

Growth Curve Example with Time Invariant Covariate

SPSS

(Note that the variable CAGE1 is age at baseline centered around the mean value of 70.75 years).

Syntax

```
*genlinmixed requires string id variable.
STRING id (A4).
COMPUTE id = STRING(rid, F4.0).

*time was a nominal variable, convert it to scale.
variable level time (scale).

GENLINMIXED
/ DATA_STRUCTURE SUBJECTS=id
/ FIELDS TARGET= depress
/ TARGET_OPTIONS DISTRIBUTION=NORMAL LINK=IDENTITY
/ BUILD_OPTIONS DF_METHOD=SATTERTHWAITE COVB=ROBUST
/ FIXED EFFECTS= time cage1 time*cage1 USE_INTERCEPT=TRUE
/ RANDOM EFFECTS=time USE_INTERCEPT=TRUE SUBJECTS=id
COVARIANCE_TYPE=UNSTRUCTURED.

*for nonrobust standard errors, the MIXED procedure specification would look like this.
*MIXED depress WITH time cage1
/ METHOD = REML
/ PRINT = SOLUTION TESTCOV HISTORY
/ FIXED = time cage1 time*cage1 | SSTYPE(3)
/ RANDOM = INTERCEPT time | SUBJECT(rid) COVTYPE(UN)
/ CRITERIA=DFMETHOD(KENWARDROGER).
```

Generalized Linear Mixed Models

Model Summary

Target	depress Summed CESD score	
Probability Distribution	Normal	
Link Function	Identity	
Information Criterion	Akaike Corrected	4919.874
	Bayesian	4938.010

Information criteria are based on the -2 log likelihood (4911.817) and are used to compare models. Models with smaller information criterion values fit better.

Coefficients of Determination

Pseudo-R Square Measures	Marginal	.028
	Conditional	.599

Fixed Coefficients ^a

Model Term	Coefficient	Std. Error	t	Sig.	95% Confidence Interval	
					Lower	Upper
Intercept	13.010	.6076	21.412	<.001	11.813	14.207
time	-1.911	.2883	-6.629	<.001	-2.479	-1.343
cage1	-.053	.0936	-.565	.573	-.237	.132
time*cage1	.006	.0464	.127	.899	-.086	.097

Probability distribution: Normal

Link function: Identity

a. Target: Summed CESD score

Covariance Parameters

Random Effect

Random Effect Covariance	Estimate	Std. Error	Z	Sig.	95% Confidence Interval	
					Lower	Upper
UN (1,1)	57.333	8.547	6.708	<.001	42.807	76.787
UN (2,1)	-4.624	3.505	-1.319	.187	-11.494	2.245
UN (2,2)	1.738	2.460	.707	.480	-.109	27.837

Covariance Structure: Unstructured

Subject Specification: id

Sample Write-up

A growth curve model was tested using age as a time-invariant predictor. Age was centered around the baseline sample mean to improve interpretation of the intercept, and slopes for time were allowed to vary across individuals. Satterthwaite degrees of freedom and robust standard errors were used. The cross-level interaction between age and time was included in the model. The intercept value was 13.01, which was the average on the depression measure at baseline for participants of average age. Baseline values on depression for participants of the average age varied significantly across individuals, $\tau_0^2 = 57.33$, $z = 6.708$, $p < .001$. The fixed effect for time also was significant, $\gamma_{10} = -1.91$, $p < .001$, and indicated that depression scores declined by nearly two points on average each year. Growth curves did not vary significantly across individuals, however, $\tau_1^2 = 1.74$, $z = .707$, $p = .24$. There was a slight tendency for depression at baseline to be associated with decline in depression over time, but the covariance between intercept and slope was nonsignificant, $\tau_{01} = -4.62$, $z = -1.319$, $p = .187$. Age did not significantly predict depression scores, $\gamma_{01} = -.053$, $p = .573$, and there was no significant Age \times Time interaction, $\gamma_{11} = .006$, $p = .899$. *(Had the interaction been significant, a plot and simple slopes would be added here to explore the interaction.)*

```
R
> library(lme4)
> #age as a time-invariant covariate
> #center age first
> mydata$cage1 <- mydata$age1 - mean(mydata$age1)
> #double check centering was done correctly, mean of new variable should be zero
> #SummaryStats(mydata)
> model <- lmer(depress ~ time + cage1 + + time*cage1 + (time|rid), data = mydata, REML=TRUE)
> summary(model)
Linear mixed model fit by REML. t-tests use Satterthwaite's method ['lmerModLmerTest']
Formula: depress ~ time + cage1 + +time * cage1 + (time | rid)
Data: mydata

REML criterion at convergence: 4911.8

Scaled residuals:
   Min       1Q   Median       3Q      Max
-3.0144 -0.4805 -0.1322  0.3465  5.0172

Random effects:
 Groups   Name                Variance Std.Dev. Corr
 rid      (Intercept)          57.332    7.572
          time                 1.739    1.319   -0.46
 Residual                    35.763    5.980
Number of obs: 702, groups: rid, 234

Fixed effects:
              Estimate Std. Error      df t value      Pr(>|t|)
(Intercept) 13.010084   0.610221 231.999900  21.320 < 0.0000000000000002 ***
time        -1.911226   0.289568 231.998584   -6.600  0.000000000276 ***
cage1       -0.052865   0.096394 231.999895   -0.548    0.584
time:cage1   0.005896   0.045742 231.998579    0.129    0.898
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
          (Intr) time  cage1
time      -0.544
cage1     0.000  0.000
time:cage1 0.000  0.000 -0.544
> VarCorr(model) #provides variances in variance form
Groups   Name                Std.Dev. Corr
rid      (Intercept)          7.5718
          time                 1.3188   -0.463
 Residual                    5.9802
> rand(model) #LR test compared to empty using mixture distribution
ANOVA-like table for random-effects: single term deletions

Model:
depress ~ time + cage1 + (time | rid) + time:cage1
      npar  logLik    AIC    LRT Df Pr(>Chisq)
<none>      8 -2455.9 4927.8
time in (time | rid)  6 -2457.0 4925.9  2.101  2    0.3498
> confint(model) #profile likelihood intervals for better random effects tests
Computing profile confidence intervals ...
      2.5 %      97.5 %
.sig01    6.45034252  8.66090097
.sig02   -1.00000000  1.00000000
.sig03    0.00000000  2.55124051
```

```
.sigma      5.47673312  6.48896111
(Intercept) 11.81428865 14.20587940
time       -2.47866724 -1.34378575
cage1      -0.24175918  0.13602902
time:cage1 -0.08373987  0.09553195
```

```
> library(MLMusingR)
> robust_mixed(model) #get robust SE estimates
```

