ANCOVA Example (One Level-1 Predictor Assuming Homogeneous Slopes)

The example below uses data from the High School and Beyond data set (HSB). The model tests differences across schools in math achievement, controlling for student SES. This model is equivalent to a random effects ANCOVA assuming homogeneous slopes for the covariate.

SPSS

```
 MIXED mathach WITH ses
/METHOD = REML
/PRINT = SOLUTION TESTCOV
/FIXED =  ses | SSTYPE(3)
/RANDOM = INTERCEPT  | SUBJECT(schoolid) COVTYPE(UN).
```

Fixed Effects

Type III Tests of Fixed Effects [a]

Source	Numerator df	Denominator df	F	Sig.
Intercept	1	148.330	4534.089	<.001
ses	1	6838.142	511.174	<.001

a. Dependent Variable: mathach.

Estimates of Fixed Effects [a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval Lower Bound	Upper Bound
Intercept	12.657	.188	148.330	67.336	<.001	12.286	13.029
ses	2.390	.106	6838.142	22.609	<.001	2.183	2.597

a. Dependent Variable: mathach.

Covariance Parameters

Estimates of Covariance Parameters [a]

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval Lower Bound	Upper Bound
Residual		37.034	.625	59.218	<.001	35.829	38.281
Intercept [subject = schoolid]	Variance	4.768	.655	7.278	<.001	3.642	6.241

a. Dependent Variable: mathach.

This p-value should be halved

Remember to **halve the p-value** for the intercept variance test. In this case, half of p < .001 is either p < .0005 or just p < .001.

Or, alternatively retest the model using the subcommand `/CRITERIA=CIN(90)` but only use the 90% intervals for the intercept variance test.

Estimates of Covariance Parameters [a]

Parameter		Estimate	Std. Error	Wald Z	Sig.	90% Confidence Interval Lower Bound	Upper Bound
Residual		37.034	.625	59.218	<.001	36.020	38.077
Intercept [subject = schoolid]	Variance	4.768	.655	7.278	<.001	3.803	5.977

a. Dependent Variable: mathach.

R

```
> #random effects ANCOVA model (homogeneous slopes)
> model2 <- lme(mathach ~ ses, random = ~ 1|schoolid, data = mydata, method="REML")
> summary(model2)
Linear mixed-effects model fit by REML
  Data: mydata
       AIC      BIC    logLik
  46653.17 46680.69 -23322.58

Random effects:
 Formula: ~1 | schoolid
        (Intercept) Residual
StdDev:    2.183615 6.085589

Fixed effects:  mathach ~ ses
                Value Std.Error   DF  t-value p-value
(Intercept) 12.657480 0.1879851 7024 67.33234       0
ses          2.390196 0.1057191 7024 22.60893       0
 Correlation: 
    (Intr)
ses 0.003 

Standardized Within-Group Residuals:
        Min          Q1         Med          Q3         Max 
-3.12607345 -0.72720308  0.02188316  0.75771694  2.91911572 

Number of Observations: 7185
Number of Groups: 160 
> #nlme provides standard deviations of the random effects by default, use VarCorr to obtain variances
> VarCorr(model2)
schoolid = pdLogChol(1) 
            Variance  StdDev  
(Intercept)  4.768174 2.183615
Residual    37.034399 6.085589
> #obtain confidence intervals for fixed effects but ignore the random effects CIs (incorrect)
> intervals(model2)
Approximate 95% confidence intervals

 Fixed effects:
                lower      est.     upper
(Intercept) 12.288973 12.657480 13.025988
ses          2.182955  2.390196  2.597437

 Random Effects:
  Level: schoolid 
                   lower     est.    upper
sd((Intercept)) 1.908434 2.183615 2.498475

 Within-group standard error:
   lower     est.    upper 
5.985709 6.085589 6.187137 
> #use 90% CI for the intercept variance
> #could use intervals(model2,.90,"var-cov") from nlme but variances are standard deviations
> #the predict means package and varcomp function will give variances
> library(predictmeans)
> varcomp(model2, ci=TRUE, level=.90)
                                  vcov        SE     lower     upper
schoolid.schoolid.var((Intercept))  4.768174 0.6337066  3.803334  5.977778
sigma_sq                          37.034399 0.6249069 36.019872 38.077500
```

HLM

Summary of the model specified
Step 2 model
Level-1 Model

$MATHACH_{ij} = \beta_{0j} + \beta_{1j}*(SES_{ij}) + r_{ij}$

Level-2 Model

$\beta_{0j} = \gamma_{00} + u_{0j}$
$\beta_{1j} = \gamma_{10}$

Mixed Model

$MATHACH_{ij} = \gamma_{00}$
$+ \gamma_{10}*SES_{ij} + u_{0j}+ r_{ij}$

Final Results - Iteration 6
Iterations stopped due to small change in likelihood function

$\sigma^2 = 37.03440$

τ
INTRCPT1,β_0 4.76815

Random level-1 coefficient	Reliability estimate
INTRCPT1,β_0	0.843

The value of the log-likelihood function at iteration 6 = -2.332167E+04

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	12.657481	0.187984	67.333	159	<0.001
For SES slope, β_1					
INTRCPT2, γ_{10}	2.390199	0.105719	22.609	7024	<0.001

Final estimation of fixed effects
(with robust standard errors)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	12.657481	0.187330	67.568	159	<0.001
For SES slope, β_1					
INTRCPT2, γ_{10}	2.390199	0.119309	20.034	7024	<0.001

Final estimation of variance components

Random Effect	Standard Deviation	Variance Component	*d.f.*	χ^2	*p*-value
INTRCPT1, u_0	2.18361	4.76815	159	1037.09077	<0.001
level-1, r	6.08559	37.03440			

Note that the random effects tests are correct in HLM, so no adjustment is needed.

Statistics for current covariance components model
Deviance = 46643.331427
Number of estimated parameters = 2

A second model was tested to examine the variability of math achievement across schools controlling for student socioeconomic status (SES). The slopes representing the relationship between SES and math achievement were nonvarying and assumed to be equal across schools. Results indicated that the adjusted mean for math achievement was 12.66, which represents the math achievement score for a student with average SES.[1] SES was significantly related to math achievement scores (γ_{10} =2.39, SE_{γ} = .19, p < .001),[2] indicating that math achievement scores increased by over two points with each standard deviation increase in SES. Schools varied significantly in math achievement scores after controlling for SES, τ_0^2 = 4.77, χ^2(159) = 1037.09, p < .001. (*Values and significance test were taken from the HLM output and may differ slightly from the SPSS and R output values*).

[1] The SES variable in the HSB data set is standardized and so the intercept is defined as the predicted value for math achievement when the predictor, SES is at the mean (here when SES equals 0, given the standardized scaling). We will discuss predictor scaling in greater detail later.
[2] I usually recommend use of the robust standard errors for the fixed effects, so I report the robust value here. In HLM, the nonrobust value is .12 and, in SPSS and R, it is .11. We will discuss this later in the course.