# Worksheet 4 Solutions

1, FALSE: $(3^m)(3^n) \neq 3^{m \cdot n}$

For example, if $m=2$ and $n=3$, we have

$$3^2 \cdot 3^3 = (3\cdot 3)(3\cdot 3\cdot 3) = 3^5$$

and $3^{2\cdot 3} = 3^6$.

2, (a) $\log_3\left(\frac{1}{9}\right)$

$$3^{\square} = \frac{1}{9}$$

I know $3^2 = 9$. But I want the reciprocal,

$$3^{-2} = \frac{1}{9}.$$

So $\boxed{\log_3\left(\frac{1}{9}\right) = -2}$

(b) $\ln(e^5)$

The ln and the e cancel, so $\ln(e^5) = 5$

<u>OR</u>

$\ln(e^5) = \log_e(e^5)$.

Thus I need to know $e^{\square} = e^5$

e to what power is $e^5$? 5!

$$\boxed{\ln(e^5) = 5}$$

2, (c) $\log_5(\sqrt{5})$

First, $\sqrt{5} = 5^{\frac{1}{2}}$

Thus $\log_5(5^{\frac{1}{2}})$ is asking "5 to what power is $5^{\frac{1}{2}}$?"

So $\boxed{\log_5(5^{\frac{1}{2}}) = \frac{1}{2}}$

3, (a) $\log 10 = (\log 2)(\log 5)$

$\boxed{\text{FALSE}}$

By properties of logs,

$\log 10 = \log(2 \cdot 5) = \log 2 + \log 5.$

(b) $\ln\left(\frac{e}{6}\right) = \ln e + \ln 6$

$\boxed{\text{FALSE}}$

By properties of logs,

$\ln(e/6) = \ln(e) - \ln(6)$

(c) $\log_4\left(\frac{1}{7}\right) + \log_4(7) = 0$

$\boxed{\text{TRUE}}$

By addative property of logs,

$\log_4\left(\frac{1}{7}\right) + \log_4(7) = \log_4\left(\frac{1}{7} \cdot 7\right) = \log_4(1)$

"4 to what power is 1?"

$4^0 = 1$ thus $\log_4(1) = 0.$ ✓

3. (d) $\ln(-e) = -1$

$\boxed{\text{FALSE}}$

$\ln(-e) = \log_e(-e)$

What power do I put on e to get $-e$?

Well, not $-1$. Because $e^{-1} = \frac{1}{e} \neq -e$.

Also, $\ln(-e)$ is undefined, because logs, including ln, can't have negative inputs.

4. (a) $\frac{2e^{-x}}{2} = \frac{8}{2}$

$e^{-x} = 8$

$\log_e(e^{-x}) = \log_e(8)$

$-x = \ln(8)$

$\boxed{x = -\ln(8)}$

Side Note: $-\ln(8) \neq \ln(-8)$

(b) $3^{2x} = 3^{5x-3}$

$\cancel{\log_3}(3^{2x}) = \cancel{\log_3}(3^{5x-3})$

$2x = 5x - 3$

$-2x \quad -2x$

$3 = 3x$

$\boxed{1 = x}$

4, (c) $\log_3(1-x)=1$

(i) To "undo" a log, I need to make it an exponent:

$$3^{(\log_3(1-x))} = 3^1$$

$$1-x = 3$$
$$+x \qquad +x$$
$$1-3 = x$$
$$\boxed{-2 = x}$$

(ii) or, I can use the definition:

If $a^t = K$ then $\log_a K = t$

Here $a=3$
$K = 1-x$
$t = 1$

Thus
$$3^1 = 1-x$$
$$\boxed{-2 = x}$$

4, (d) $\ln 2x + \ln 3x = \ln 6$

Property of logs: Sum of 2 logs is the log of their product.

$$\ln(2x \cdot 3x) = \ln(6)$$
$$\ln(6x^2) = \ln(6)$$

Undo the ln, make exponent one

$$e^{\ln(6x^2)} = e^{\ln(6)}$$

$$6x^2 = 6$$
$$x^2 = 1$$
$$x = \pm 1$$

But $-1$ is not a solution since $\ln(2(-1))$ is undefined.

