustrated in the

cross-level as in the handout "Cross-level Interaction Example: Predicting Random Slopes," where we found that there was a significant cross-level interaction between the sector variable and the (group-centered) SES variable. I did not center the binary sector variable but that might also be desirable. I recommend centering the level-2 variable whenever there is not a meaningful or desirable zero-point on the scale, and this is more likely to be the case in general than not.

SPSS

The asymptotic covariance matrix of fixed effects can be obtained in SPSS by adding COVB to the /PRINT subcommand.

```
MIXED mathach WITH cses sector  
/CRITERIA=MXITER(1000) SCORING(1)  
/METHOD = REML  
/PRINT = SOLUTION TESTCOV HISTORY COVB  
/FIXED = cses sector cses*sector| SSTYPE(3)  
/RANDOM = INTERCEPT cses | SUBJECT(schoolid) COVTYPE(UN).
```

Covariance Matrix for Estimates of Fixed Effects^a

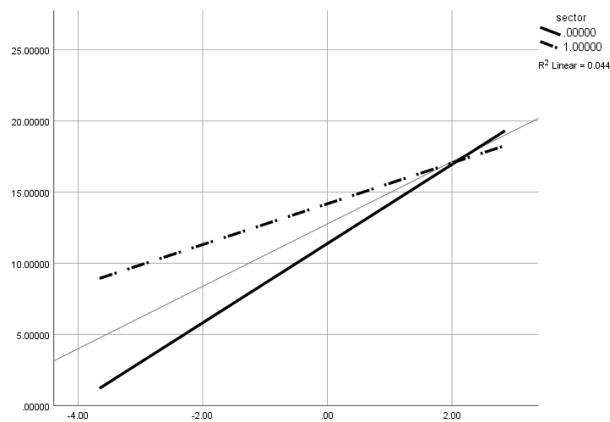
Parameter	Intercept	cses	sector	cses * sector
Intercept	.086	.012	-.086	-.012
cses	.012	.024	-.012	-.024
sector	-.086	-.012	.193	.027
cses * sector	-.012	-.024	.027	.055

a. Dependent Variable: mathach.

Note that the values in the matrix may be printed in scientific notation. For example, the variance error for γ_{00} might be 8.5848×10^{-2} instead of .085848.¹ You may notice that the values are close to the values obtained from the HLM and R programs but they may not be exactly the same.

For plotting the interaction, I modified the syntax I used earlier and replaced schoolid with sector.

```
GGRAPH  
/GRAPHDATASET NAME="GraphDataset" VARIABLES= cses mathach sector  
/GRAPHSPEC SOURCE=INLINE.  
BEGIN GPL  
SOURCE: s=userSource( id( "GraphDataset" ) )  
DATA: cses=col( source(s), name( "cses" ) )  
DATA: mathach=col( source(s), name( "mathach" ) )  
DATA: sector = col(source(s), name("sector"), unit.category())  
ELEMENT: point( position(cses * mathach))  
ELEMENT: line(position(smooth.linear(cses * mathach)), shape(sector))  
END GPL.
```



¹ It may be advisable to use more decimal places for greater precision. In the SPSS output, click on the Fixed Covariance Matrix for Estimates of Fixed Effects Box and then highlight the cells of the table in the panel at the left side and then increase the number of decimals in the Cell Format tab in the panel at the right side.

To remove the scatterplot points, click on the graph in the chart editor, choose the Options tab, Bin Element, Marker Tab and change the fill and border to the red diagonal line option (no fill/border).

My level-2 predictor (`sector`) was dichotomous. If there is a continuous level-2 predictor, you can create a categorical variable for the purposes of plotting. Assuming my centered level-2 variable was `csizes` and that the standard deviation for `csizes` was 604.1725, then this line of code would produce a new variable (`sizespts`) with three values that would replace `sector` in the above graphing code.

```
recode csizes (lo through -604.1725=-1) (-604.1725 thru 604.1725=0) (604.1725 thru hi=1) into sizespts.
```

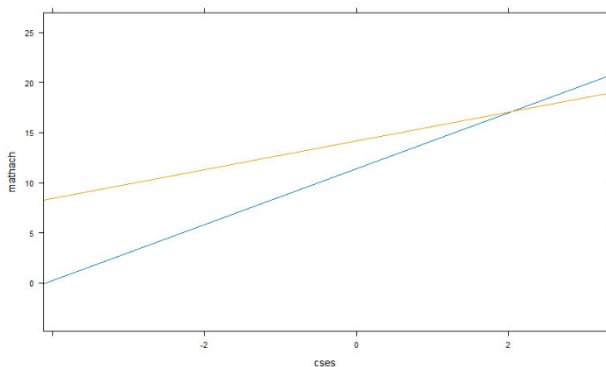
R
 To obtain the coefficient variance estimates in R, the `vcov()` function can be used to obtain a matrix of similar values:

```
> model1 <- lmer(mathach ~ cses + sector + cses*sector + (cses|schoolid), data = mydata, REML = TRUE)
> summary(model1)
```

