EXPERIMENT 6
LOW-PASS FILTERS, 3dB BREAKPOINT and COMPARATORS

I. PURPOSE:
This laboratory session pursues two main objectives. First, to build a low-pass RC filter and measure the output voltage (magnitude and phase) as a function of its frequency. A log-log plot (Bode plot) of the output signal vs frequency will help familiarize with the concepts of “3dB breaking point” and “decrease of output levels per octave and per decade.” Second, familiarize with the functioning of the operational amplifiers, and its application in comparator circuits. Comparators that uses no feedback and a Schmitt Trigger comparator (that uses positive feedback) will be built.

II. THEORETICAL CONSIDERATIONS

II.1 The scale of decibels
Comparison of signal amplitudes
Comparison of Power levels

II.2 Single-stage low–pass filter
Output voltage (magnitude and phase)

II.3 Bode-plots and the 3dB breakpoint

II.4 Voltage drop at high frequencies (in decibels) in single low-pass filter
II.4A Roll off per octave
II.4B Roll off per decade

II.5 Dual-stage low–pass filter

II.1. The scale of decibels
Comparison of signal amplitudes
Let’s consider two signals of amplitudes $A_1$ and $A_2$ respectively.

The ratio of these two signals is: $\frac{A_1}{A_2}$

Because we often we deal with ratios that change by many order of magnitude during a testing procedure, the decibel scale is frequently used. By definition,

$$\left( \frac{A_1}{A_2} \right)_{dB} = 20 \log_{10} \frac{A_1}{A_2}$$  \hspace{1cm} (1)

\[
\begin{align*}
\log_{10} 1 &= 0, \quad &\log_{10} 2 &= 0.3010, \quad &\log_{10} 3 &= 0.477, \\
\log_{10} 5 &= 0.6989, \quad &\log_{10} A^n &= n \log_{10} A, \quad &\log_{10} AB &= \log_{10} A + \log_{10} B
\end{align*}
\]
Example: \( \frac{A_1}{A_2} = 2 \) is equivalent to +6 dB

because \[ \left( \frac{A_1}{A_2} \right)_{dB} \equiv 20 \log_{10} 2 = 6 \]

We say, \( A_1 \) is +6 dB relative to \( A_2 \).

Example: A signal \( A_1 \) 10 times as large as \( A_2 \) is +20 dB

A signal \( B_1 \) one-tenth as large as \( B_2 \) is -20 dB

**Comparison of Power levels**

If \( P_1 \) and \( P_2 \) represent the power of two signal levels, their ratio in decibels is defined by,

\[
\left( \frac{P_1}{P_2} \right)_{dB} \equiv 10 \log_{10} \frac{P_1}{P_2}
\]

(2)

**II.2 Low-pass filter**

![Fig.1](image)

A low pass filter is obtained with a resistor and a capacitor connected in series while the output is taken across the capacitor.

**Output voltage (magnitude and phase)**

In Fig.1, assuming

a zero output impedance of the driving source, (which provides \( v_{in} \),) and

an infinite input impedance of the loading device. (to which \( v_{out} \) is connected,) we obtain,

\[
v_{out} = \frac{v_{in}}{R + Z_C} = \frac{v_{in}}{R + \frac{1}{j\omega C}} = \frac{1}{j\omega CR + 1} v_{in}
\]

\[
v_{out} = \frac{1}{\sqrt{(\omega RC)^2 + 1}} e^{j\varphi} v_{in} \quad \text{where} \quad \varphi = \tan^{-1}(-\omega RC)
\]

(3)

That is, the output voltage \( v_{out} \) lags the input voltage \( v_{in} \).

The ratio of the input and output voltage magnitude is given by,

\[
\left| \frac{v_{out}}{v_{in}} \right| = \frac{1}{\sqrt{(\omega RC)^2 + 1}}
\]

(4)
II.3 Bode-plots and the 3dB breakpoint

RC is the characteristic time response of the low-pass RC circuit.

At $\omega = 1/RC$:

- The ratio $|v_{out}/v_{in}|$ drops to $1/\sqrt{2} = 0.7$
  
  In the decibels scale, this ratio is equal to $20 \log_{10}(0.7) = -3$;
  
  that is $|v_{out}/v_{in}|_{\text{dB}} = -3$ dB

- The change in phase is $45^\circ$.

