

Name:

SOLUTIONS

INSTRUCTIONS

- No notes
- No Homework
- Graphing calculators are allowed
- Read the directions carefully. Be sure to show all of your work. Partial credit will be given for partially correct solutions.
- Please ask clarifying questions if you are ever unsure.
- If you are stumped on a question, I might provide hints in return for a point deduction.

1. True/False. (3 points each)

a) The vertex of a parabola with the equation $f(x) = -2(x + 1)^2 + 5$ is $(1, 5)$.

FALSE The vertex is $(-1, 5)$

b) $x = 2$ is a zero of the function $f(x) = x^3 - x^2 - 4x + 4$.

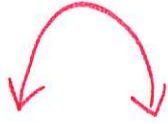
TRUE

Check: $f(2) = 2^3 - 2^2 - 4 \cdot 2 + 4$
 $= 8 - 4 - 8 + 4$
 $= 0$

c) The function $g(x) = -x^2 + 3x - 2$ has no global minimum.

TRUE

It is a downward facing parabola:
keeps going down



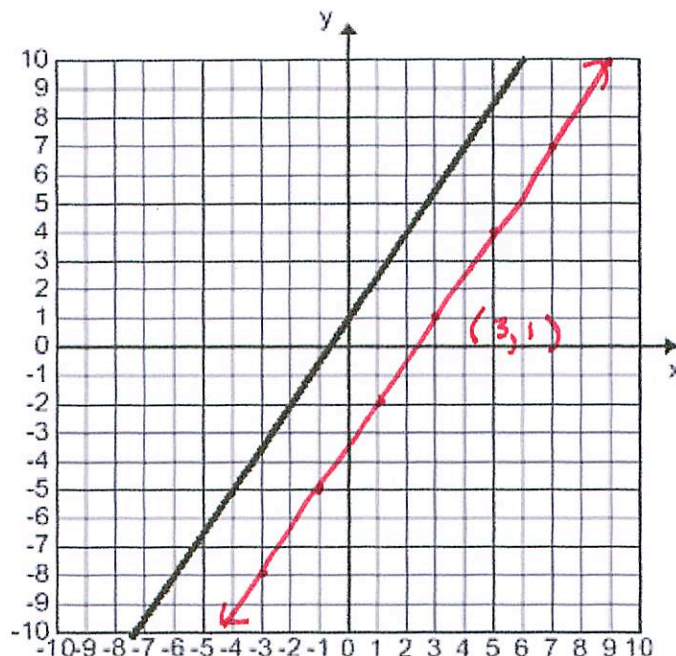
d) The following relation is a function:

x	-1	3	4	-2	3
y	4	5	4	8	5

TRUE

For each input there is only one output: plug in 3 get out 5 every time

2. (10 points) The line below has the equation $y = \frac{3}{2}x + 1$.



- a) Find an equation of a new line that is parallel to this line that goes through the point (3, 1).

$$m = \frac{3}{2}$$

$$y = \frac{3}{2}x + b$$

plug in (3, 1) to
find b:

$$1 = \frac{3}{2}(3) + b$$

$$1 = \frac{9}{2} + b$$

$$\frac{2}{2} - \frac{9}{2} = b$$

$$-\frac{7}{2} = b$$

$$y = \frac{3}{2}x - \frac{7}{2}$$

- b) Find the exact x and y intercepts of your new line. Show all of your work.

$$x\text{-int: } y = 0$$

$$0 = \frac{3}{2}x - \frac{7}{2}$$

$$\frac{7}{2} = \frac{3}{2}x \cdot \frac{2}{3}$$

$$\frac{7}{3} = x$$

$$y\text{-int: } x = 0$$

$$y = 0 - \frac{7}{2}$$

$$y = -\frac{7}{2}$$

$$(0, -\frac{7}{2})$$

- c) Draw your new line on the graph above.

3. (10 points) My car has a 13 gallon gas tank and uses 3 gallons per hour when driving at 60 miles per hour. After filling up the gas tank, I drive at 60 miles per hour until the gas runs out. Let $G = f(t)$ be the amount of gasoline in the tank at time t hours after filling the tank.

a) After two hours of driving, how much gas do I have?

$$13 - 2 \cdot 3 = 13 - 6 = 9 \text{ gallons}$$

b) Find a linear equation for f .

$$G = 13 - 3t$$

↑ ↖ how much I lose each hour
starting amount

c) What is the G -intercept? What does it represent in this situation?

$$\text{when } t=0, G=13$$

I start with 13 gallons

d) What is the t -intercept? What does it represent in this situation?

$$\text{when } G=0, 0 = 13 - 3t$$

$$\begin{array}{l} 3t = 13 \\ \boxed{t = \frac{13}{3}} \end{array}$$

I run out of gas in $\frac{13}{3} = 4 \frac{1}{3}$ hour

e) What domain makes sense for this situation?

$$0 \leq t \leq \frac{13}{3}$$

These are the times I have gas in the tank.

