EXPERIMENT 4
INPUT and OUTPUT IMPEDANCE

I. PURPOSE
To familiarize with the concept of input and output impedance of transistors

II. THEORETICAL CONSIDERATIONS
   II.A The concept of circuit loading
   II.B The emitter-follower circuit
       II.B.1 Calculation of the effective (load) input impedance $Z_{in}$ in the emitter-follower circuit
       II.B.2 Calculation of the effective (source) output impedance $Z_{out}$ in the emitter-follower circuit
       II.B.3 Comparing the output and input voltages

II.A The concept of circuit loading
A voltage source has intrinsically an internal resistance ($r_{source}$). Attaching the source to a circuit whose load resistance $R_L$ is less than or even comparable to the internal resistance $r_{source}$ will reduce the output voltage $V_{ab}$ considerably; this is illustrated through expression (1) and the corresponding graph in Fig 1.

\[
V_{out} = V_{in} \cdot \frac{1}{1 + \frac{1}{r_{source}/R_{Load}}}
\]  

(1)

![Fig. 1](image_url)

*Fig. 1* “Circuit loading” refers to the undesirable reduction of the open-circuit voltage $V_{in}$ by the load.

Solution to avoid “loading” the circuit: Use $R_{Load} >> r_{source}$
(Rule of thumb: $R_{Load} > 10 r_{source}$)
Connecting circuits one after another

In electronic circuits, stages are connected one after another.

i) Sometimes it is OK to load the circuit, as far as we know how much the loaded is, and particularly if $Z_{in}$ is going to be constant.

ii) Of course, it is always better to have a “stiff source” ($Z_{out} \ll Z_{in}$), so that signal levels do not change when a load is connected.

iii) However, there are situations in which it is rather required to have $Z_{out} = Z_{in}$. That is the case in radiofrequency circuits to avoid signal reflections.

So, be aware to respond accordingly depending on the situation.

![Amplifier Diagram](image1)

**Fig. 2** Amplifiers are typically characterized by their effective output and input impedances. This is particularly important for analysis when cascading them one after another.

II.B The emitter follower circuit

The emitter-follower circuit (see Fig.3) will be used as a test example throughout this lab session. It is called an **emitter follower** because the output terminal (the emitter) follows the input (the base) except by a diode drop voltage:

$$V_E = V_B - 0.6V$$

![Emitter Follower Diagram](image2)

**Fig. 3** Emitter follower circuit.

At first glance this circuit may appear useless, until one realizes (as we will see later) that it has input impedance much larger than the output impedance. This means (see Fig. 4 below),

- The circuit requires less power from the signal source (of voltage $V_{in}$ and some internal
impedance $r_{source}$) to drive a given load $R_E$ than would be the case if the signal source were to drive the load directly.” The effective impedance of the source, $Z_{out}$ decreases.”

Or equivalently

- A signal ($V_{in}$) of some internal impedance $r_{source}$ can drive a load of comparable or even lower impedance ($R_E$) without loss of amplitude (that is, without the detrimental effects of the voltage divider effects.) The effective load impedance, $Z_{in}$ increases”.

The objective of this lab is to clarify/understand the two statements made above.

![Fig.4 Comparison between circuits that present different effective load to the signal source $V_{in}$ that has some internal resistance $r_{source}$. The difference is made by the presence of the transistor (circuit at the left). In this laboratory session, we show that the circuit on the left tolerates higher values of $R_E$ (without loading the circuit) than the circuit on the right.](image)

**II.B.1 Calculation of the effective (load) input impedance $Z_{in}$ in the emitter-follower circuit**

In the circuit shown in Fig.5 below, we are looking for an expression for $Z_{in} = \Delta v_B / \Delta i_B$ in terms of load impedance $R_E$.

First, an expression for $\Delta i_B$ is obtained from the charge conservation, $I_E = I_B + I_C$. For the case of ac-input signals this implies,

$$\Delta i_E = \Delta i_B + \Delta i_C$$

In experiment #3 we found, $I_C = \beta I_B$, or $\Delta i_C = \beta \Delta i_B$; hence

$$\Delta i_E = (\beta +1) \Delta i_B$$

or,

$$\Delta i_B = \Delta i_E / (\beta +1)$$

(3)
Next, we want an expression for $\Delta v_B$ that relates to $\Delta i_E$ and the load impedance $R_E$.

