

Math 111 Midterm A

July 20, 2010

Name:

SOLUTIONS

INSTRUCTIONS

- No notes
- No Homework
- Graphing calculators are allowed
- Read the directions carefully. Be sure to show all of your work.
- Please ask clarifying questions if you are ever unsure.
- If you are stumped on a question, I might provide hints in return for a point deduction.

1. True/False. (3 points each)

a) The following relation is a function:

$x$	-1	3	4	-2	3
$y$	4	5	4	8	5

True

For every input there is one output.  
For the input 3, the outputs match.

b) The vertex of a parabola with the equation  $f(x) = -2(x+1)^2 + 5$  is (1, 5).

FALSE

The vertex is at (-1, 5)

c) The function  $g(x) = -x^2 + 3x - 2$  has no global minimum.

True

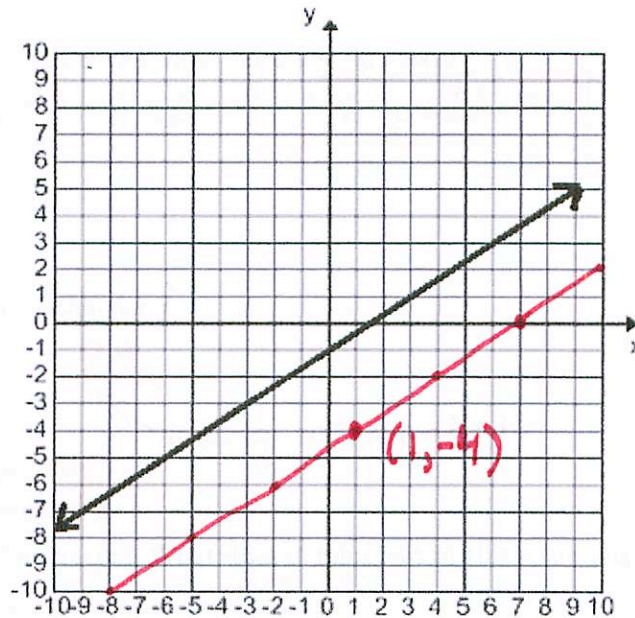
It is an upside down parabola:  
No global min b/c it keeps  
going down.

d)  $x = 2$  is a zero of the function  $f(x) = x^3 - x^2 - 4x + 4$ .

True

$$\begin{aligned} \text{Check: } f(2) &= 2^3 - 2^2 - 4 \cdot 2 + 4 \\ &= 8 - 4 - 8 + 4 \\ &= 0 \end{aligned}$$

2. (10 points) The line below has the equation  $y = \frac{2}{3}x - 1$ .



- a) Find an equation of a new line that is parallel to this line that goes through the point  $(1, -4)$ .

$m = \frac{2}{3}$   
 $y = \frac{2}{3}x + b$   
 plug in  $(1, -4)$  to solve for  $b$   
 $-4 = \frac{2}{3} + b$   
 $-\frac{12}{3} - \frac{2}{3} = -\frac{14}{3} = b$   
 $y = \frac{2}{3}x - \frac{14}{3}$

- b) Find the exact  $x$  and  $y$  intercepts of your new line. Show all of your work.

Let  $x = 0$   
 $y = -\frac{14}{3}$   
 $(0, -\frac{14}{3})$   
 Let  $y = 0$   
 $0 = \frac{2}{3}x - \frac{14}{3}$   
 $\frac{3}{2} \cdot \frac{14}{3} = \frac{2}{3}x \cdot \frac{3}{2}$   
 $7 = x$

- c) Plot your new line on the graph above.

3. (10 points) My car has a 13 gallon gas tank and uses 2 gallons per hour when driving at 65 miles per hour. After filling up the gas tank, I drive at 65 miles per hour until the gas runs out. Let  $G = f(t)$  be the amount of gasoline in the tank at time  $t$  hours after filling the tank.

a) After two hours of driving, how much gas do I have?

$$13 - 2 \cdot 2 = 13 - 4 = \boxed{9 \text{ gallons}}$$

b) Find a linear equation for  $f$ .

$$G = 13 - 2t$$

$\uparrow$  starting amount       $\nwarrow$  losing at 2 gal/hr

c) What is the  $G$ -intercept? What does it represent in this situation?

$G$  int. is when  $t = 0$

$G = 13$  is the gallons when I start.

d) What is the  $t$ -intercept? What does it represent in this situation?