Output omitted

```
> #obtain the asymptotic covariance values from the lme4 model for simple slopes
> library(merDeriv)
> vcov(model1)
4 x 4 Matrix of class "dgeMatrix"
      (Intercept)      cses      sector cses:sector
(Intercept) 0.08584760 0.01191263 -0.08584760 -0.01191263
cses         0.01191263 0.02401251 -0.01191263 -0.02401251
sector       -0.08584760 -0.01191263 0.19319402 0.02716079
cses:sector  -0.01191263 -0.02401251 0.02716079 0.05464643
```

```
> library(lattice)
> xyplot(mathach~cses, mydata, group = sector, type=c('r'))
```



If `csizes` is the continuous level-2 variable with standard deviation equal to 604.1725, then this base code function `cut` can be used to create a categorical variable with three values that can replace `sector` in the graphing code above.

```
mydata$sizespts <- cut(mydata$csizes, breaks = c(-Inf, -604.1725, 604.1725, +Inf), labels = c("-1SD", "mean", "+1SD"))
```

The `regelper` package has a function called `simple_slopes` that can be used with `lme4` results.

```
> simple_slopes(model1,
+   levels=list(sector=c(0, 1,'sstest'))) # test at values 0 and 1 for z variable
      cses sector Test Estimate Std. Error    df t value      Pr(>|t|) Sig.
1  sstest    0    2.8029    0.1539 142.9668 18.2131 < 0.00000000000000022 ***
2  sstest    1    1.4615    0.1739 162.3563  8.4043 0.00000000000020666 ***
3 -0.666583 sstest 3.6936    0.4229 152.0228  8.7336 0.000000000000004165 ***
4 -0.005995 sstest 2.8075    0.4363 155.5971  6.4344 0.000000001454909662 ***
5  0.654593 sstest 1.9214    0.4990 155.4003  3.8507      0.0001718 ***
```

sector=c(0,1, 'sstest') requests the simple slopes for levels of sector equal to 0 and 1. The simple slopes tests appear in the first two rows of the results and match the computations I obtained above with the online computation tool.

There is a reghelper graphing tool as well.

```
> graph_model(model1, y=mathach, x=cses, lines=sector)
```

Graph is omitted as it resembles the one above.

For a continuous level-2 variable, the reghelper simple_slopes function is easier if you want to use -1SD, 0, and +1 SD for the values. Just specify the model results. The example below is for the model with centered school size, CS es*csiz.

```
> library(reghelper)
> simple_slopes(model2) # test at default -1SD and +1SD
   cses      csiz  Test Estimate Std. Error    df t value      Pr(>|t|) Sig.
1 -0.666583  sstest   -0.0009   0.0004  157.1846  -2.1435    0.03362    *
2 -0.005995  sstest   -0.0005   0.0004  157.8698  -1.3028    0.19454
3  0.654593  sstest   -0.0001   0.0004  157.0743  -0.3399    0.73437
4  sstest -604.172499  1.8381    0.1806  157.7916  10.1798 < 0.0000000000000002 ***
5  sstest      0      2.1735    0.1258  154.4076  17.2753 < 0.0000000000000002 ***
6  sstest  604.172499  2.5089    0.1700  155.9938  14.7599 < 0.0000000000000002 ***
```

By default, the function tests both simple slopes (at levels of x and levels of z). The last three rows give the simple slopes for the -1SD, 0, and +1SD values of csiz.

For graphing, the default also uses -1SD and +1SD.

```
> graph_model(model2, y=mathach, x=cses, lines=csiz)
```

HLM

To obtain the necessary information about covariance matrix of coefficients in HLM, before running the model, choose **Output Settings** on the **Other Settings** menu. Then check the **Print variance-covariance matrices** box.

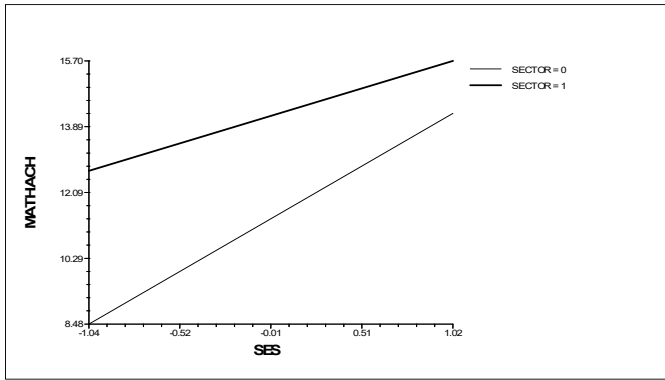
Below are the standard fixed effects estimates and random effects from the analysis. The variance covariance matrix of the regression estimates information are printed in a separate file called gamvc.dat stored in the folder where the .mdm and .hlm files are located.