For the case of ac-signals, expression (2) gives $\Delta v_E = \Delta v_B$

$$\Delta v_E = R_E \Delta i_E$$

$$\Delta v_B = R_E \Delta i_E$$

Equating (3) and (4) one obtains,

$$Z_{in} = \frac{\Delta v_B}{\Delta i_B} = (\beta + 1)R_E$$

Since $\beta$ is typically of the order of 100, then $Z_{in}$ is $\sim 100$ times greater than $R_E$. For $R_E = 0.5 \, \text{kΩ}$, $Z_{in} \sim 50 \, \text{kΩ}$

[Notice the mathematical derivation above is independent of the particular accessory to the circuit (see Fig. 5) that may be used to experimentally measure $\Delta i_B$ and $\Delta v_B$. It only requires that the transistor is properly biased (i.e. working in the active region, as to justify the use of $I_C = \beta I_B$.)]

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**Fig. 5** Evaluation of the effective (load) input impedance $Z_{in}$ of the emitter-follower circuit.

**Fig. 6** HIGHER (LOAD) INPUT IMPEDANCE. The presence of the transistor has the net effect to increasing the (load) impedance $R_E$ by a factor of $\beta + 1$. “The emitter-follower has high input impedance”.
II.B.2 Calculation of the effective (source) output impedance \( Z_{out} \) in the emitter-follower circuit

In the circuit shown in Fig.7 below, we are looking for an expression for 
\[ Z_{out} = \frac{(v_{in} - \Delta v_B)}{\Delta i_E} \] in terms of source impedance \( r_{source} \).

**Fig.7** Evaluation of the effective (source) output impedance \( Z_{out} \) of the emitter-follower circuit.

In the circuit on the left side of Fig.7, applying expression (2) for the case of ac-voltages gives 
\[ \Delta v_E = \Delta v_B . \] Accordingly,
\[ v_{in} = r_{source} \Delta i_B + R_E \Delta i_E \]
Using \( \Delta i_E = (\beta+1)\Delta i_B \)
\[ v_{in} = r_{source} \Delta i_E / (\beta+1) + R_E \Delta i_E \]
\[ v_{in} = ( r_{source} / (\beta+1) + R_E ) \Delta i_E \] \hspace{1cm} (6)

In the circuit on the right side of Fig.7,
\[ v_{in} = (Z_{out} + R_E) \Delta i_E \] \hspace{1cm} (7)

From (6) and (7),
\[ Z_{out} = r_{source} / (\beta+1) \] \hspace{1cm} (8)

**Fig.8** LOWER (SOURCE) OUTPUT IMPEDANCE. The presence of the transistor has the net effect to reducing the (source) output impedance \( r_{source} \) by a factor of \((\beta+1)\). “The emitter-
followed lowers the (source) output impedance”.

II.B.3 Comparing the output and input voltages

We have obtained in two separate sections, II.B.1 and II.B.2, explicit expressions (5) and (8) for the corresponding values of $Z_{out}$ and $Z_{in}$ of a emitter-follower circuit. Both expressions can in fact be obtained from a single more compact expression that compares the input and output voltages, as shown below.

![Emitter-follower circuit](image_url)

**Fig.9** Emitter-follower circuit.

Notice in Fig.9,

$$v_{in} - r_{source} \Delta i_B - \Delta v_E = 0$$

Since $\Delta v_E = v_{out}$,

$$v_{in} - v_{out} = r_{source} \Delta i_B$$  \hspace{1cm} (9)

To make $R_E$ intervene in expression (9), we use

$$v_{out} = R_E \Delta i_E$$

Since $\Delta i_E = (\beta +1) \Delta i_B$

$$v_{out} = R_E (\beta +1) \Delta i_B$$

$$\Delta i_B = v_{out} / R_E (\beta +1)$$  \hspace{1cm} (10)

Replacing (10) in (9),

$$v_{in} - v_{out} = r_{source} v_{out} / R_E (\beta +1)$$

$$v_{in} = v_{out} (1 + \frac{r_{source}}{(\beta +1)R_E})$$

$$v_{out} = \frac{1}{1 + \frac{r_{source}}{R_E} \frac{1}{(\beta +1)}} v_{in}$$  \hspace{1cm} (For circuit in Fig.9 that, uses a transistor)  \hspace{1cm} (11)
Compare this last expression for $\Delta v_{out}$ with the case in which the transistor circuit were not used (Fig.1):

$$v_{out} = v_{in} \frac{1}{1 + \frac{r_{source}}{R_E}}$$

*(For circuit in Fig.1, no transistor used)* \hfill (12)

Notice in (11), one obtains the same result whether

- Considering an effective (source) output impedance $r_{source} / (\beta+1)$ and load impedance $R_E$, or
- Considering a (source) output impedance $r_{source}$ and an effective input impedance $R_E(\beta+1)$.

