

Mediation Analysis with Logistic Regression

Mediation is a hypothesized causal chain in which one variable affects a second variable that, in turn, affects a third variable. The intervening variable, M , is the mediator. It “mediates” the relationship between a predictor, X , and an outcome. Graphically, mediation can be depicted in Figure 1.1 below:

Figure 1.1

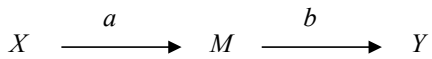


Figure 1.2

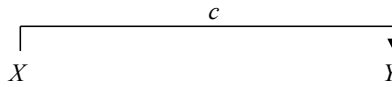
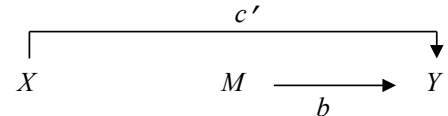


Figure 1.3



Paths a and b are called direct effects. The mediational path, in which X leads to Y through M , is called the *indirect effect*.¹

Baron and Kenny (1986) proposed a widely cited method of investigating mediation through a series of three simple regression models, establishing a significant relationship for each unstandardized regression coefficient, a , b , and c , depicted in Figures 1.1. and 1.2. Mediation was then indicated by results from a third, multiple regression model, with both X and M predicting Y . *Full mediation* is indicated by the results if the relationship between X and Y was eliminated entirely when M is controlled (i.e., c' is non-significant), and partial mediation is indicated by the reduction but not the elimination of the X to Y relationship. This approach is a less precise and potentially inaccurate method of testing for mediation, because a true mediational relationship may exist if some of these relationships are not found or if there is a more complex set of associations among the three variables (MacKinnon, Fairchild, & Fritz, 2007).

The current approach (e.g., MacKinnon, 2008) involves the computation of an *indirect effect coefficient* that is a function of the compound pathway a and b from Figure 1.3 together (Note: the b path in the product is the partial regression coefficient of Y regressed on X when M is also in the model). In ordinary least squares regression, the difference between the direct effect of X on Y with and without M , $c - c'$ from separate regression models depicted in Figures 1.2 and 1.3 (Judd & Kenny, 1981), and the product of the two paths from the model shown in Figure 1.3, ab (Sobel, 1962), are equivalent. In either case, the indirect effect coefficient gives the change in Y for each unit change in X as mediated through M . A significant indirect effect coefficient is evidence consistent with a mediational hypothesis, but it does not “prove” that the causal pathway is the reason for the associations. Combined with experimental, longitudinal, or other design features, one may eliminate some of the alternative explanations, however, and strengthen the case for a possible causal pathway.

Mediation Analysis with Logistic Regression

Because of the nonlinear nature of logistic regression, the two methods for calculating the indirect effect, ab and $c - c'$, are no longer equivalent (Winship & Mare, 1983) if either M or Y are binary, particularly with rarer outcomes. The discrepancy between the methods can also vary with the presence of covariates in the model. There are several possible solutions to the problem (see also Iacobucci, 2012a; Imai, Keele, & Tingley, 2010; Kenny, 2013; MacKinnon, Lockwood, Brown, & Wang, 2007; Valeri & VanderWeele, 2013), one of which is to standardize the variables, and then compute the ab product. Special attention needs to be paid to the variance of the binary dependent variable when standardizing either path coefficient. Standardizing the a path, for example, would use the usual formula for converting an unstandardized coefficient to a standardized coefficient using the standard deviations of X and M for the first path, $a^* = a(S_X/S_M)$, but using a special computation of the standard deviation of a binary M variable, $s_M = \sqrt{a^2 s_x^2 + \pi^2 / 3}$. The mathematical constant π divided by 3 is an estimate of the binomial distribution variance. If Y is also binary, an analogous standardization of the second path, b , would be

¹ This is perhaps the most common notation. MacKinnon and colleagues (e.g., MacKinnon 2008) use α and β for the direct paths between the predictor X and mediator M and between the mediator and the outcome Y and τ and τ' for the paths between X and Y .

used. The standardization method nearly eliminates the discrepancy between ab and the $c - c'$ indirect coefficient computations (MacKinnon & Dwyer 1993; MacKinnon et al., 2007). The product of the standardized paths, $(a^*)(b^*)$, is then divided by a standard error estimate computed as a function of the standard deviations of the two variables (Sobel, 1962) or computed from a bootstrap resampling approach.