4. (10 points) Show all of your work by hand. Stating "I graphed it" is not valid work.

Let $f(x) = -\frac{1}{3}x^2 + \frac{4}{3}x + 4$.

(a) Find the vertex of $f(x)$.

x-coord: $\frac{-b}{2a} = \frac{-\frac{4}{3}}{2 \cdot (-\frac{1}{3})} = \frac{-\frac{4}{3}}{-\frac{2}{3}} \cdot \frac{3}{3} = \frac{-4}{-2} = 2$

y-coord: $y = -\frac{1}{3}(2^2) + \frac{4}{3} \cdot 2 + 4$
 $= -\frac{4}{3} + \frac{8}{3} + 4 = -\frac{4}{3} + \frac{8}{3} + \frac{12}{3} = \frac{16}{3}$

$(2, \frac{16}{3})$

(b) Find the x and y-intercepts of $f(x)$.

y-int is when $x=0$
 $y = -\frac{1}{3} \cdot 0 + \frac{4}{3} \cdot 0 + 4$

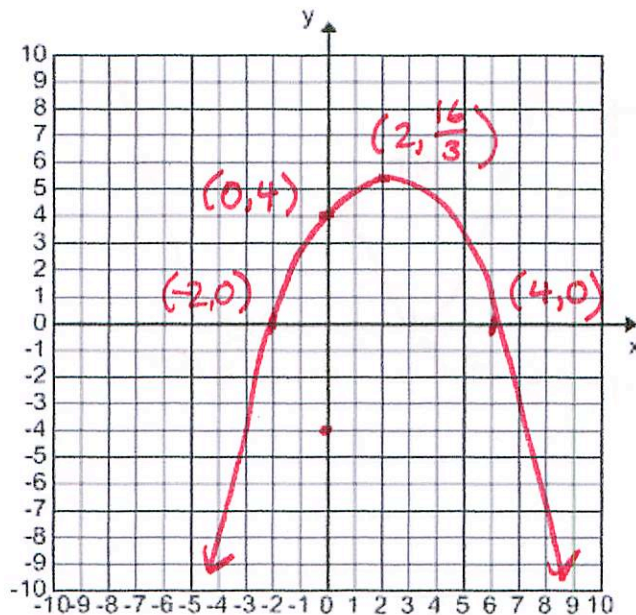
$(0, 4)$

x-int is when $y=0$

$\frac{-\frac{4}{3} \pm \sqrt{(\frac{4}{3})^2 - 4 \cdot (-\frac{1}{3})(4)}}{2(-\frac{1}{3})} = \frac{-\frac{4}{3} \pm \sqrt{\frac{16}{9} + \frac{16}{3}}}{-\frac{2}{3}}$

$= \frac{-\frac{4}{3} \pm \sqrt{\frac{16}{9} + \frac{48}{9}}}{-\frac{2}{3}} = \frac{-\frac{4}{3} \pm \frac{8}{3}}{-\frac{2}{3}} \cdot \frac{3}{3} = \frac{-4 \pm 8}{-2} =$

(c) Graph $f(x)$. Be sure to label the vertex and all intercepts.

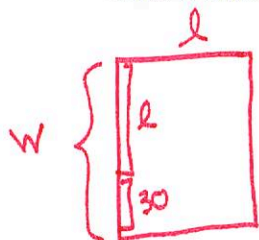


$\frac{-4+8}{-2} = -2$

$\frac{-4-8}{-2} = 6$

x-int
 $(-2, 0)$
 $(6, 0)$

5. (8 points) I am building a house with a rectangular floor. I want the width of the floor to be 30 feet longer than the length of the floor. The total area of the floor should be 9976 ft². What should the dimensions of the floor be?



$$w = l + 30$$

$$A = l \cdot w$$

$$A = l \cdot (l + 30)$$

$$A = l^2 + 30l$$

$$\text{Set } 9976 = l^2 + 30l$$

$$0 = l^2 + 30l - 9976$$

$$\frac{-30 \pm \sqrt{30^2 - 4 \cdot 1 \cdot (-9976)}}{2} = \frac{-30 \pm 202}{2} = \begin{matrix} 86 \\ \text{or} \\ -146 \end{matrix}$$

