The Doppler Effect

Case 1: Observer in motion  
Source and medium at rest

Observer in motion

If the observer were at rest, he would detect $f$ crests per second.

If the observer moves with speed $V_{obs}$, how many crests does he detect per second?

source emits $f$ "crests" per second.

Stationary source
There are \( \frac{V_{\text{obs}} \cdot 3 \text{ sec}}{\lambda} \) crests in this segment.

So, in 1 second the observer detects \( (f + \frac{V_{\text{obs}}}{\lambda}) \) crests. This is

\[
f' = f + \frac{V_{\text{obs}}}{\lambda}
\]

Now, let \( V \) the speed of the sound when the medium is at rest (no wind).

It follows that \( V = \lambda f \).

Replacing this last expression into the previous one we obtain:

\[
f' = f + \frac{V_{\text{obs}}}{V} f \quad \text{or} \quad f' = f \frac{V + V_{\text{obs}}}{V}
\]

\( V \) : speed of sound
Observer approaching the source

\[ f' = f \frac{\nu + |v_{\text{obs}}|}{\nu} \]

Observer moving away from the source

\[ f' = f \frac{\nu - |v_{\text{obs}}|}{\nu} \]
Case 2: Source in motion
Observer and medium at rest

**Moving source**

- Source emits one crest every $T$ seconds, $T = \frac{1}{f}$
- New wavelength $\lambda'$
- Velocity of the source $V_s$

**Stationary observer**

- Notice: wavelength $\lambda'$ will vary according to the speed of the source.

**V': velocity of the sound when the medium is at rest**

**Important:** Once a wavefront (crest) is emitted, it travels at speed $V$ toward the observer, independent of the motion of the source.

**Diagram:**
- Two observers: stationary observer and stationary observer
- Time $t = 0$
- Time $t = T$
- Velocity $V_s$
the distance between the causts A and B is

\[ \lambda' = VT' = v_s T = (V - v_s) T \]

but \( T = \frac{1}{f} \)

\( f \) being the frequency at which the causts are being emitted by the source

\[ \lambda' = \frac{V - v_s}{f} \]

\[ \lambda' = \frac{\lambda}{V} \]

thus, the observer receives 1 caust every \( \frac{\lambda}{V} \) seconds. That is:

\[ T' = \frac{\lambda'}{V} = \frac{V - v_s}{f V} \]

\[ f' = f \frac{V}{V - v_s} \]

\( V_s \) : speed of the source

\( V \) : speed of sound

\( V_s \) : velocity of the source

\( V \) : velocity of the sound

(when the medium is at rest)

Stationary observer
Summary: \( V \) (speed of sound) remains constant.

Moving source

\[ f' = f \frac{V}{V - |v_s|} \]

Frequency perceived by the observer

Source approaching the observer

Stationary observer

\[ f' = f \frac{V}{\sqrt{V^2 + |v_s|^2}} \]

Source moving away from the observer
**Exercise**

\[ f' = f \frac{V + V_{obs}}{V - V_s} \]

Using the previous results, show explicitly that \( V \): velocity of the sound when the medium (air) is at rest.

**Solution**

When the observer is at rest, he receives \( \frac{V}{\lambda} \) causts in 1 sec. \( \text{①} \)

When the observer moves with speed \( V_{obs} \), he receives an additional number of causts:

\[ \frac{V_{obs}}{\lambda} \] \( \text{②} \)
Adding (1) and (2)

\[ f' = \frac{V}{\lambda} + \frac{V_{\text{obs}}}{\lambda} \]

\[ = \frac{V + V_{\text{obs}}}{\lambda} = \frac{V + V_{\text{obs}}}{V - V_s} \]

**Hint**

\[
\begin{pmatrix}
  f'
  \\
  f
\end{pmatrix} = f
\]

numerator

\[ \text{observer} \]

source

denominator

observer

source
EXAMPLE: A rocket moves at a speed of 242 m/s directly towards a stationary pole (through stationary air) while emitting sound waves at frequency $f = 1250$ Hz.

a) What is the frequency $f'$ measured by a detector that is attached to the pole?

b) Some of the sound reaching the pole reflects back to the rocket as an echo. What frequency $f''$ does a detector on the rocket detects for the echo?

Case: Source moving and approaching a stationary detector

$$f' = f \frac{343 \text{ m/s}}{343 \text{ m/s} + |V_R|}$$

Case: Source stationary and detector approaching the source

$$f'' = f' \frac{343 \text{ m/s} + |V_R|}{343 \text{ m/s}}$$
\[ f'' = f \frac{343 \text{ m/s} + |V_R|}{343 \text{ m/s} - |V_R|} \]

**Example**

- Police car
- Source: stationary
- Observer
- Source: stationary

\[ f' = f \frac{343 \text{ m/s} + |V_R|}{343 \text{ m/s}} \]

\[ f'' = f' \frac{343 \text{ m/s}}{343 \text{ m/s} - |V_R|} \]

So, \[ f'' = f \frac{343 \text{ m/s} + |V_R|}{343 \text{ m/s} - |V_R|} \]
Another way to find the signal frequency detected by the police car.

\[
\begin{align*}
\text{At } t = 0 & \\
\text{Signal travels with speed } c & \\
< t_1 + v & \\
L_0 & \\
t_0 & = \frac{L_0}{c + v} \\
\end{align*}
\]

\[2t_1 = \frac{2L_0}{c + v}\] time taken by the beam to leave the source and reflect back to the source (the police).

\[
\begin{align*}
\text{At } t = T & \\
\text{Second beam} & \\
2 \frac{L_0 - VT}{c + v} & \end{align*}
\]

time taken by the second beam to leave from the time it leave the source and return back to the source.
Reception receives a pulse at $t_1' = \frac{2l_0}{c+v}$

Reception receives a second pulse at $t_2' = T + 2\frac{l_0 - vT}{c+v}$

$$t_2' - t_1' = R\left(\frac{vT}{c+v}\right) + T$$

$$= T\left(1 - \frac{2v}{c+v}\right)$$

$$t_2' - t_1' = T\frac{c+v - 2v}{c+v} = T\frac{c - v}{c+v}$$

So, $T' = T\frac{c - v}{c+v}$

$$\Rightarrow \frac{1}{f'} = \frac{1}{f}\frac{c-v}{c+v}$$
Doppler Effect and Relativity

observer moving

\[ f' = f \frac{V + u}{V} = f \left(1 + \frac{u}{V}\right) \]  \tag{1}

source moving

\[ f' = f \frac{V}{V - u} = f \frac{1}{1 - \frac{u}{V}} \]  \tag{2}

For the particular case that \( \frac{u}{V} \ll 1 \) we obtain

\[ f' = f \left(1 + \frac{u}{V} + \left(\frac{u}{V}\right)^2 + \ldots \right) \]  \tag{2'}
where we have used the "Binomial theorem"
\[(1+x)^n = 1 + nx + \frac{1}{2}n(n-1)x^2 + \ldots\]
with \(n = -1\) and \(x = \frac{\mu}{\nu}\)

Notice:
- There is not symmetry between (1) and (2)
  It means, the Doppler frequency depends on whether the observer is moving, or whether it is the source that is in motion.
- Symmetry in the expressions for the Doppler frequency occurs only when \(\mu\) (the relative velocity between the observer and the source) is very small compared to the speed of the wave
We cannot use the previous expressions for light signals. This has to do with Einstein's second postulate of Special Relativity.

Both observers A and B see the light beam travel with speed c (no matter what the value of \(\mu\)).