Comparison of Notation for Growth Curve Models

Each model below is the same basic growth curve model with no covariates and a random slope expressed in the notation used in our various reading sources.

Snijders & Bosker (2012) Notation

Separate Equations:

(level 1) $Y_{ti} = \beta_{0i} + \beta_{1i}(t - t_0) + R_{ti}$

(level 2) $\begin{array}{l} \beta_{0i} = \gamma_{00} + U_{0i} \\ \beta_{1i} = \gamma_{10} + U_{1i} \end{array}$

Single Equation

$$Y_{ti} = \gamma_{00} + \gamma_{10}\left(t - t_0\right) + U_{0i} + U_{1i}(t - t_0) + R_{ti}$$

The subscript $_{ti}$ is used to designate time point, t, nested within individual, i. Instead of using an x or other variable, the time predictor is expressed as $t\text{-}t_0$ to indicate that the first time point is coded as 0 and so that the intercept represents baseline scores on the dependent variable. For example if t = 1, 2, 3, 4, then $t\text{-}t_0$ = 0, 1, 2, 3. The value of the intercept is then equal to the value of Y when $t\text{-}t_0$ is equal to 0, leading to an interpretation of the intercept as the average baseline value of the dependent variable.

Raudenbush & Bryk (2002) Notation

Separate Equations:

(level 1) $Y_{ti} = \pi_{0i} + \pi_{1i}a_{ti} + e_{ti}$

(level 2) $\begin{array}{l} \pi_{0i} = \beta_{00} + r_{0i} \\ \pi_{1i} = \beta_{10} + r_{1i} \end{array}$

Single Equation

$$Y_{ti} = \beta_{00} + \beta_{10}a_{ti} + r_{0i} + r_{1i}a_{ti} + e_{ti}$$

The subscript $_{ti}$ is used to designate time point, t, nested within individual, i. Instead of using $t\text{-}t_0$ for the time variable as Snijders and Bosker do, a new variable letter, a_{ti} is used to represent the special time predictor. a was picked for *age,* but can represent any time predictor. The rationale for using π for the regression coefficients at level 1 and β for the regression coefficients at level 2 is to keep the βs associated with the person level and to allow for γs to be used for a group level should there be a nested data structure also. A summation sign, Σ, is often used for a general formula to indicate that any number of other predictors (terms) can be added to the equation.

Singer & Willett (2003) Notation

Separate Equations:

(level 1) $Y_{ij} = \pi_{0i} + \pi_{1i} TIME_{ij} + \varepsilon_{ij}$

(level 2) $\begin{aligned} \pi_{0i} &= \gamma_{00} + \zeta_{0i} \\ \pi_{1i} &= \gamma_{10} + \zeta_{1i} \end{aligned}$

Single Equation

$$Y_{ij} = \gamma_{00} + \gamma_{10} TIME_{ij} + \zeta_{0i} + \zeta_{1i} TIME_{ij} + \varepsilon_{ij}$$

The subscript ij used by Singer and Willett differs from the other two notations, because j represents the time point and i represents the person. Thus, ij can be read as "person i at time point j." ε (epsilon) is used for the level-1 error term and ζ (zeta) is used for the level-2 error term.

Hox (2010; Hox, Moerbeek, & van de Schoot, 2018) Notation

Separate Equations:

(level 1) $Y_{ti} = \pi_{0i} + \pi_{1i} T_{ti} + e_{ti}$

(level 2) $\begin{aligned} \pi_{0i} &= \beta_{00} + u_{0i} \\ \pi_{1i} &= \beta_{10} + u_{1i} \end{aligned}$

Single Equation

$$Y_{ti} = \beta_{00} + \beta_{10} T_{ti} + u_{0i} + u_{1i} T_{ti} + e_{ti}$$

The subscript $_{ti}$ is used to designate time point, t, nested within individual, i. The time predictor is T_{ti}. Compared with the Raudenbush and Bryk notation, the π and the β are switched, so that the π represents the level-1 coefficient and β represents the level-2 coefficient.