In general

*The frequency at which the output voltage drops by -3 dB is referred to as the "-3 dB breakpoint" of a filter (or of any circuit that behaves as a filter).*

![Bode plot](image)

**Fig. 2** Left: Frequency response of a low pass filter. Right: The right figure displays the same data but in a logarithmic scale.

II.4 Voltage drop at high frequencies (in decibels)

Let’s express the ratio of voltages in decibels. From expression (4) we obtain,

$$
|v_{out}/v_{in}|_{\text{dB}} = 20 \log_{10} \frac{|v_{out}|}{|v_{in}|} = 20 \log_{10} \frac{1}{\sqrt{\omega RC}^2 + 1} = -10 \log_{10} (\omega RC)^2 + 1
$$

At high frequencies, $\omega \gg 1/RC$,

$$
|v_{out}/v_{in}|_{\text{dB}} = 20 \log_{10} \frac{|v_{out}|}{|v_{in}|} \xrightarrow{\text{large } \omega} -20 \log_{10} (\omega RC)
$$

That is,

$$
|v_{out}/v_{in}|_{\text{dB}} \xrightarrow{\text{for large } \omega} -20 \log_{10} (\omega) - 20 \log_{10} (RC)
$$

(5)
We expect, then, that at large frequencies (i.e. for $\omega \gg 1/RC$), a plot of $\frac{\mathcal{V}_{\text{out}}}{\mathcal{V}_{\text{in}}}_{\text{dB}}$ vs $\log_{10}(\omega)$ should linearly decrease with a slope of 20. This is in fact verified in the figure below.

**Low pass filter**

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{1 + (\omega RC)^2}^{1/2}$$

$$\phi = \tan^{-1}(-\omega RC)$$

**Fig. 3** Linear decrease of the output signal (in decibels) at high frequencies.

### II.4A Roll-off per octave

We are familiar with the fact that in music, in **one octave** the **frequency is doubled**. (6)

Similarly in our case of electrical signal changes (as described above,) we can ask ourselves how much does the voltage ratio in expression (5) changes (in decibels) per octave?

**Answer:**

In one octave (a range where the frequency doubles) the horizontal axis in Fig. 4 changes by

$$[\log_{10}(2\omega) - \log_{10}(\omega)] = \log_{10}(2) = 0.3010.$$ .

Since the slope is -20, a change in the horizontal axis by 0.30 will give a vertical change equal to $-20 \times 0.30 = -6$ dB

Accordingly, we say:

**In a simple RC low-pass filter the output voltage drops -6 dB per octave** (7)

**Fig. 4** Roll-off pr octave. **Left:** Linear drop of the output/input voltage ratio (in decibels) at high frequencies in a logarithmic scale. **Right:** Same data but in a zoomed-in scale.
Equivalently,

\[ \text{in a simple } \text{RC low-pass filter the output voltage decreases by a factor of 2 per octave.} \] \hfill (8)

(This follows from the fact that for \((A_i/A_0) = 1/2\) one has \((A_i/A_0)_{dB} = 20 \log_{10}(1/2) = -6 \text{ dB}.\)

**II.4B Decibels per decade**

In a decade, the frequency changes by a factor of 10.

How much does the voltage ratio in expression (5) changes (in decibels) per decade?

Answer:

In one decade (a range where the frequency changes by a factor of 10) the horizontal axis in Fig. 5 changes by

\[
10 \log_{10} \left( \text{log}_{10}(10) - \log_{10}(\omega) \right) = \log_{10}(10) = 1.
\]

Since the slope is -20, a change of 1 in the horizontal axis will correspond to a vertical axis equal to -20 x 1 = -20 dB

Accordingly, we say:

\[ \text{In a RC low-pass filter the output voltage drops -20 dB per decade} \] \hfill (9)

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**Fig. 5** Response from a single-stage low-pass filter
Equivalently,

*in a simple $RC$ low-pass filter the output voltage decreases by a factor of 10 per decade.*

\[(A_1 / A_2) = 1/10 \quad \text{one has} \quad (A_1 / A_2)_{dB} = 20 \log_{10}(1/10) = -20 \text{ dB.}\]

II.5 Dual-stage low–pass filter

**TASK:** Derive an expression for $v_{out}$ in terms of $v_{in}$.

![Two-stage low-pass filter](image)

**III. EXPERIMENTAL CONSIDERATIONS**

**III.1 RC FILTER**

**III.1A Single-stage filter**

Implement an RC Low-Pass filter. (You may use $C = 0.22 \mu F$ and $R = 1 K\Omega$; of course, feel free to try other values.) Use a signal generator to provide a sinusoidal input signal $v_{in}$ of ~ 400 mV amplitude, and check with the oscilloscope whether the output voltage leads or lags the input voltage. Suggestion: Monitor $v_{in}$ and $v_{out}$ in the oscilloscope’s channels 1 and 2, respectively.

