Math 111: Midterm 1 Review

Prerequisite material (see review section for additional problems)

1. Simplify the following:

$$\frac{20a^{-2}b}{4a^{-2}b^{-1}}$$

$$(-2x^{-3}y^{-2})^2$$

$$8^{\frac{2}{3}} + \left(\frac{1}{4}\right)^{\frac{1}{2}}$$

$$(x+2)^2 - (x-5)$$

2. Factor the following:
   a) $x^2 + 2x - 3 = 0$
   b) $x^5 - 2x^3 - 6x = 0$
   c) $3x^2 - 9x = 0$

3. Steve is throwing an awesome New Year's Eve party. At 9:00 p.m. there are 21 people at the party. By midnight the number of women at the party has tripled, and the number of men at the party has doubled, with a total of 48 people at the party. How many men and women were at the party at 9:00?

1.1: Numbers, Data and Problem Solving (pages 10-13)

1. Convert the following into scientific notation:
   (a) $145,000,000$
   (b) $0.00037 + 0.00024$

2. The speed of light is about 186,000 miles per second. The Milky Way galaxy has an approximate diameter of 6×10^{17} miles. Estimate the number of years to the nearest thousand it takes for light to travel across the galaxy.

3. Calculate the average number of days in a month for 2009.

4. Mr. Franklin owns 240 shares of stock, 40% of which are in computers. If he buys 10 more shares of computers, what percentage of his stock will be in computers?

1.2: Visualization of Data (pages 25-28)

1. Find the midpoint of the line segment from $(4, -1)$ and $(-2, 3)$.

2. Find a value of x such that the distance between the points $(-1, 2)$ and $(x, -10)$ is 12.

3. Consider the following table

x	-1	0	4	-2
y	4	6	2	-2

   (a) List the data as a set of ordered pairs.
   (b) Plot the data on a graph.
   (c) Find the distance between the first and last ordered pair.

1.3: Functions and Their Representations (pages 42-45)

1. How can you tell if a relation is a function?

2. Is the following relation a function? Why or why not? $(1, 4), (-1, -4), (3, 7), (7, 7), (-1, -7), (3, 3)$

3. Draw a graph of a function. Then draw a graph of something that is not a function. Why is the first a function and the other not a function?

4. Is height a function of shoe size?

5. Find a function that computes the number of dimes in x dollars.

6. I am buying fabric to make a Halloween costume at a price of \$4 per yard. Write the cost of fabric as a function of yards. Find $f(3)$ and describe what it means in words.

7. If $f(x) = 2x^2 - 5$ find:
   (a) $f(2)$
   (b) $f(a - 1)$

8. Find the domain of the following functions:
(a) $f(x) = 3x^2 - 7$
(b) $g(x) = \frac{3x}{4x-2}$
(c) $h(x) = \sqrt{5 - 2x}$

1.4: Functions and Their Rates of Change (pages 58-61)

1. Consider the following data set:

x	-2	0	4	6
y	1	0	-2	-3

(a) Does the following data set represent a linear function?
(b) Find an equation that fits the data set.
(c) Graph the data set.

2. I throw an apple into the air, and its height in feet in the air is given by the equation $h(t) = -5t^2 + 20t + 5$, where t is time measured in seconds.
(a) Find $h(0)$. What does this mean in words?
(b) Find the average rate for change from $t = 1$ to $t = 2$. What are the units for the average rate of change?

3. Find the difference quotient for the function $g(x) = 5x^2 - 1$.

2.1: Linear Functions and Regression (pages 82-86)

1. The IRS has a standard mileage rate used to compute expense deductions for businesses. These are the rates from the years listed.

Year	*Mileage Rate*
1999	31.4
2000	32.5
2002	36.5
2003	36.0
2004	37.5

(a) Is this relation a function? What is the domain? What is the range?
(b) Is this relation a linear function?

2. I bought a cool, green geo metro in 2003 for \$1980, and now it is worth \$660. Assume that the value of the car and its age are related linearly.
(a) Write an equation that models this situation, with year and an input and value of

the car as an output.
(b) How much was the car be worth in 2005?
(c) In what year will the car be worth \$0?

(d) What is the domain of this function?

3. Consider the equation $2y = 6x + 4$.
   (a) If $x = \frac{1}{2}$, what is the value for y?
   (b) If $y = -5$, what is the value for x?
   (c) Is $x = \frac{4}{6}$ and $y = 4$ a solution to the equation?
   (d) Graph the equation, labeling the x-intercept, y-intercept and one other point.