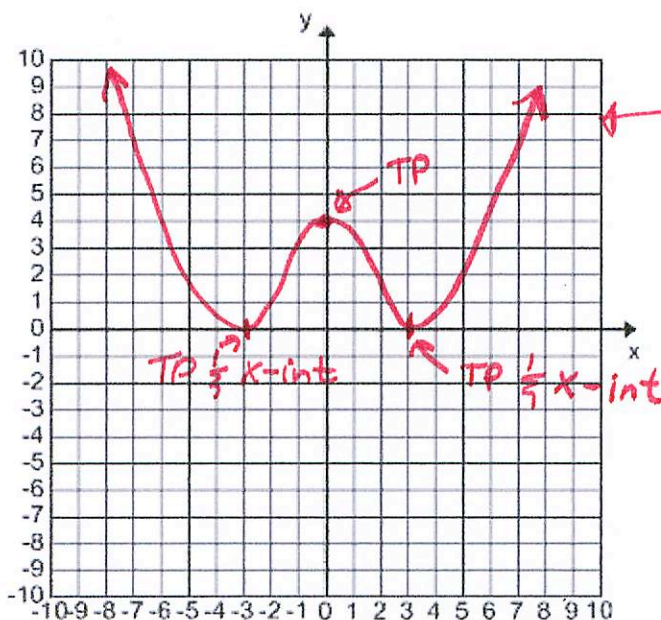
$$x = 86$$

$$y = 86 + 30 = 116$$

$$\boxed{\begin{matrix} w = 116 \\ l = 86 \end{matrix}}$$

6. (5 points) Sketch a graph of an even polynomial that goes through the point (0, 4) with exactly 3 turning points and 2 x-intercepts. Be sure to label the x-intercepts and turning points.

negative values don't make sense for this problem

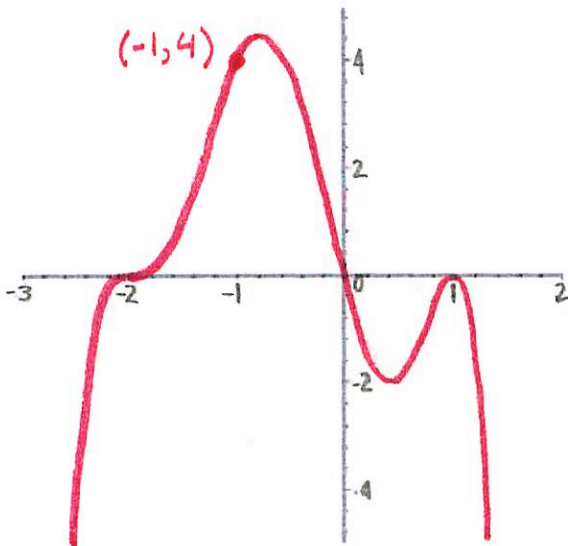


It is even because it is symmetric about the y-axis

7. (5 points) (a) For the polynomial below, list the x-intercepts and state the multiplicity of each x-intercept.

$x = -2$ mult. odd (3?): touches & goes through
 $x = 0$ mult. 1: goes straight through
 $x = 1$ mult. even (2?): bounces off

- (b) Give a possible equation of the polynomial in factored form.



$$y = a(x+2)^3(x+0)^1(x-1)^2$$

Solve for a: plug in (-1, 4)

$$4 = a(-1+2)^3(-1)^1(-1-1)^2$$

$$4 = a(1)^3(-1)(-2)^2$$

$$4 = a \cdot 1 \cdot -1 \cdot 4$$

$$\frac{4}{-4} = a = -1$$

$$y = (x+2)^3(x)(x-1)^2$$

8. Challenge Question: (Only look at this question after you have finished all of the other questions!) a) If $f(x^2 - 4x + 4) = 2 - x$ find $f(3x)$.

If I plug in $x^2 - 4x + 4$, I get out $2 - x$.

$$\text{But } x^2 - 4x + 4 = (x-2)^2$$

$$\text{Thus } -\sqrt{x^2 - 4x + 4} = -(x-2) = 2-x.$$

- b) What is the domain of f ?

Domain: $x > 0$

$$f(x) = -\sqrt{x}$$

$$\text{So } f(3x) = -\sqrt{3x}$$