$t$  int. is when  $G = 0$

$$\begin{aligned} 0 &= 13 - 2t \\ 2t &= 13 \\ t &= 6.5 \end{aligned}$$

The gas will run out in  $6\frac{1}{2}$  hours

e) What is the slope of the model? What does it mean in this situation?

Slope is  $-2$ . I lose 2 gallons every hour.

f) What domain makes sense for this situation?

Domain: ~~no~~ input values:  $t$  values that work.  
 In this case, only values  $0 \leq t \leq 6.5$ ,  
 because those are the times that there is  
 gas in your tank.

4. (10 points) Show all of your work by hand. Stating "I graphed it" is not valid work.

Let  $f(x) = -\frac{1}{3}x^2 + \frac{4}{3}x + 4$ .

(a) Find the vertex of  $f(x)$ .

x-coord:  $\frac{-b}{2a} = \frac{-\frac{4}{3}}{2 \cdot (-\frac{1}{3})} = \frac{-\frac{4}{3}}{-\frac{2}{3}} \cdot \frac{3}{3} = \frac{-4}{-2} = 2$

y-coord:  $y = -\frac{1}{3}(2^2) + \frac{4}{3} \cdot 2 + 4$   
 $= -\frac{4}{3} + \frac{8}{3} + 4 = -\frac{4}{3} + \frac{8}{3} + \frac{12}{3} = \frac{16}{3}$

$(2, \frac{16}{3})$

(b) Find the x and y-intercepts of  $f(x)$ .

y-int is when  $x=0$   
 $y = -\frac{1}{3} \cdot 0 + \frac{4}{3} \cdot 0 + 4$

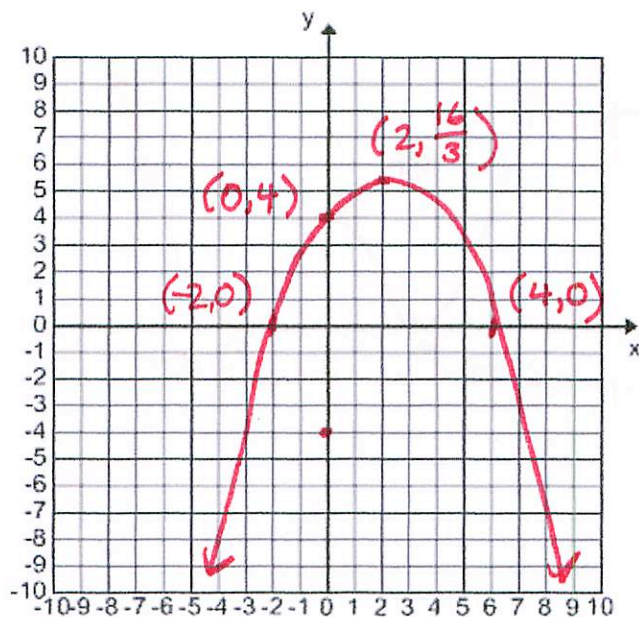
$(0, 4)$

x-int is when  $y=0$

$\frac{-\frac{4}{3} \pm \sqrt{(\frac{4}{3})^2 - 4 \cdot (-\frac{1}{3}) \cdot 4}}{2(-\frac{1}{3})} = \frac{-\frac{4}{3} \pm \sqrt{\frac{16}{9} + \frac{16}{3}}}{-\frac{2}{3}}$

$= \frac{-\frac{4}{3} \pm \sqrt{\frac{16}{9} + \frac{48}{9}}}{-\frac{2}{3}} = \frac{-\frac{4}{3} \pm \frac{8}{3}}{-\frac{2}{3}} \cdot \frac{3}{3} = \frac{-4 \pm 8}{-2} =$

(c) Graph  $f(x)$ . Be sure to label the vertex and all intercepts.

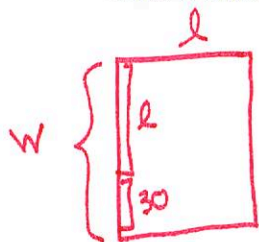


$\frac{-4+8}{-2} = -2$

$\frac{-4-8}{-2} = 6$

x-int  
 $(-2, 0)$   
 $(6, 0)$

5. (8 points) I am building a house with a rectangular floor. I want the width of the floor to be 30 feet longer than the length of the floor. The total area of the floor should be 9976 ft<sup>2</sup>. What should the dimensions of the floor be?