2.2: Equations of Lines (pages 99-105)

1. For each of i to iv find the following information:
   a) Find an equation for the line.
   b) Graph the line.
   c) Find the x and y intercepts of the line.

   i) A line of slope -3 and y-intercept of $\frac{2}{3}$.
   ii) A line through the points $(-3, 5)$ and $(1, 1)$.
   iii) A line parallel to the line $2x + y = 9$ and y-intercept of -3.
   iv) A line through (3, -2) perpendicular to the line $y = 3x - 12$.

2.3: Functions and Their Representations (pages 118-123)

1. Solve the following equations for x using algebra:
   a) -$\frac{4}{3}(x + 4) = \frac{5x}{12} - \frac{1}{4}$

   b) $2(-3x + 2) = 4(x - 1) + 12$

2. Find a point on the line $y = -\frac{1}{2}x + 8$ that is also on the line $y = 4x - 4$.

3. Suppose you can rent a midsize car from Company A for \$250 per week with unlimited milage. A similar car can be rented from Company B for \$150 per week plus \$0.25 per mile. When is Company B a better deal?

2.4: Linear Inequalities (pages 134-139)

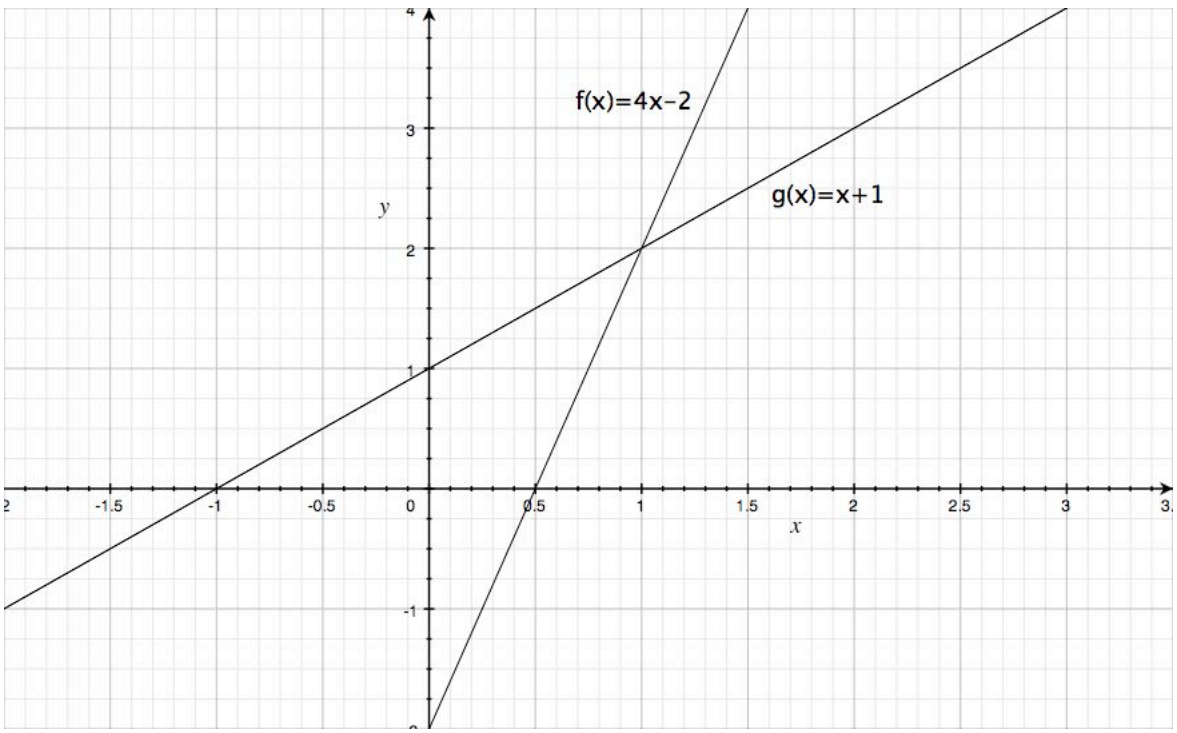
1. Rewrite the following inequalities as intervals and graph them on a number line:

   (a) $1 < x \leq 5$
   (b) $x \geq -3$
   (c) $x < -3$

2. Solve the following inequalities. Write the answer in interval notation and graph the solution on a number line.

   (a) $-8 \leq 1 - 3(x - 2) \leq 13$
   (b) $x - \frac{3}{2} \leq 5$
   (c) $-5 \leq -2x < 3$

3. Use the following graph to find the values of x for which $x + 1 \leq 4x - 2$. Give your solution in interval and inequality notation.