Using Odds Ratios or Proportion Mediated for the Magnitude of the Indirect Effect

An odds ratio for the indirect effect can be defined, but its interpretation becomes considerably more complex. For a simpler case, when both M and Y are binary, the odds ratio for the indirect effect represents the odds of Y given X for the value of M_1 as compared with the value M_0 (VanderWeele & VanderWeele, 2010). VanderWeele & VanderWeele (2010) also suggest corrections to the odds ratio for rarer outcomes (generally with outcome $\hat{\pi} < .10$). If the predictor is continuous in the a or b path, the odds ratio involves a unit change interpretation that adds additional complexity. The odds ratio may also be unstable if the relationships among the variables are weak (MacKinnon, Warsi, & Dwyer, 1995). One can estimate the *proportion mediated*, $ab/(c' + ab)$, as a way to gauge the magnitude of the indirect effect, but MacKinnon and colleagues (2007) find that it also is unstable with sample sizes less than 500. Vanderweele (2016) suggests a computation of the proportion mediated using odds ratios for the direct (θ_{direct}) and indirect ($\theta_{indirect}$) effects, where proportion mediated = $\theta_{direct}(\theta_{indirect} - 1)/(\theta_{direct} * \theta_{indirect} - 1)$, although this approach has not been evaluated extensively.

Probit Models

An additional proposed solution is to estimate the models involving a binary outcome with probit analysis. Probit regression, discussed in the next section of the course, is an alternative to logistic regression that uses an assumed normal error distribution and lends itself to standardized coefficients more readily. The probit mediational approach still performs better with the product approach than the difference approach ($c - c'$) to the indirect coefficient, but it also requires rescaling. Coefficient standardization is a bit more straightforward with probit than logit, so rescaling prior to computation of the indirect path is potentially clearer. The probit method appears to perform relatively well with sample sizes of 200 or more given the correct model and when distributional assumptions are met (MacKinnon et al., 2007).

Software Examples

There are several ways to test mediation in current software programs,² although I will only illustrate use of Hayes' PROCESS macro in SPSS, R and SAS³ and the probit (WLSMV) approach in Mplus and lavaan in R. The PROCESS macro and the Mplus methods allow the user to specify more than one mediator (as well as combinations of moderators and mediators) as well as covariates, but I will keep the illustrations simple here. Structural equation modeling packages, can also be used with some greater flexibility including multiple predictors and mediators and latent variables. Some packages, such as Mplus (Muthén & Muthén, 1998-2012; see also Chapter 8 in Muthén, Muthén, & Asparouhov, 2017 for detail and illustrations), have incorporated Sobel and bootstrap approaches to the standard errors and statistical tests. The Mplus approach can be used with the diagonal weighted least squares approach (estimator=WLSMV), which is a probit analysis and for which standardized coefficients are available (addressing the scaling issue described above).

The examples below use negative exchanges (w1neg), depression (w1cesd9), and heart disease (w1hheart) from the LLSSE study (also used in the "Logistic Regression" handout). The hypothesized mediational model is that negative exchanges lead to depression which, in turn, lead to heart disease, w1neg → w1cesd9 → w1hheart.

² There are other software approaches available including macros developed by Valeri and Vanderweele (2013). The `mediations` function from the `mediation` package in R is yet another possibility (and there are several other R functions that simulate the PROCESS macro or domediation).

³ Warning: I am not clear on the approach of the PROCESS macro to standardization for the computation of the product ab in the logistic case (or whether there is one).

SPSS

*I set the temporary directory here to make sure temporary files have a legitimate place to go.
cd "c:\jason\temp".

```
insert file='C:\Jason\SPSSWIN\macros\process.sps'.
execute.
process y = wlhheart
  / x = wlneg
  / m = wlcesd9
  /total=1
  /boot=10000
  /seed=10000
  /model=4
  /stand=1.
execute.
```

At the bottom of the output, the “Indirect effect of X on Y” gives the indirect effect and its confidence limits (BootLLCI,BootULCI) using the bootstrap standard error method.