### III. EXPERIMENTAL CONSIDERATIONS

#### III.1 Making stiffer sources: Emitter follower

- **III.1A Input impedance of the emitter-follower circuit**
- **III.1B Output impedance of the emitter-follower circuit**

#### III.2 Matching impedance: measuring the 50 Ω output impedance.

### III.1 Making stiffer (i.e. low output impedance) sources: Emitter follower

**TASK.** Build a stiff emitter-follower

![Emitter follower circuit diagram](image)

*Fig.10 Emitter follower. Notice, we are interested in using ac-voltages at the input*

Notice that the stiffer emitter-follower does not invalidate the calculation performed above with the simpler emitter-follower circuit. The support argument is shown in Fig.11. The voltage divider is a DC bias that provides a DC base current that bias properly the transistor to work in the active region. The connection of another AC-source produces an additional AC base current. (See figures 13 and 14 in the hand outs for Experiment #3.)
Fig. 11 Left: Stiff emitter-follower circuit. Right: Equivalent circuit, which helps to differentiate the additional (AC) base current injected by the signal generator from the DC base current established by the bias circuit.

Method of analysis
Underlying our method of analysis is to consider the emitter follower as a black box, which to the effect of measuring its effective load it will be considering having input impedance $Z_{in}$, and to the effect of driving a subsequent circuit stage it will be considered having output impedance $Z_{out}$. Our objective in this lab is to calculate and measure these two impedances (of the same emitter follower circuit.) See figures 5 and 7 above.

III.1A Input impedance of the emitter-follower circuit

Experimental procedure
- First, ensure the transistor in the emitter followers is working in the active region.
- Establish the range of frequency at which the circuit works as an emitter follower.
- Measure the input impedance of the circuit

The diagram on the left in Fig. 12 shows in a very straightforward manner that the input impedance can be determined experimentally by measuring the base-voltage $\Delta v_B$ and the input base-current $\Delta i_B$. We want to measure $\Delta v_B / \Delta i_B$.

The diagrams on the right in Fig. 12 show two optional ways of implementing the measurement. For $\Delta v_{in}$ use an ac-voltage of amplitude $\sim 20$ mV. (Notice, in the first option an additional “small” resistance $R_x$ has been introduced for the purpose of measuring $\Delta i_B$.)
Fig. 12 **Left:** Black box diagram representing the emitter-follower. Measuring $\Delta v_B / \Delta i_B$ gives the value of $Z_{in}$. **Right:** Optional experimental implementations for measuring $\Delta v_B$ and $\Delta i_B$. In the first option, the measurement of $\Delta i_B$ is implemented by using a small resistance $R_x$ and setting the oscilloscope inputs to ac-mode; invert channel 2 and measure $v_1 - v_2$ in order to obtain the input current $\Delta i_B$. In the second option, an ammeter is inserted to measure directly $\Delta i_B$. Verify also that $\Delta v_{out}$ follows $\Delta v_B$ (i.e. ensure the transistor is working in the active region.)

**TASKS:** Verify also that $\Delta v_{out}$ follows $\Delta v_B$ (i.e. ensure the transistor is working in the active region.)

Verify if the predicted value given in (5) is close to the experimental value you measured in the section above.
III.1B Output impedance of the emitter-follower circuit

Similarly to the procedure for finding the input impedance, we will consider the emitter follower circuit (Fig. 14, left diagram) as a black box (right diagram) for the purpose of finding its equivalent output impedance. A variable resistance $R_{LOAD}$ will do the trick.

Experimental procedure

In this setup, the output voltage across $R_{LOAD}$ is given by,

$$\Delta v_{out} = \frac{R_{LOAD}}{Z_{out} + R_{LOAD}} \Delta v_{equiv}$$

By choosing two different values for $R_{LOAD}, R_1$ and $R_2$, one obtains,
Fig. 15 The equivalent follower emitter circuit hooked to two different output loads.

\[
\Delta v_{\text{out};1} = \frac{R_1}{Z_{\text{out}} + R_1} \Delta v_{\text{equiv}} \quad \text{and} \quad \Delta v_{\text{out};2} = \frac{R_2}{Z_{\text{out}} + R_2} \Delta v_{\text{equiv}}
\]

Solving for \(Z_{\text{out}}\) (see proof at the end of these notes),

\[
Z_{\text{out}} = R_2 \left[ \frac{\Delta v_{\text{out};1}}{\Delta v_{\text{out};2}} - 1 \right] \left( \frac{1}{1 - \frac{R_2}{R_1} \frac{\Delta v_{\text{out};1}}{\Delta v_{\text{out};2}}} \right)
\]

9(14)

- As we expect \(Z_{\text{out}}\) to be very low (\(\leq 100 \, \Omega\)) compared to the input impedance, you may use \(R_1\) and \(R_2\) to be also both low.

