1. A solid brass ball of mass 0.28g will roll smoothly along a loop-the-loop track when released from rest along the straight section. The circular loop has a radius $R=14.0$ cm, and the ball has radius $r<<R$.

1A Use the graph below to show ALL the forces acting on the small solid brass ball when it passes by the points P and Q, respectively.

1B What is the value of $h$ if the ball is on the verge of leaving the track when it reaches the top of the loop (give your answer in cm)?

   - a) 38
   - b) 28
   - c) 45
   - d) 59
   - e) NA

2 A block of mass $M= 1$ Kg, at rest on a horizontal frictionless table, is attached to a rigid support by a spring of constant $k=5$ N/cm. Another block of mass $m=250$ grams is fired into the 1 Kg block with a speed of 10 m/s and becomes attached to the block.

2A The speed of the block immediately after the collision is equal to (in m/s).

   - a) 0.1
   - b) 5
   - c) 0.25
   - d) 2
   - e) NA

2B The amplitude of the resulting simple harmonic motion (in meters).

   - a) 0.1
   - b) 0.5
   - c) 0.25
   - d) 0.2
   - e) NA
3. A horizontal force $F$ is applied at the axle of a wheel of radius $R=50$ cm and mass $M=1.5$ Kg, in order to raise it over an obstacle of height $h=20$ cm.

3A In the space provided above, draw the free body diagram for the wheel, indicating all the individual forces acting on the wheel, label them with the proper name (consider the wheel has already been lifted a bit).

3B The minimum value for the magnitude of the force $F$ necessary to raise the wheel over the obstacle is (in Newtons):

a) 14.7  

b) 19.6  

c) 25.6  

d) 4.9  

e) NA

4A A ladder of length $L=2$ m and weighing 40 N rests against a vertical wall, making an angle of $60^\circ$ with the floor. Draw all the vector forces acting on the ladder and find the normal reaction force $F_3$ acting on the ladder. The ladder is provided with rollers at A so that friction with the vertical wall is negligible.

a) $F_3= 40.2$ N  

b) $F_3 = 32.5$ N  

c) $F_3= 320$ N  

d) 11.6 N  

e) NA

4B A rectangular bar of uniform mass density, shown in the figure, is resting in EQUILIBRIUM. Draw the vector forces acting on the bar at the points A and B, AND find their corresponding magnitudes. The weight of the bar is 40 N and its length is 12 meters.
5. Two masses (M= 0.25 Kg) are attached to three springs of different spring-constants (k₀, 2k₀, and k₀, respectively) as shown in the figure. The figure shows the system in equilibrium position.

5A. In the diagrams below, show the two possible modes of oscillation.

For the assumed initial elongations (compatible with the corresponding mode) draw the free-body-diagram (showing ALL the forces acting on each mass.)

**MODE-1**

Figure shows the total force on each mass

**MODE-2**

Figure shows the forces on the mass at the left only

5A. Calculate the corresponding frequencies of oscillation for each mode.
ANSWER:  \( \omega_1 = \sqrt{\frac{k_o}{M}} = 4 \text{ rad/s} \) \( \omega_2 = \sqrt{\frac{5k_o}{M}} = 8.9 \text{ rad/s} \)

6. When middle C on the piano (frequency 262 Hz) is struck, it loses half its energy after 4 seconds.
   6A What is the decay time constant \( \tau \) (in seconds)?
   a) 3.2 \hspace{1cm} b) 9.1 \hspace{1cm} c) 5.8 \hspace{1cm} d) 4 \hspace{1cm} e) NA

   6B What is the Q factor for this piano wire?
   a) 200 \hspace{1cm} b) 9500 \hspace{1cm} c) 720 \hspace{1cm} d) 4300 \hspace{1cm} e) NA

7. An object of mass 1.5 Kg on a spring of force constant 600 N/m loses 3% of its energy in each cycle.
   7A What is the Q factor for this system?
   a) 210 \hspace{1cm} b) 6507 \hspace{1cm} c) 572 \hspace{1cm} d) 31 \hspace{1cm} e) NA

   7B What is the approximate value of the period of the oscillations (in seconds)?
   a) 4.2 \hspace{1cm} b) 52 \hspace{1cm} c) 17 \hspace{1cm} d) 0.3 \hspace{1cm} e) NA

8. In the overhead view of the figure at the right side, five forces of the same magnitude act on a merry-go-around for the strange; it is a square, of side equal to 2 m, that can rotate about point "P" at mid-length along one of the edges. Rank the forces acting on the square according to the magnitude of the torque they create about point P, greatest first