Run MATRIX procedure:

***** PROCESS Procedure for SPSS Version 3.4 *****

Written by Andrew F. Hayes, Ph.D. www.afhayes.com
Documentation available in Hayes (2018). www.guilford.com/p/hayes3

```
Model : 4
  Y : wlhheart
  X : wlneg
  M : wlcesd9
```

Sample
Size: 692

Custom
Seed: 10000

OUTCOME VARIABLE:
wlcesd9

| Model Summary | R | R-sq | MSE | F | df1 | df2 | p |
|---------------|-------|-------|---------|---------|--------|----------|-------|
| | .3404 | .1159 | 19.7753 | 90.4255 | 1.0000 | 690.0000 | .0000 |

| Model | coeff | se | t | p | LLCI | ULCI |
|----------|--------|-------|---------|-------|--------|--------|
| constant | 3.4324 | .2083 | 16.4783 | .0000 | 3.0235 | 3.8414 |
| wlneg | 2.8439 | .2991 | 9.5092 | .0000 | 2.2567 | 3.4311 |

OUTCOME VARIABLE:
wlhheart

Coding of binary Y for logistic regression analysis:

| wlhheart | Analysis |
|----------|----------|
| .00 | .00 |
| 1.00 | 1.00 |

| Model Summary | -2LL | ModelLL | df | p | McFadden | CoxSnell | Nagelkrk |
|---------------|----------|---------|--------|-------|----------|----------|----------|
| | 623.6956 | 2.0224 | 2.0000 | .3638 | .0032 | .0029 | .0049 |

| Model | coeff | se | Z | p | LLCI | ULCI |
|----------|---------|-------|----------|-------|---------|---------|
| constant | -1.7485 | .1495 | -11.6926 | .0000 | -2.0416 | -1.4554 |
| wlneg | .0282 | .1853 | .1523 | .8789 | -.3349 | .3913 |
| wlcesd9 | .0279 | .0215 | 1.2978 | .1944 | -.0142 | .0701 |

These results are expressed in a log-odds metric.

```
***** DIRECT AND INDIRECT EFFECTS OF X ON Y *****
Direct effect of X on Y
  Effect      se      Z      p      LLCI      ULCI
  .0282     .1853     .1523    .8789    -.3349     .3913

Indirect effect(s) of X on Y:
      Effect      BootSE    BootLLCI    BootULCI
wlcesd9      .0794      .0580      -.0229      .2050

***** ANALYSIS NOTES AND ERRORS *****

Level of confidence for all confidence intervals in output:
 95.0000

Number of bootstrap samples for percentile bootstrap confidence intervals:
 1000

NOTE: Total effect model not available with dichotomous Y
NOTE: Effect size option not available with dichotomous Y
NOTE: Direct and indirect effects of X on Y are on a log-odds metric.

----- END MATRIX -----
```

R with PROCESS

I omit the output from the R run here, because it is virtually identical in appearance to the SPSS output.

```
source('c:/jason/R/macros/process.R',echo=FALSE)
process(data=d,y="wlhheart",x="wlneg",m="wlcesd9",total=1,boot=1000,seed=10000,model=4,stand=1)
```

R lavaan (probit)