$$w = l + 30$$

$$A = l \cdot w$$

$$A = l \cdot (l + 30)$$

$$A = l^2 + 30l$$

$$\text{Set } 9976 = l^2 + 30l$$

$$0 = l^2 + 30l - 9976$$

$$\frac{-30 \pm \sqrt{30^2 - 4 \cdot 1 \cdot (-9976)}}{2} = \frac{-30 \pm 202}{2} = \begin{matrix} 86 \\ \text{or} \\ -146 \end{matrix}$$

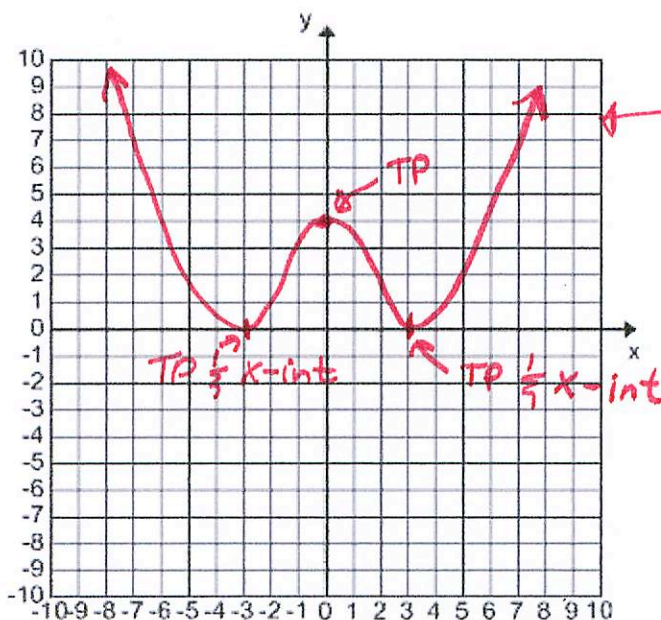
$$x = 86$$

$$y = 86 + 30 = 116$$

$$\boxed{\begin{matrix} w = 116 \\ l = 86 \end{matrix}}$$

6. (5 points) Sketch a graph of an even polynomial that goes through the point (0, 4) with exactly 3 turning points and 2 x-intercepts. Be sure to label the x-intercepts and turning points.

*negative values don't make sense for this problem*

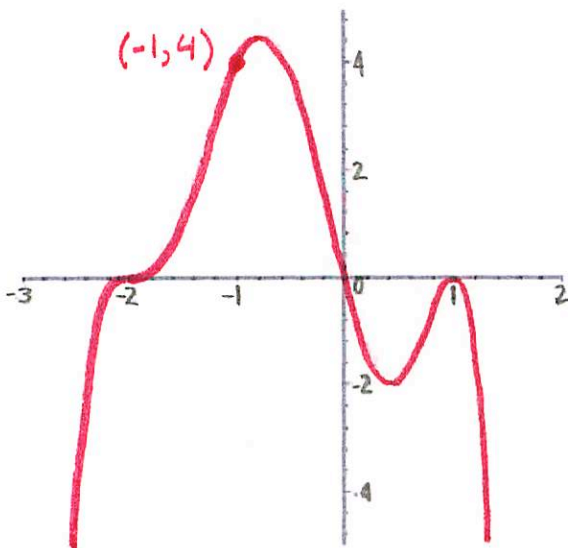


*It is even because it is symmetric about the y-axis*

7. (5 points) (a) For the polynomial below, list the x-intercepts and state the multiplicity of each x-intercept.

$x = -2$  mult. odd (3?): touches & goes through  
 $x = 0$  mult. 1: goes straight through  
 $x = 1$  mult. even (2?): bounces off

- (b) Give a possible equation of the polynomial in factored form.



$$y = a(x+2)^3(x+0)^1(x-1)^2$$

Solve for a: plug in (-1, 4)

$$4 = a(-1+2)^3(-1)^1(-1-1)^2$$

$$4 = a(1)^3(-1)(-2)^2$$

$$4 = a \cdot 1 \cdot -1 \cdot 4$$

$$\frac{4}{-4} = a = -1$$

$$y = (x+2)^3(x)(x-1)^2$$

8. Challenge Question: (Only look at this question after you have finished all of the other questions!) a) If  $f(x^2 - 4x + 4) = 2 - x$  find  $f(3x)$ .

If I plug in  $x^2 - 4x + 4$ , I get out  $2 - x$ .

But  $x^2 - 4x + 4 = (x-2)^2$

Thus  $-\sqrt{x^2 - 4x + 4} = -(x-2) = 2-x$ .

- b) What is the domain of  $f$ ?

Domain:  $x > 0$

$$f(x) = -\sqrt{x}$$

$$\text{So } f(3x) = -\sqrt{3x}$$