- Alternatively, in Fig. 15, \(R_1\) can be the input impedance of the oscilloscope (that is, you just hook the output of the emitter-follower to the oscilloscope. Take \(R_1\) equal to infinity. Then use a low value resistance \(R_2\).
  
  i) Connect the output to the oscilloscope (assumed here that the impedance of the oscilloscope is infinite; \(R_{\text{LOAD}} = R_1 = \infty\)). This allows measuring the amplitude of \(\Delta v_{\text{equiv}}\).

\[
\Delta v_{\text{out};1} = \frac{R_{\text{LOAD}}}{Z_{\text{out}} + R_{\text{LOAD}}} \Delta v_{\text{equiv}} = \frac{R_1}{Z_{\text{out}} + R_1} \Delta v_{\text{equiv}} \quad \overset{R_1 \to \infty}{\Rightarrow} \quad \Delta v_{\text{equiv}}
\]

That is,

\[
\Delta v_{\text{out};1} = \Delta v_{\text{equiv}} \quad (\text{measured by simply connecting the output of the transistor to the oscilloscope}.)
\]

ii) Using an arbitrary external resistance \(R_2\) (typically low values work better) as \(R_{\text{LOAD}}\) measure the corresponding \(\Delta v_{\text{out};2}\).

Then solve for \(Z_{\text{out}}\) using expression (13).

\[
\Delta v_{\text{out};2} = \frac{R_2}{Z_{\text{out}} + R_2} \Delta v_{\text{equiv}}
\]
\[
(Z_{\text{out}} + R_i) = \frac{R_2 \Delta v_{\text{equiv}}}{\Delta v_{\text{out,2}}} \\
Z_{\text{out}} = R_2 \left[ \frac{\Delta v_{\text{equiv}}}{\Delta v_{\text{out,2}}} - 1 \right] = R_2 \left[ \frac{\Delta v_{\text{out,1}}}{\Delta v_{\text{out,2}}} - 1 \right]
\]

- Be aware of the role played by the capacitor impedance. Choose the proper frequency such that \( a \) the capacitor impedance is minimum, but at the same time \( b \) the used frequency is within the frequency-bandwidth response of the transistor.

### III.2. Matching Impedance

**50 Ω Output Impedance**

This is a real resistor, physical placed at the output (as literally drawn in the figure.) A high quality resistor, with the lowest reactance possible, is preferred.

Alternatively, you can think this is the Thevenin’s equivalent circuit.

**50 Ω Impedance Cable**

This is a 50 Ω complex impedance cable

**50 Ω Input Impedance**

MEASUREMENTS

Some signal generators are specified to have 50 Ω output impedance AND expect to be connected to 50Ω input impedance devices. ONLY if the latter requirement is satisfied, the reading of the output voltage will coincide with the actual output voltage.
The input and output impedance create a voltage divider, hence the voltage readings (2 V_{pp} and 1 V_{pp}) shown in the figure.

Notice in the case above that if, for example the output voltage were 1 V_{pp} (1 volt peak-to-peak) the voltage at A would be 2 V_{pp}.

**HOW TO TEST IF AN EQUIPMENT HAS A 50Ω OUTPUT IMPEDANCE OR NOT?**

Task: Verify whether or not your signal generator has a 50Ω output impedance.

i)

ii)

Now, connect a 50 Ω resistor as shown in the figure below. If the voltage drops to 50%, then the output impedance is 50Ω.
Appendix

Proof of expression (14):

\[ \Delta v_{out} = \frac{R_{LOAD}}{Z_{out} + R_{LOAD}} \Delta v_{equiv} \]

When using \( R_{LOAD} = R_1 \) and \( R_{LOAD} = R_2 \) respectively, one obtains,

\[ \Delta v_{out;1} = \frac{R_1}{Z_{out} + R_1} \Delta v_{equiv} \quad \text{and} \quad \Delta v_{out;2} = \frac{R_2}{Z_{out} + R_2} \Delta v_{equiv} \]

\[ \frac{\Delta v_{out;1}}{\Delta v_{out;2}} = \frac{R_1 (Z_{out} + R_2)}{R_2 (Z_{out} + R_1)} \]

\[ (Z_{out} + R_1) \frac{R_2 \Delta v_{out;1}}{R_1 \Delta v_{out;2}} = (Z_{out} + R_2) \]

\[ Z_{out} \left[ \frac{R_2 \Delta v_{out;1}}{R_1 \Delta v_{out;2}} - 1 \right] + R_1 \frac{R_2 \Delta v_{out;1}}{R_1 \Delta v_{out;2}} = R_2 \]

\[ Z_{out} \left[ \frac{R_2 \Delta v_{out;1}}{R_1 \Delta v_{out;2}} - 1 \right] = R_2 \left[ 1 - \frac{\Delta v_{out;1}}{\Delta v_{out;2}} \right] \]

\[ Z_{out} = R_2 \left[ \frac{1}{1 - \frac{\Delta v_{out;1}}{\Delta v_{out;2}}} \right] \]

\[ \frac{R_2 \Delta v_{out;1}}{R_1 \Delta v_{out;2}} - 1 \]