

Math 111: Final Review

Suggested Directions: Start by reviewing the new material with the first portion of the review sheet. Then take every quiz again as if it were a test. No book. No notes. Limit yourself to 10-15 minutes per quiz. After you are done, check your work with the online copies at web.pdx.edu/~mccaffrc

If you miss a problem, review that section of the book by doing old homework problems or the previous review packet.

4.4 Complex Numbers

1. What does the mathematical symbol i equal?
2. Simplify the following
 - (a) $(-4 + 2i) - (7 + 35i)$
 - (b) $(3+4i)(2-i)$
3. Find the solutions to the following equations:
 - (a) $2x^2 - x + 7 = 0$
 - (b) $x^2 = 3x - 5$
4. $f(x)$ is a degree 4 polynomial with no real zeros. How many x-intercepts does $f(x)$ have? Sketch a possible graph of $f(x)$.

4.5 Rational Functions

$$f(x) = \frac{(x+2)(x-1)}{(2x+5)(x-1)(x+3)}$$

- (a) Find the vertical asymptotes of $f(x)$.
- (b) Find the horizontal asymptotes of $f(x)$.
- (c) Find the holes of $f(x)$.
- (d) Find the x-intercepts of $f(x)$, or explain why they don't exist.
- (e) Find the y-intercept of $f(x)$, or explain why it doesn't exist.
- (f) Sketch a graph of $f(x)$.

5.1 Combining Functions

1. Let $f(x) = 4x - x^2$ and $g(x) = x + 3$. Find:
 - (a) $(g + g)(2)$
 - (b) $(f \circ g)(x)$
 - (c) $(g \circ f)(1)$

2. Given the following tables of $f(x)$ and $g(x)$:

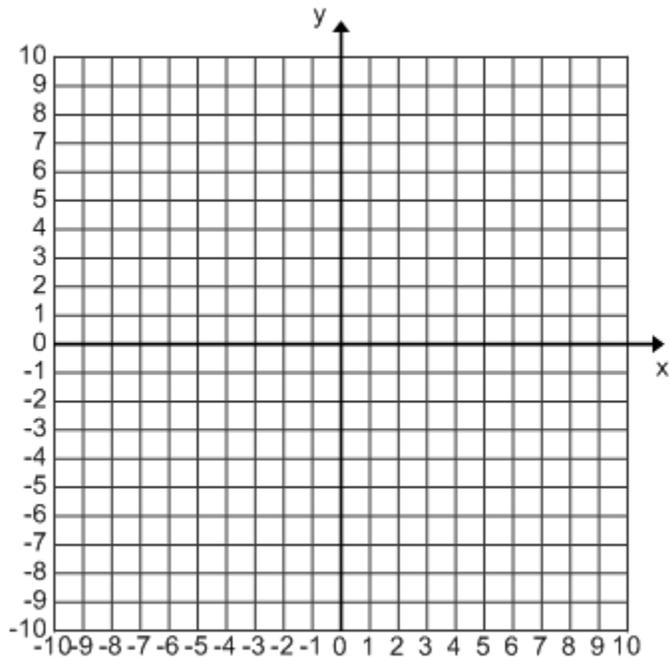
x	1	3	4	6
$f(x)$	2	6	5	7

x	2	3	4	7
$g(x)$	4	2	6	0

- (a) Find $(g \circ f)(1)$.
- (b) Find $(f \circ g)(4)$.
- (c) Find $(f \circ f)(3)$.

5.2 Inverse Functions

- 1. Give a definition of function. Give a definition of 1 to 1 function. How can you tell when a function is 1 to 1?
- 2. Graph the inverse of the following function on the same axis.



- 3. Find the inverses of the following:
 - (a) Take three steps forward and hop once.
 - (b) $f(x) = 2x - 5$
 - (c) $g(x) = \frac{2}{\sqrt{x}}$

4. True or False: $f \circ f^{-1}(x) = x$. Test your assumption using the function $f(x) = 2x - 5$ from 2(c).

5.3 Exponential Functions

1. Simplify the following:
 - (a) 2^{-3}
 - (b) $4^{\frac{1}{6}} 4^{\frac{1}{3}}$
2. Sketch a graph of the following:
 - (a) $f(x) = 2^x$
 - (b) $f(x) = \frac{1}{2}^x$
 - (c) $f(x) = -2^x$
3. \$950 at 5% is compounded monthly. How much is in the account after 2 years?
4. My pokeman trading card was worth 3 dollars, but it loses half of its value every month. Find C and a so that $f(x) = Ca^x$ models this situation.