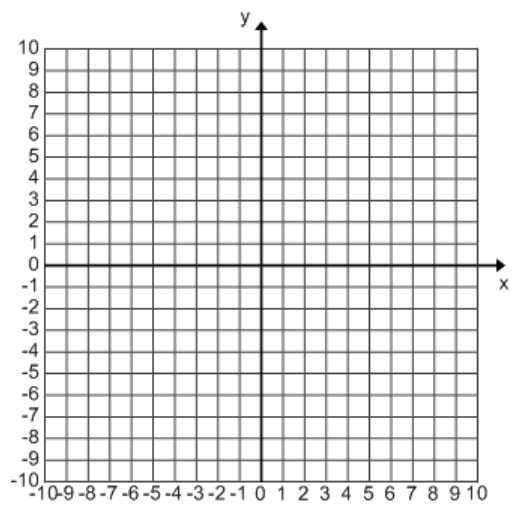


2.5: Piecewise Functions and Absolute Values

1. Consider the following piecewise function:

$$f(x) = \begin{cases} \frac{1}{3}x - 2 & \text{if} \quad -7 \leq x \leq -2 \\ -2 & \text{if} \quad -2 < x < 1 \\ -x + 1 & \text{if} \quad 1 \leq x \leq 6 \end{cases}$$

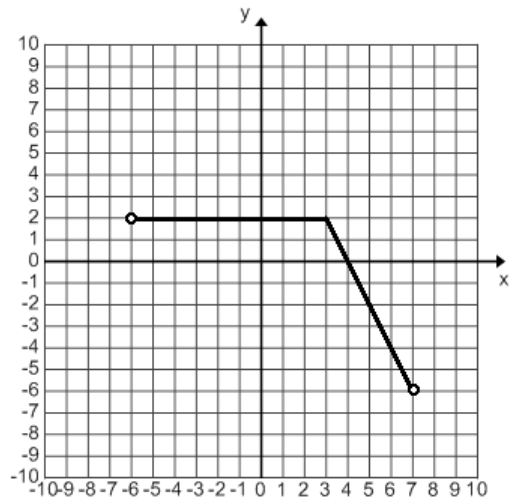
   (a) Find $f(-3)$, $f(0)$ and $f(2)$.
   (b) Is $f(x)$ continuous?
   (c) Graph $f(x)$ on the grid below.



2. Solve for x: $|2x - 5| \leq 3$.
   (a) Write your solution graphically on a number line.
   (b) Write your solution using interval notation.

3. Let $g(x) = |x - 3|$.
   (a) Graph $g(x)$.
   (b) Write $g(x)$ as a piecewise function.

4. Find a piecewise equation for the graph below.



3.1: Quadratic Functions and Models

1. Find the vertex of a parabola with the equation $f(x) = 3(x + 4) - 7$.

2. Let $f(x) = x^2 + 10x + 7$.
   (a) Rewrite $f(x)$ in vertex form by completing the square.
   (b) Find the y-intercept of $f(x)$. (NOTE: Find the Y-intercept).
   (c) Does this parabola open up or down? How can you tell?

3. Write an equation of a quadratic function whose graph has a vertex of $(-1, 3)$, is short and fat, and opens up.

4. A farmer is building a rectangular pig-pen along the side of a barn. He has 200 meters of fence material. He only needs to build three sides of the pen, because the barn forms the fourth side. What are the dimensions of the pen that maximize the area?

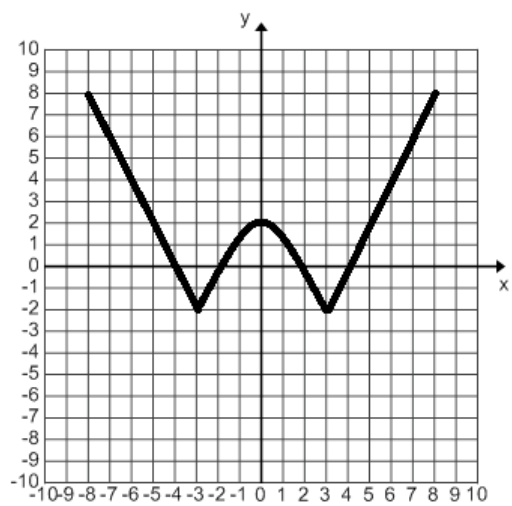
5. Koller Company is selling vacuum cleaners, but they haven't decided how much the vacuums should cost. Their profit function is given by $P(v) = -\frac{1}{2}v^2 + 50v$, where $P(v)$ is profit in thousands of dollars, and v is the price of the vacuums.
   (a) What price should they sell the vacuums in oder to make \$800,000 in profit?
   (b) How much should they sell vacuums for to maximize profit?
   (c) What is the maximum profit in part (b)?