4, (e) $\log 5 = 2 - \log(x+2)$

Method 1

$\log 5 - 2 = -\log(x+2)$

$-\log 5 + 2 = \log(x+2)$ undo this log

$10^{(-\log 5 + 2)} = \cancel{10^{\log}}(x+2)$

$10^{(-\log 5 + 2)} = x + 2$

Exponent rule: $10^{x+y} = 10^x \cdot 10^y$

$10^{(-\log 5 + 2)} - 2 = x$ OK to stop here

$10^{-\log 5} \cdot 10^2 - 2 = x$

$k \log 5 = \log 5^k$

$10^{\log 5^{-1}} \cdot 100 - 2 = x$

$\cancel{10}^{\cancel{\log} \frac{1}{5}} \cdot 100 - 2 = x$

$\frac{1}{5} \cdot 100 - 2 = x$

$20 - 2 = x$

$\boxed{18 = x}$

4 (e) Method 2

$\log 5 = 2 - \log(x+2)$

$\log 5 + \log(x+2) = 2$

$\log(5 \cdot (x+2)) = 2$

$\log(5x+10) = 2$

$\cancel{10^{\log}}(5x+10) = 10^2$

$5x+10 = 100$

$5x = 90$

$\boxed{x = 18}$

5, (a)

| t | z |
|---|---|
| 2 | 1000 |
| 7 | 7600 |

Linear: $f(t) = mt + b$

slope initial value

Slope: $\frac{\Delta y}{\Delta x} = \frac{7600-1000}{7-2} = \frac{6600}{5} = 1320$

$f(t) = 1320t + b$

Plug in point to find initial value (b)

$1000 = 1320(2) + b$

$1000 = 2640 + b$

$-1640 = b$

$\boxed{f(t) = 1320t - 1640}$

5 (b) Method 1: Find constant multiple using the jumping method.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1000. | | | | | 7600 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |

$1000(K) = 7600$

$K = \frac{7600}{1000} = 7.6$ ; I need to multiply by 7.6 to get from 1000 to 7600.

But that is 5 days, 5 jumps.

I need to find a number so that if I jump (multiply) by it 5 times, it is the same as multiplying by 7.6

So $j \cdot j \cdot j \cdot j \cdot j = 7.6$

$j^5 = 7.6$

$j = (7.6)^{1/5} \approx 1.5$

Now I need to find the initial value. All that I know is at $t=2$, $z=1000$ and $t=7$, $z=7600$.

$f(t) = C(1.5)^t$

Plug in a point to find the initial value.

$1000 = C(1.5)^2$

$1000 = C(2.25)$

$444.4 \approx C$

$f(t) \approx 444.4(1.5)^t$

5(b) (con't)

Another way to find the initial value:
Just jump backwards.

$$\underset{0}{C_0} \longrightarrow \underset{1}{C_1} \longrightarrow \underset{2}{1000}$$

(I am labeling the starting amount $C_0$, the amount at time 1 $C_1$.)

I know that I need to multiply by 1.5 for each jump. Thus $C_1(1.5) = 1000$

So $C_1 = \frac{1000}{1.5} = 666.\overline{66}$

Again, $C_0(1.5) = C_1$

So $C_0(1.5) = 666.\overline{66}$

$$\boxed{C_0 \approx 444.4}$$

Another method:

No need to find the initial value;

~~Just shift the graph~~

I can "pretend" that day 2 is the "start". In that case my equation would be

$$f(t) = 1000(1.5)^t$$

But day 2 isn't the start; I need to shift the graph forward 2 so that the day 2 value matches for $t=2$. Thus

$$f(t) = 1000(1.5)^{t-2}$$

(Notice that if I put $t=2$ in, I get my day 2 value of 1000).

5 (c) Check the day 10 values in the functions:

Linear: If $t=10$

$$f(10) = 1320(10) - 1640$$
$$= 11,560$$

Exponential: If $t=10$

$$f(10) \approx 444.4\,(1.5)^{10}$$
$$\approx 25626.34 \approx 25,630$$

Exponential is a better fit

(d) Find the inverse of $f(t) = 444.4\,(1.5)^t$

$$z = 444.4\,(1.5)^t$$

$$\frac{z}{444.4} = 1.5^t$$

$$\log_{1.5}\left(\frac{z}{444.4}\right) = \cancel{\log_{1.5}}\,(\cancel{1.5}^t)$$

$$\boxed{\log_{1.5}\left(\frac{z}{444.4}\right) = t}$$

Let $z = 100,000$

$$\log_{1.5}\left(\frac{100000}{444.4}\right) \approx \log_{1.5}(225)$$

To put in calculator (already an approximation)

$$\frac{\log(225)}{\log(1.5)} = 13.357$$

YIKES! Only 13 days!
Better get your machete!