5.4 More Logs

1. Simplify the following:
 - (a) $\log\left(\frac{1}{1000}\right)$
 - (b) $\log_9(3)$
 - (c) $\ln(1)$
2. Solve the following equations for x :
 - (a) $3^x = 81$
 - (b) $2 \cdot 10^x = 66$
 - (c) $\ln(x) = 2$
 - (d) $\log_3(5x) = 10$
3. Graph the function $f(x) = \log_2(x)$. (Hint: $f^{-1}(x) = 2^x$.)

5.5 Properties of Logs

1. Simplify the following using log rules:

(a) $\log_3(5) + \log_3(2)$

(b) $\frac{\log_4(8)}{\log_4(2)}$

(c) $\ln(6) - \ln(2)$

2. True or False

(a) $\ln(x^4) - \ln(x^2) = 2\ln(x)$

(b) $\log(x^2) + \log(x^3) = 5\log(x)$

5.6 Exponential and Logarithmic Equations

1. Solve the following for x :

(a) $2e^{-x} = 8$

(b) $7^{x^2} = 7^{4x-3}$

(c) $3 \cdot 2^x = -12$

(d) $5\ln(x) = 10$

(e) $\log_3(1 - x) = 1$

2. My car loses 20% of its value every year. This year it is worth \$5000 dollars.

(a) How much will the car be worth in 1 year? In 3 years? In 10 years?

(b) Find an equation for the value of the car as a function of the year.

(c) When will the car be worth \$3000 dollars?

3. How long does it take for \$1000 to double its value if the interest rate is 12% compounded continuously?

4. I decided to start a rabbit farm. At the start of the year there were 10 rabbits, and after 3 months there were 45 rabbits.

(a) Assume the number of rabbits can be modeled by a linear equation. Find an equation for the number of rabbits, R , as a function of time, t , (in months).

(b) Assume that the number of rabbits can be modeled by an exponential equation. Find an equation for the number of rabbits, R , as a function of time, t , (in months).

(c) Compare the graphs of these two equations. When does the linear model predict a greater number of rabbits? When does the exponential model predict a greater number of rabbits? When do the models predict the same number of rabbits?

5. Janet decided to rid her home of ants by setting lots of ant traps. Three weeks into the process, there were 1200 ants in her home. Six weeks later (on week 9) there were only 900 ants. Assume that this function can be modeled using an exponential function.

(a) Write an equation for the number of ants in Janet's house as a function of time.

(b) How many ants were in Janet's house to begin with? At week 20 how many ants

are in Janet's house?

(c) When will there be fewer than 100 ants?

Review Material

1. Solve for x : $5(3x + 2) = 7x - (x + 2)$
2. Factor the following expression: $x^2 - x - 12$
3. Use the properties of exponents to simplify the following expression:

$$\frac{10x^3y^{-4}}{15x^5y^2}$$

4. Rewrite in standard notation: 6.21×10^{-5}
5. True or False (Please show your work)
 - (a) The point $(2, 5)$ lies on the graph $f(x) = 2x + 5$.
 - (b) The data in the following table are linear.

x	-1	0	3	5
y	10	8	2	-2

- (c) The midpoint of the line segment connecting $(1, 2)$ and $(5, -4)$ is $(3, -1)$.
6. Find the difference quotient of the function $f(x) = 2x + 1$.
(Hint: The difference quotient is the average rate of change between x and $x + h$.)
 7. You are 5 years older than my sister Susan.
 - (a) Find a function f that computes your age as a function of Susan's age x .
 - (b) Find $f(20)$, and explain what this means in words.
 - (c) What is the domain of this function? Explain.
 8.
 - (a) Solve the following inequality for x : $-2(x + 3) \leq \frac{1}{4}$
 - (b) Give your solution graphically on a number line.

