Growth Curve Models

Overview
Growth curve models investigate change over three or more time points. The statistical model is a special case of multilevel models for which multiple measurements are nested within a person. Generally, the way in which the models are estimated and the software implementation is identical to the group-nesting circumstance. There are several special considerations and interpretations for growth curve models, however.

Fundamental Concepts
Growth curve models examine change in the dependent variable as a function of time for each individual. Below is the example of a single growth curve (linear slope) for one individual.

![Individual Growth Curve](image)

This individual growth curve can be represented by a regression formula, with a dependent variable measured at each time point, \( Y_{ij} \), for each individual, \( i \). The slope, \( \pi_{1i} \), represents the change in the dependent variable for each unit increase in time, \( TIME_{ij} \). The intercept, \( \pi_{0i} \), represents the value of \( Y_{ij} \) when \( TIME_{ij} \), (i.e., time) equals zero. If time is coded, 0, 1, 2, ..., then the intercept represents the value of \( Y_{ij} \) at the beginning of the study.¹

\[
Y_{ij} = \pi_{0i} + \pi_{1i}TIME_{ij} + \varepsilon_{ij}
\]

This coding for the time variable does not have to be used, and there are some instances in which the middle (centered), last time point, or other value may be of interest instead. Moreover, coding does not have to be in unit increments. For example, if a year is skipped in an otherwise annual survey, codes would likely follow suit.

Conceptually, intercepts and slopes for each individual can be used as dependent variables in second-level regression equations.

\[
\begin{align*}
\pi_{0i} &= \gamma_{00} + \zeta_{0i} \\
\pi_{1i} &= \gamma_{10} + \zeta_{1i}
\end{align*}
\]

¹ For now, I am following the notation in your assigned reading by Singer and Willett (2003), but I will discuss other notations including the notation used by Hox and the Snijders and Bosker text soon.
\( \gamma_{00} \) (gamma) is the average intercept (baseline score). The two levels can be combined into a single multilevel equation.

\[
Y_{ij} = \gamma_{00} + \gamma_{10}TIME_{ij} + \zeta_{0i} + \zeta_{1i}TIME_{ij} + \varepsilon_{ij}
\]

where \( \gamma_{10} \) now becomes the average slope.

Random effects can be estimated as in the group-nested model, where, in this context, the random effect for the intercept represents the variance of baseline values (for the 0, 1, 2 \ldots coding) across individuals and variance of the slopes represents individual differences in rates of change over time.

The models can be expanded in a variety of ways, including incorporating level-1 and level-2 covariates, cross-level interactions, and nonlinear effects and they have, therefore, become an invaluable tool for investigating change in longitudinal studies.

**References**