Standard error type = CR2
 Degrees of freedom = Satterthwaite

```
      Estimate  mb.se  robust.se  t.stat  df      Pr(>t)
(Intercept)  13.010  0.610    0.610  21.322 232 <0.0000000000000002 ***
time         -1.911  0.290    0.290  -6.600 232 <0.0000000000000002 ***
cage1        -0.053  0.096    0.094  -0.561 110      0.58
time:cage1    0.006  0.046    0.047  0.126 110      0.90
```

 Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

For write-ups using R, report the profile likelihood confidence intervals for determination of significance of the random effects instead of p-values and significance tests, because they should provide a more accurate determination of significance than halving p-values. I also recommend reporting robust standard errors for the fixed effects unless the number of cases is small (e.g., < 50-100).

HLM

Summary of the model specified

Level-1 Model

$$DEPRESS_{ij} = \beta_{0j} + \beta_{1j}(TIME_{ij}) + r_{ij}$$

Level-2 Model

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(CAGE1_j) + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}(CAGE1_j) + u_{1j}$$

Mixed Model

$$DEPRESS_{ij} = \gamma_{00} + \gamma_{01}CAGE1_j + \gamma_{10}TIME_{ij} + \gamma_{11}CAGE1_jTIME_{ij} + u_{0j} + u_{1j}TIME_{ij} + r_{ij}$$

Final Results - Iteration 2

Iterations stopped due to small change in likelihood function

$\sigma^2 = 35.76270$

τ

```
INTRCPT1, $\beta_0$   57.33260 -4.62430
TIME, $\beta_1$       -4.62430  1.73956
```

τ (as correlations)

```
INTRCPT1, $\beta_0$   1.000 -0.463
TIME, $\beta_1$      -0.463  1.000
```

Random level-1 coefficient	Reliability estimate
INTRCPT1, β_0	0.658
TIME, β_1	0.089

The value of the log-likelihood function at iteration 2 = -2.455908E+003

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard error	t-ratio	Approx. d.f.	p-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	13.010084	0.610222	21.320	232	<0.001
CAGE1, γ_{01}	-0.052865	0.096394	-0.548	232	0.584
For TIME slope, β_1					
INTRCPT2, γ_{10}	-1.911226	0.289569	-6.600	232	<0.001
CAGE1, γ_{11}	0.005896	0.045742	0.129	232	0.898

**Final estimation of fixed effects
 (with robust standard errors)**

Fixed Effect	Coefficient	Standard error	t-ratio	Approx. d.f.	p-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	13.010084	0.607609	21.412	232	<0.001
CAGE1, γ_{01}	-0.052865	0.093586	-0.565	232	0.573
For TIME slope, β_1					
INTRCPT2, γ_{10}	-1.911226	0.288329	-6.629	232	<0.001
CAGE1, γ_{11}	0.005896	0.046426	0.127	232	0.899

Final estimation of variance components

Random Effect	Standard Deviation	Variance Component	d.f.	χ^2	p-value
INTRCPT1, u_0	7.57183	57.33260	232	678.31403	<0.001
TIME slope, u_1	1.31892	1.73956	232	254.56979	0.148
level-1, r	5.98019	35.76270			

Statistics for current covariance components model

Deviance = 4911.816705

Number of estimated parameters = 4