9. Graph the following piecewise function:

$$f(x) = \begin{cases} -4 & \text{if } -6 \leq x \leq -1 \\ 2x & \text{if } -1 < x < 2 \\ -x + 3 & \text{if } 2 \leq x \leq 7 \end{cases}$$

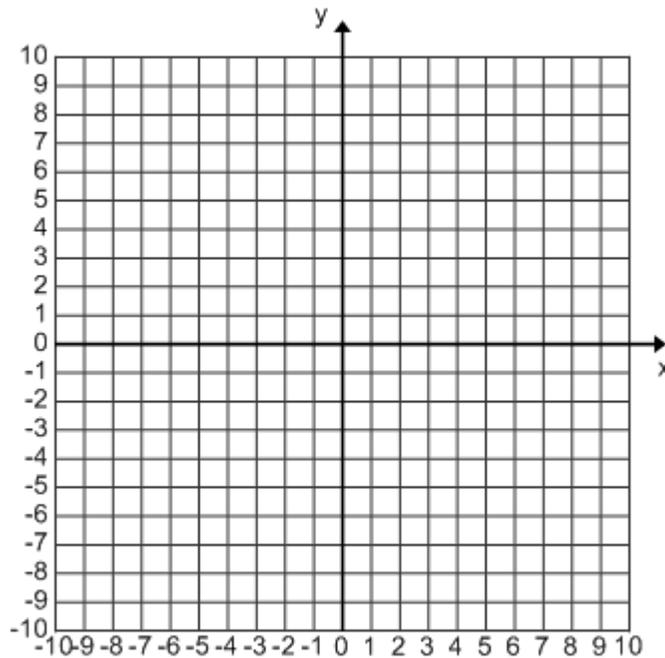
10. Let $g(x) = |2x + 3|$.

- (a) Find the value of x so that $g(x) = 0$.
- (b) Find the values of x so that $g(x) > 1$.
- (c) Write $g(x)$ as a piecewise function.
- (d) Graph $g(x)$.

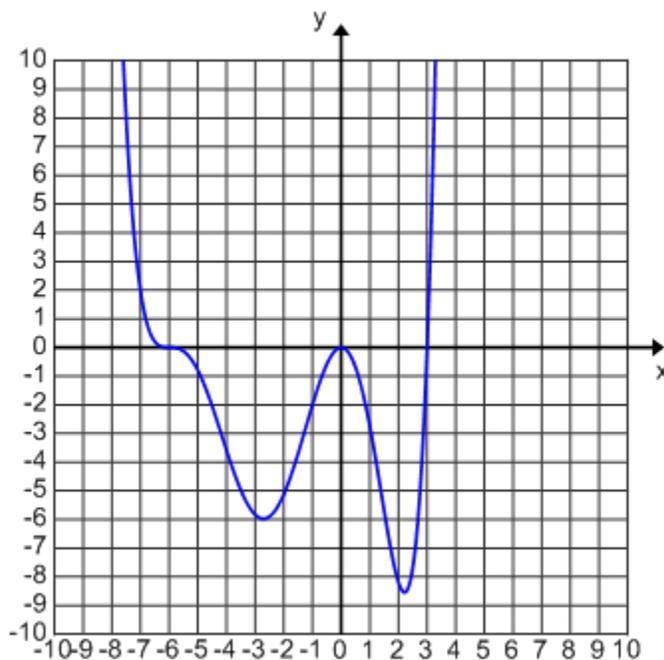
11. Let $f(x) = -2x + x + 3$.

- (a) Find the x -intercepts of $f(x)$.
- (b) Find the vertex of $f(x)$. (Include both x and y coordinates.)

12. Below is the graph of $g(x) = |x - 3|$.



- (a) Find the value of $g(x)$ when $x = 2$.
- (b) Graph the function $y = g(x) - 3$.



- List the approximate intervals where $g(x)$ is increasing. (Hint: your intervals should be given in terms of x .)
- Label any local maximums and local minimums, or explain why they don't exist.
- Label any global maximums or global minimums, or explain why they don't exist.
- Is this function odd, even, or neither? How can you tell?
- What is a possible degree of $g(x)$? How can you tell?
- List the zeros of $g(x)$.
- Using the zeros of $g(x)$ found above, write a possible equation for $g(x)$. Do not simplify.

