

## Multilevel Regression Equations

Multilevel modeling is based on two levels of regression equations. The first level represents the relationship between  $X$  and  $Y$  within a particular group. One can conceptualize that there is a separate regression equation within each group. Hence, to keep track of which equation belongs to which group, there is a new subscript added,  $j$ , to represent the index for groups. The first level (within-group) MLR regression equation is

**Level 1** (within-group, micro)

$$Y_{ij} = \beta_{0j} + \beta_{1j}X_{ij} + R_{ij}$$

The second-level equation involves a prediction of the intercept and slope from the first equation. The second-level equation is structured in exactly the same way as any regression equation, although the symbols used are different. There are also two equations, one that predicts the intercepts from each group, and one that predicts the slope for each group.

**Level 2** (between-group, macro)

$$\beta_{0j} = \gamma_{00} + \gamma_{01}z_j + U_{0j}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}z_j + U_{1j}$$

where  $\gamma_{00}$  and  $\gamma_{10}$  are intercepts,  $\gamma_{01}$  and  $\gamma_{11}$  are slopes, and  $U_{0j}$  and  $U_{1j}$  are errors or residuals.

### Single multilevel equation

The level 1 and level 2 equations can be combined into a single equation by plugging the level-2 equations into the level-1 equations.

$$Y_{ij} = (\gamma_{00} + \gamma_{01}Z_j + U_{0j}) + (\gamma_{10} + \gamma_{11}Z_j + U_{1j})X_{ij} + R_{ij}$$

Then rearranging the terms, we have:

$$Y_{ij} = \gamma_{00} + \gamma_{01}Z_j + \gamma_{10}X_{ij} + \gamma_{11}Z_jX_{ij} + U_{0j} + U_{1j}X_{ij} + R_{ij}$$

Notice that the last three terms of the equation,  $U_{0j} + U_{1j}X_{ij} + R_{ij}$ , if considered as one error term, transform this more complicated equation into a regular regression equation:

$$Y_{ij} = \gamma_{00} + \gamma_{01}Z_j + \gamma_{10}X_{ij} + \gamma_{11}Z_jX_{ij} + error$$

The only special term remaining is  $\gamma_{11}Z_jX_{ij}$ , which is an interaction between  $X$  and  $Z$ .

This gets pretty complex quickly...so we will build up to these full equations slowly.