- Make a Bode plot of the magnitude and phase of $v_{out} / v_{in}$ as a function of the angular frequency. Make sure to get data over several decades of frequency values. To measure the phase, use the same technique used in Experiment-1.
- Determine experimentally the frequency at which the 3dB breaking point occurs. Compare your experimental results with the expected theoretical value.
- Determine experimentally the change of the output signal in dB per octave and per decade.

**III.1B Two-stage low-pass filter.**
Implement two RC Low-Pass filters in cascade. Repeat the same measurement requested in part III.1A.

\[
\begin{align*}
\mathcal{V}_{\text{in}} & \quad R \quad C \quad R \quad \mathcal{V}_{\text{out}} \\
\end{align*}
\]

Fig. 8 Two low-pass RC filters.

- Make a Bode plot of the magnitude and phase of \( \mathcal{V}_{\text{out}} / \mathcal{V}_{\text{in}} \) as a function of the angular frequency. Make sure to get data over several decades of frequency values.
  To measure the phase, use the same technique used in Experiment-1.
- Determine experimentally the frequency at which the 3dB breaking point occurs. Compare your experimental results with the expected theoretical value.
- Determine experimentally the change of the output signal in dB per octave and per decade.

III.2 OPERATIONAL AMPLIFIERS and COMPARATORS

III.2A Comparator (with no feedback)

III.2B Comparator with positive feedback: Schmitt trigger

Op amps are widely used to amplify dc or ac signals. In this lab session, we will use LM358AP whose terminal connections are shown in Fig. 9.

\[
\mathcal{V}_{\text{out}} = A \left( \mathcal{V}_{\text{in}(+)} - \mathcal{V}_{\text{in}(-)} \right)
\]

\[
\begin{align*}
+\mathcal{V}_{\text{CC}} & \quad \text{Positive supply} \\
-\mathcal{V}_{\text{CC}} & \quad \text{Negative supply} \\
\end{align*}
\]

Fig. 9 Left: Op-amp terminals. Center: Pin assignment for the dual op-amp LM358AP. Use \( \mathcal{V}_{\text{CC}} = 12 \text{V} \). Right: Input/output volatge characteristics.
APPLICATIONS: COMPARATORS and SCHMITT TRIGGER

Many applications require knowing which of two signals is larger, upon which a terminal may need to be activated (or deactivated). In a digital voltmeter, for example, in order to convert an analog signal to a digital number, the unknown voltage is applied to one input of a comparator, while a linear ramp is applied to the other input. A digital counter counts the cycles of a clock while the ramp is less than the unknown voltage and display the result when both signals are equal. The resultant count is proportional to the unknown voltage. In this session we will build first a simpler comparator and then a more reliable Schmitt Trigger.

III.2A Comparator (with no feedback)

The circuit shown in the figure compares two voltages ($V_{in}$ and $V_+$) and amplifies the difference between them. The resistors $R_1$ and $R_2$ establish the switching point at which the output will change state ($V_{CC}$ to $-V_{CC}$, or vice versa.)

Set up such a circuit using resistors in the 1KΩ to 100KΩ range, determine the trigger point voltage, and examine the levels at the output and inputs. Sketch a plot of $V_{out}$ vs $V_{in}$.

How would you use that circuit to measure an unknown resistor?

Comparator with positive feedback: Schmitt trigger

The simple comparator presents disadvantage when the input voltage is noisy, causing the output to make several transitions as the input passes through the trigger point. This problem can be solved by using positive feedback.

The effect of $R_1$ and $R_2$ is to make the circuit have two thresholds, depending of the output state.

Set up such a circuit using resistors in the 1KΩ to 30KΩ range, determine the trigger point voltage, and examine the levels at the output and inputs. Sketch a plot of $V_{out}$ vs $V_{in}$.

Verify that the output depends both on the input voltage and on its recent history, an effect called hysteresis.
Fig. 11 Schmitt trigger circuit. Notice a fraction of the output voltage feedbacks to the positive input of the operational amplifier.