13. Completely factor the following polynomial: $f(x) = x^3 - x^2 + 2x - 2$.

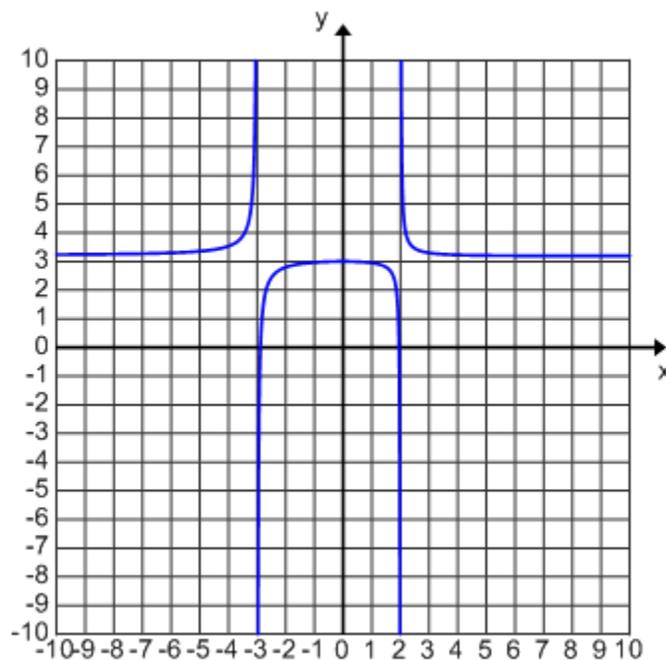
14.

$$h(x) = \frac{x^2}{(3x + 5)(x - 4)}$$

- Find $h(5)$.
- Find the horizontal asymptotes of $h(x)$.
- Find the vertical asymptotes of $h(x)$.
- Graph $h(x)$.

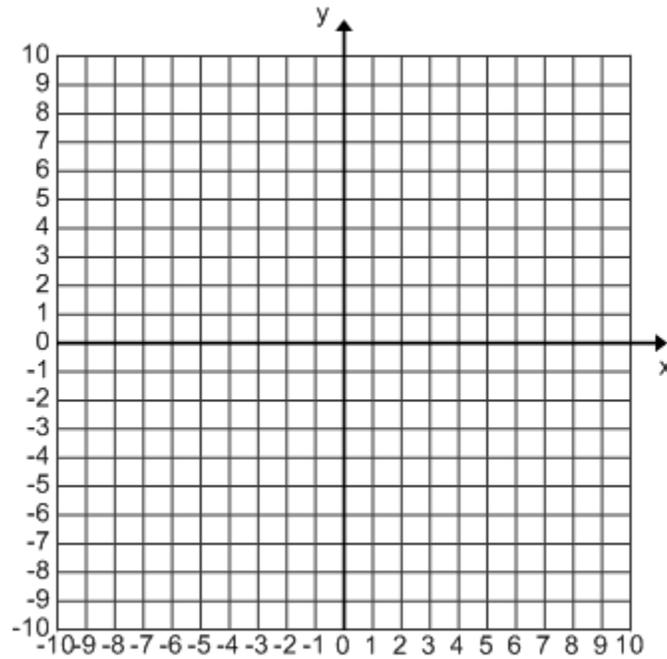
15. A graph of $g(x)$ is given below.

- List the horizontal asymptotes of $g(x)$. How can you tell?
- List the vertical asymptotes of $g(x)$. How can you tell?



- (c) Find a possible equation for $g(x)$. (Use a calculator to check).
16. Find the inverse of $h(t) = 3t - \frac{2}{3}$.
17. Let $f(x) = 2x^2 + 1$ and $g(x) = 4\sqrt{x}$.
- Find $g \circ f(2)$.
 - Find $f \circ g(x)$.
18. The half-life of Plutonium is 127 years.
- What percentage of Plutonium is remaining after 300 years?
 - Write an equation for the percentage of remaining Plutonium as a function of time (in years).
 - When does the percentage of remaining Plutonium reach 10%?
 - Write an equation that models the YEAR (time is output) as a function of PERCENTAGE PLUTONIUM.

19. Let $k(x)$ be given by the graph below.



- (a) What is the domain of $k(x)$?
- (b) Is $k(x)$ a function? Explain.
- (c) Is $k(x)$ a one-to-one function? Explain.

20. Simplify the following:

(a) $\log_5\left(\frac{1}{25}\right) =$

(b) $\ln(e^{-3}) =$

(c) $\log\left(\frac{1}{10}\right) =$