The lavaan package in R is a structural equation modeling package. The estimator='dwls' option gives a robust diagonal weighted least squares approach and gives probit estimates (same as WLSMV in Mplus). The ab coefficient in the output is the indirect effect coefficient.

```
library(lavaan)

model = '
  wlcesd9 ~ a*wlneg
  wlhheart ~ b*wlcesd9 + c*wlneg

#indirect effect
  ab := a*b

#total effect
  total := c + (a*b)
,

#fit = sem(model, data = mydata, missing = 'listwise', se = 'bootstrap', estimator='ml',
#          link='probit')

fit = sem(model, data = mydata, missing = 'listwise', se = 'bootstrap',
          estimator='dwls', ordered=c("wlhheart"), parameterization="theta")
summary(fit,fit.measures=TRUE, rsquare=TRUE, standardized=TRUE)

parameterestimates(fit)Parameter Estimates:

Information                                Observed
Standard Errors                            Bootstrap
Number of requested bootstrap draws        1000
Number of successful bootstrap draws       1000

Regressions:
      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
wlcesd9 ~
  wlneg      (a)   2.844   0.423   6.721   0.000   2.844   0.341
```

```
wlhheart ~
  wlcesd9   (b)   0.016   0.011   1.487   0.137   0.016   0.077
  wlneg     (c)   0.015   0.101   0.146   0.884   0.015   0.008

Intercepts:
      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
.wlcesd9      3.432   0.210  16.310   0.000   3.432   0.727
.wlhheart      0.000
      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
Thresholds:
  wlhheart|t1      1.048   0.087  12.026   0.000   1.048   1.045

Variances:
      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
.wlcesd9      19.718   1.743  11.313   0.000  19.718   0.884
.wlhheart      1.000
      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
Scales y*:
  wlhheart      0.997
      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
R-Square:
      Estimate
  wlcesd9      0.116
  wlhheart      0.006

Defined Parameters:
      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
ab      0.047   0.034   1.388   0.165   0.047   0.026
total    0.061   0.104   0.588   0.557   0.061   0.035
```

```
> parameterestimates(fit)
      lhs op   rhs label   est   se      z pvalue ci.lower ci.upper
1  wlcesd9 ~   wlneg   a  2.844 0.423  6.721  0.000   2.061   3.724
2  wlhheart ~ wlcesd9  b  0.016 0.011  1.487  0.137  -0.006   0.037
3  wlhheart ~   wlneg   c  0.015 0.101  0.146  0.884  -0.203   0.199
4  wlhheart |   t1      1.048 0.087 12.026  0.000   0.874   1.218
5  wlcesd9 ~~ wlcesd9      19.718 1.743 11.313  0.000  16.295  23.152
6  wlhheart ~~ wlhheart      1.000 0.000    NA    NA    1.000   1.000
7  wlhheart ~*~ wlhheart      0.997 0.000    NA    NA    0.997   0.997
8  wlcesd9 ~1      3.432 0.210 16.310  0.000   2.997   3.848
9  wlhheart ~1      0.000 0.000    NA    NA    0.000   0.000
10   ab :=      a*b   ab  0.047 0.034  1.388  0.165  -0.015   0.118
11  total :=  c+(a*b) total 0.061 0.104  0.588  0.557  -0.169   0.251
```

SAS

I omit the output from the PROCESS macro in the SAS run here, because it is virtually identical in appearance to the the SPSS output.

```
OPTIONS MSTORED SASMSTORE=macros;
%include "c:\jason\sas\macros\process.sas";
%process(data=one, y=wlhheart, x=wlneg, m=wlcesd9, total=1, boot=1000, seed=10000, model=4, stand=1);
run;
```

Mplus

Without going into great detail on setting up models in Mplus, I will point out that the `on` statement is for variable Y regressed on X . I have omitted some of the output and the `model indirect` command that specifies the indirect effect. The analysis type by default when any categorical variables are declared is WLSMV which is a robust diagonal weighted least squares approach and gives probit estimates.

```
title: Data from social exchanges study;
data: file=heart.dat; format=free;
listwise=on;
variable: names = wlhheart wlneg wlcesd9;
missing = all (-99);
categorical = wlhheart;
analysis: type=general;
bootstrap = 1000;
! at least 500 bootstrap samples are recommended;
```

```

model:  wlcesd9 on wlneg;
        wlhheart on wlneg wlcesd9;

! the following command gives the indirect path coefficient test;
Model indirect:  wlhheart ind wlneg;

output:  stdyx cinterval(bootstrap);
! cinterval (bootstrap) gives bootstrap confidence intervals.

```

MODEL RESULTS

| | Estimate | S.E. | Est./S.E. | Two-Tailed P-Value |
|-------------------------------|----------|-------|-----------|-----------------------|
| W1CESD9 ON W1NEG | -0.042 | 0.135 | -0.313 | 0.754 |
| W1HHEART ON W1NEG | 0.020 | 0.040 | 0.493 | 0.622 |
| W1CESD9 | 0.020 | 0.011 | 1.915 | 0.056 |
| Intercepts W1CESD9 | 4.412 | 0.188 | 23.434 | 0.000 |
| Thresholds W1HHEART\$1 | 1.057 | 0.080 | 13.216 | 0.000 |
| Residual Variances W1CESD9 | 22.177 | 1.967 | 11.276 | 0.000 |

STANDARDIZED MODEL RESULTS

STDYX Standardization

| | Estimate | S.E. | Est./S.E. | Two-Tailed P-Value |
|-------------------------------|----------|-------|-----------|-----------------------|
| W1CESD9 ON W1NEG | -0.014 | 0.040 | -0.343 | 0.731 |
| W1HHEART ON W1NEG | 0.031 | 0.059 | 0.520 | 0.603 |
| W1CESD9 | 0.096 | 0.050 | 1.922 | 0.055 |
| Intercepts W1CESD9 | 0.937 | 0.034 | 27.942 | 0.000 |
| Thresholds W1HHEART\$1 | 1.056 | 0.080 | 13.269 | 0.000 |
| Residual Variances W1CESD9 | 1.000 | 0.002 | 435.185 | 0.000 |