The first row provides the fixed effects found in the output for γ_{00} , γ_{01} , γ_{10} and γ_{11} . These values are given as labels of the rows in columns of the 4 x 4 matrix, which appear in the order γ_{00} , γ_{01} , γ_{10} and γ_{11} .

```
  11.3938363    2.8074648    2.8024488   -1.3406339
  8.5722700E-002 -8.5722700E-002  1.1633490E-002 -1.1633490E-002
 -8.5722700E-002  1.9291038E-001 -1.1633490E-002  2.6525923E-002
  1.1633490E-002 -1.1633490E-002  2.4499393E-002 -2.4499393E-002
 -1.1633490E-002  2.6525923E-002 -2.4499393E-002  5.5709414E-002
```

For example, the variance error for γ_{00} is 8.57227×10^{-2} or .0857227.

Using HLM's graphing feature (**File** → **Graph Equations** → **Model Graphs**), and choosing SES as X focus and SECTOR as Z-focus, we get the following graph which helps illustrate the above simple slope results.



Simple Slope Significance Tests

Preacher and colleagues present an online calculator to compute the tests of simple slopes at <http://www.quantpsy.org/interact/hlm2.htm>. Case 3 is the cross-level interaction, and one simply inputs the appropriate values from the output and the asymptotic covariance matrix obtained in the gamvc.dat file. Below is a screen shot of the data I input into the calculator.

Case 3: x_1 : focal predictor; w_1 : moderator

$\hat{y} = \hat{\gamma}_{00} + \hat{\gamma}_{10}x_1 + \hat{\gamma}_{01}w_1 + \hat{\gamma}_{11}w_1x_1$					
Regression Coefficients		Coefficient Variances		Conditional Values	
$\hat{\gamma}_{00}$	11.3938363	$\hat{\gamma}_{00}$.085722700	$w_{1(1)}$	0
$\hat{\gamma}_{10}$	2.8024488	$\hat{\gamma}_{10}$.024499393	$w_{1(2)}$	1
$\hat{\gamma}_{01}$	2.8074648	$\hat{\gamma}_{01}$.19291038	$w_{1(3)}$	
$\hat{\gamma}_{11}$	-1.3406339	$\hat{\gamma}_{11}$.055709414	$x_{1(1)}$	
Coefficient Covariances		Degrees of Freedom*		$x_{1(2)}$	
$\hat{\gamma}_{00,01}$	-.085722700	df_{int}	158	$x_{1(3)}$	
$\hat{\gamma}_{10,11}$	-.024499393	df_{slp}	158	Other Information	
$\hat{\gamma}_{00,10}$.011633490	Reset		α	.05
$\hat{\gamma}_{01,11}$.026525923	Calculate			
<input type="checkbox"/> Check this box if w_1 is dichotomous					
Status:		Okay so far..			

I chose two values of the moderator SECTOR, 0 and 1, but with a continuous variable, you might use the value that corresponds with -1 S.D. below the mean of w , the mean of w , and +1 S.D. above the mean of w for estimating simple slopes of x . After calculating, these are the results, retaining the relevant output:

CASE 3 TWO-WAY INTERACTION SIMPLE SLOPES OUTPUT

Your Input

```
=====
w1(1)      = 0
w1(2)      = 1
Intercept  = 11.3938363
x1 Slope   = 2.8024488
w1 Slope   = 2.8074648
w1x1 Slope = -1.3406339
alpha      = 0.05
df(int)    = 158
df(slp)    = 158
```

Asymptotic (Co)variances

```
=====
var(g00) 0.0857227
var(g10) 0.02449939
var(g01) 0.19291038
var(g11) 0.05570941
cov(g00,g01) -0.0857227
cov(g10,g11) -0.02449939
cov(g00,g10) 0.01163349
cov(g01,g11) 0.02652592
```

```
Region of Significance on w (level-2 predictor)
=====
w1 at lower bound of region = 1.6391
w1 at upper bound of region = 2.9958
(simple slopes are significant *outside* this region.)

Simple Intercepts and Slopes at Conditional Values of w
=====
At w1(1)...
  simple intercept = 11.3938(0.2928), t=38.9154, p=0
  simple slope    = 2.8024(0.1565), t=17.9044, p=0
At w1(2)...
  simple intercept = 14.2013(0.3274), t=43.3766, p=0
  simple slope     = 1.4618(0.1767), t=8.2746, p=0
```

SECTOR 0 (public) had a steeper slope (stronger relationship) between SES and math achievement, $\gamma = 2.802$, $p < .001$ than SECTOR 1 (Catholic), $\gamma = 1.462$, $p < .01$.

References

- Bauer, D.J., & Curran, P.J. (2005). Probing interactions in fixed and multilevel regression: Inferential and graphical techniques. *Multivariate Behavioral Research*, 40, 373-400.
- Preacher, K.J., Curran, P.J., & Bauer, D.J. (2006). Computational tools for probing interactions in multiple linear regression, multilevel modeling, and latent curve analysis. *Journal of Educational and Behavioral Statistics*, 31, 437-448.