3.2: Quadratics and their Intercepts

1. Solve for x: $x^2 = 3x$.
2. Let $f(x) = 2 - 3x + 8x^2$. Find all real values of x such that $f(x) = 0$.
3. Write an equation of a quadratic function that has x-intercepts of $(-2, 0)$ and $(4, 0)$. Is this the only possible equation?

3.4: Transformations of Graphs

1. Given the graph of $g(x)$

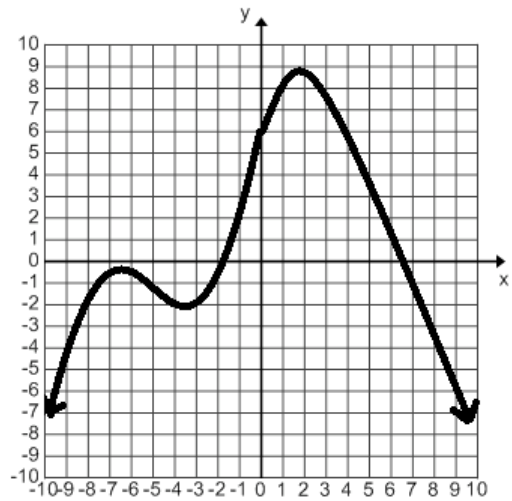


   (a) Graph $g(x) + 3$ and $g(x + 3)$.
   (b) Graph $-g(x)$.

2. Sketch a graph of the function $f(x) = \frac{1}{2}(x + 2)^2$.

4.1: Nonlinear Functions and Their Graphs

1. Given the function $g(x)$ below:
   (a) List the intervals where $g(x)$ is increasing.
   (b) Label all local maximums and local minimums.
   (c) Label any global maximums or global minimums, or explain why they don't exist.



2. State whether the following function is odd, even or neither. Then graph the function.
   (a) $f(x) = x^2 - 2$
   (b) $g(x) = 3x + x^3$
   (c) $h(x) = x^3 + 1$

3. Give an example of a function that is both even and odd, or explain why one cannot exist.

4.2: Polynomial Functions and Models

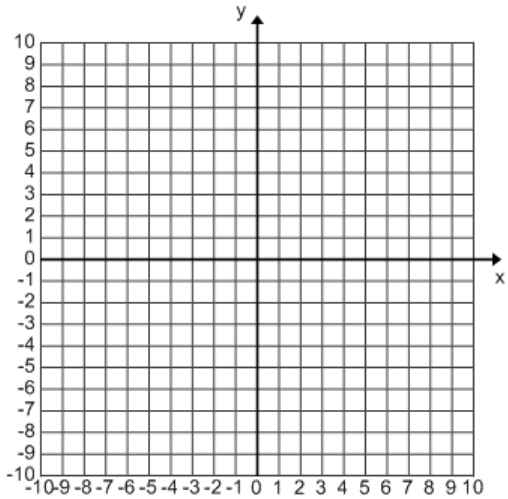
1. Let $f(x) = 3x + 2x^2 - \frac{1}{3}x^5$
   (a) What is the degree of $f(x)$?
   (b) What is the end behavior of $f(x)$?

2. Sketch the graph of an example of a degree 4 polynomial with a negative leading coefficient, exactly 3 turning points and exactly 2 x-intercepts.

3. Consider the following piecewise function:

$$f(x) = \begin{cases} 2x + 6 & \text{if} \quad -7 \leq x \leq -2 \\ 3 & \text{if} \quad -2 < x < 0 \\ -x^2 + 3 & \text{if} \quad 0 \leq x \leq 7 \end{cases}$$

   (a) Find $f(-2)$, $f(0)$ and $f(2)$.
   (b) Is $f(x)$ continuous?
   (c) Graph $f(x)$ on the grid below.



4.3: Real Zeros of Polynomial Functions

1. Let $f(x) = x^3 - 4x^2 + 3x - 2$.
   (a) Is $x = 3$ a zero of $f(x)$?
   (b) Is (x-3) a factor of $f(x)$?

2. Divide the expression
$$\frac{x^3 - 2x^2 - x + 3}{x + 1}$$

3. Factor $g(x) = 2x^3 + x^2 - 11x - 10$. Hint: $k = -2$ is a zero.

4. Draw a graph of a degree 5 function with zero $x = 3$ of multiplicity 2 and zero $x = -1$ with multiplicity 3.