R-SQUARE

| Observed Variable | Estimate | S.E. | Est./S.E. | Two-Tailed P-Value | Residual Variance |
|----------------------|----------|-------|-----------|-----------------------|----------------------|
| W1HHEART | 0.010 | 0.011 | 0.886 | 0.376 | 0.991 |
| W1CESD9 | 0.000 | 0.002 | 0.084 | 0.933 | |

TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS

| | Estimate | S.E. | Est./S.E. | Two-Tailed P-Value |
|--|----------|------|-----------|-----------------------|
|--|----------|------|-----------|-----------------------|

Effects from W1NEG to W1HHEART

| | | | | |
|----------------|--------|-------|--------|-------|
| Total | 0.019 | 0.040 | 0.470 | 0.638 |
| Total indirect | -0.001 | 0.003 | -0.271 | 0.787 |

| Specific indirect 1 | | | | |
|---------------------|--------|-------|--------|-------|
| W1HHEART | | | | |
| W1CESD9 | | | | |
| W1NEG | -0.001 | 0.003 | -0.271 | 0.787 |
| Direct | | | | |
| W1HHEART | | | | |
| W1NEG | 0.020 | 0.040 | 0.493 | 0.622 |

STANDARDIZED TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS

STDYX Standardization

| | Estimate | S.E. | Est./S.E. | Two-Tailed P-Value |
|--------------------------------|----------|-------|-----------|--------------------|
| Effects from W1NEG to W1HHEART | | | | |
| Total | 0.029 | 0.059 | 0.498 | 0.618 |
| Total indirect | -0.001 | 0.004 | -0.304 | 0.761 |
| Specific indirect 1 | | | | |
| W1HHEART | | | | |
| W1CESD9 | | | | |
| W1NEG | -0.001 | 0.004 | -0.304 | 0.761 |
| Direct | | | | |
| W1HHEART | | | | |
| W1NEG | 0.031 | 0.059 | 0.520 | 0.603 |

CONFIDENCE INTERVALS OF TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS

| | Lower .5% | Lower 2.5% | Lower 5% | Estimate | Upper 5% | Upper 2.5% | Upper .5% |
|--------------------------------|-----------|------------|----------|----------|----------|------------|-----------|
| Effects from W1NEG to W1HHEART | | | | | | | |
| Total | -0.122 | -0.084 | -0.057 | 0.019 | 0.072 | 0.081 | 0.104 |
| Total indirect | -0.007 | -0.005 | -0.004 | -0.001 | 0.006 | 0.008 | 0.015 |
| Specific indirect 1 | | | | | | | |
| W1HHEART | | | | | | | |
| W1CESD9 | | | | | | | |
| W1NEG | -0.007 | -0.005 | -0.004 | -0.001 | 0.006 | 0.008 | 0.015 |
| Direct | | | | | | | |
| W1HHEART | | | | | | | |
| W1NEG | -0.122 | -0.083 | -0.058 | 0.020 | 0.070 | 0.080 | 0.100 |

Sample Write-up (based on the process results; odds ratios were computed by hand)

Two regression models were tested to investigate whether the association between negative social exchanges and heart disease is mediated by depression symptomatology. In the first ordinary least squares regression model, negative social exchanges were significantly related to higher depression scores, $b = 2.844$, $SE = .299$, $p < .001$, 95% CI = 2.257, 3.431. In the second logistic regression model, which included negative social exchanges and depression as predictors of heart disease, neither negative exchanges, $b = .028$, $SE = .185$, ns, OR = 1.03, 95% CI -.335, .391, nor depression, $b = .028$, $SE = .022$, ns, OR = 1.028, 95% CI = -.014, .070, was significantly independently associated with heart disease. [I would interpret the ORs here if they had been significant]. The bootstrap confidence intervals derived from 1000 samples indicated that the indirect effect coefficient was not significant, $b = .079$, $SE = .058$, 95% CI = -.021, .205, which did not support the hypothesis that the relation between negative social exchanges and heart disease is mediated by depression

References and Further Reading

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