   ![Diagram of forces](image)

   a) \( F_1, F_2, F_3, F_4 \) \hspace{1cm} b) \( F_2, F_1, F_4, F_3 \) \hspace{1cm} c) \( F_2, F_1, F_4, F_3 \)

   d) \( F_1, F_2, F_4, F_3 \) \hspace{1cm} e) NA
8B The flat body in the figure below is pivoted at "O'. Three forces act in the direction shown:
F_A = 10 N at point "A", 8.0 m from "O";
F_B = 16 N at point "B", 4.0 m from "O"; and
F_C = 20 N at point "C", 3.0 m from "O";

The magnitude of the net torque about O is? (in units of N.m)

a) 6  b) 134  c) 54  d) 70  e) NA

9 In a horizontal table a mass of 0.5 Kg is attached to a spring ( k = 50 N/m). The mass is stretched 5 cm from its equilibrium position and released with zero speed at t=0.

9A Indicate which of the following function(s) describes correctly the position of the mass.

\[ x_1(t) = 5cm \ COS(10s^{-1} t + \pi / 9 ) \]  ANSWER: Yes  No
\[ x_2(t) = 5cm \ COS(20\pi \ s^{-1} t ) \]  ANSWER: Yes  No.
\[ x_3(t) = 10cm \ COS(20\pi \ s^{-1} ) \]  ANSWER: Yes  No.

9B Indicate which of the following functions describes correctly the velocity of the mass.

\[ v_1(t) = 50 \frac{cm}{s} \ SIN(10s^{-1} t ) \]  ANSWER: Yes  No
\[ v_2(t) = 100\pi \frac{cm}{s} \ COS(20\pi \ s^{-1} t ) \]  ANSWER: Yes  No.

BONUS QUESTION (5 extra points if answered correctly)
A 20 cm diameter, 2.0 Kg disk is rotating at 200 rpm.
Another 20 cm diameter, 1.0 kg circular loop is dropped straight down onto the rotating disk. Friction causes the loop to accelerate until it is “riding” on the disk.
What is the angular velocity of the combined system?
1.0 kg

2.0 kg

Answer: 100 rpm

FORMULAS

\[ \omega = 2\pi f \quad \ln 2 = 0.69 \]
\[
\cos A + \cos B = 2 \cos \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right)
\]

Simple Pendulum: Period \( T = 2\pi \sqrt{\frac{L}{g}} \) Use \( g = 10 \text{ m/s}^2 \) in this exam

Physical Pendulum: Period \( T = 2\pi \sqrt{\frac{I}{mgh}} \) Use \( g = 10 \text{ m/s}^2 \) in this exam

Where \( h \) is the distance from the pivot to the center of mass

**MOMENTUM of INERTIA (or ROTATIONAL INERTIA):** \( I = \sum_i m_i r_i^2 \)

- Thin bar of length \( L \) \( I_{CM} = \frac{1}{12} ML^2 \) (Axis perpendicular to the bar)
- Disk of radius \( R \) \( I_{CM} = \frac{1}{2} MR^2 \) (Axis perpendicular to the plane of the disk)
- Sphere of radius \( R \): \( I_{CM} = \frac{2}{5} MR^2 \) (Axis passing through the CM)

**ROTATIONAL KINETIC ENERGY** \( K_{rot} = (1/2) I \omega^2 \)

\( I \) is the Momentum of Inertia

**LINEAR MOMENTUM** \( P = m \mathbf{v} \)

**TORQUE** \( \tau = r \times F \) (\( \times \) indicates vector product)

**UNIFORM CIRCULAR MOTION** \( a = \omega^2/R \) (centripetal acceleration)

**CENTER OF MASS** \( \bar{R}_{CM} = \frac{1}{M} \sum_i m_i \bar{x}_i \)

Damped simple harmonic oscillator
\[
\frac{d^2 x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \frac{k}{m} x = 0
\]

Solution: \( x(t) = Ae^{-\alpha t} \cos (\omega t + \phi) \)
where \[ \alpha = \frac{b}{2m} \quad \omega^2 = \omega_0^2 - \left(\frac{b}{2m}\right)^2 \]

and \[ \omega_0 = \sqrt{\frac{k}{m}} \] is the natural frequency of the oscillator (in rad/sec)

\[ \tau = \frac{m}{b} \]

**Quality factor:** \[ Q = \frac{m}{b}\omega_0 = \tau \omega_0 \]

\[ Q = \tau \frac{2\pi f_o}{T} = \frac{2\pi}{T} \] (where T is the period of the oscillations)

In terms of Q, the solution to the damped simple harmonic oscillator is:

\[ x(t) = A e^{-\frac{\omega_0 t}{2Q}} \cos(\omega t + \phi) \]

Energy loss per cycle:

\[ \frac{|\Delta E|_{cycle}}{E} = \frac{T}{\tau} = \frac{2\pi}{Q} \]