The Theory of The Rainbows

I) Abstract:
The purpose of the project is to help students apply their knowledge that they learn from optics class to real life. The Theory of The Rainbows is that I will research and study its information for my project. This project is very important to me because I have a chance to apply and understand more about the law of reflection and refraction. How does a rainbow form? What makes color in rainbows? The mathematics of rainbows, and types of rainbows are that I will present in this project.

II) Introduction:
Most people have ever seen rainbows. Rainbow is an arc of spectrum colors that appears opposite of the sun in a heavy falling rain. This topic is interesting to me because I have a chance to study about the rainbow that I have had many of questions since I was child. Fortunately, I study optics class and do this project that may answer my questions. Furthermore, I also want to share the information I got about the rainbow to class.

III) Content:
1) How does rainbow form?
Rainbows are created by sunlight shines through water droplets that are suspended in the air. When the sunlight hits the surface of raindrop, it is refracted because the light changes the medium index from air to water. When the light travels to the back of the raindrop most of the light is refracted and go out the drop. Only some is reflected and when it comes out the front surface of the drop, each color is refracted again as it leaves the raindrop.

A rainbow occurs when rain in one portion of the sky. Rainbow is like an image, we can not touch it because it is not physical, but we can see it because the sunlight travel through water droplets and is split into seven colors. Below is a picture that illustrates the sunlight travel through a raindrop to create a rainbow.
2) What makes colors in rainbows?
The sunlight consists many colors. However, when all of the colors are combined together we see as white light. When the sun comes out, sunlight strikes the raindrop. It is refracted, different wavelengths refract at different angles, and therefore the various colors are separated. The colors of primary rainbow are always followed the order: red, orange, yellow, green, blue, indigo, and violet. In order to understand the origin of colors, we need only recall that light of different wavelengths is refracted by slightly different amounts as it passes from one transparent medium to another. This dispersion effect is what causes a prism to produce the spectrum colors from white light. In the case of water droplet, violet light will be refracted through a slightly greater angle than red light. This will cause the violet band in the primary rainbow to be seen below the red band. The rests of the colors lie in between red and violet.

![Diagram showing how light enters a raindrop and refracts to form a rainbow.](image)

When it rains, there are a lot of raindrops falling in the sky. Each raindrop can create only one color that your eye can see. The location of each color reflected off the back of the drop and striking your eye is always at a fixed angle measured from a line between your eye and the sun. This angle is approximately 42° measured to the top of the red band and approximately 40° to the bottom of the violet band.

![Diagram showing how raindrops create a rainbow.](image)

In order to help us easy to see the location of the rainbow angle, observer, and rainbow, I tried to use “paint” to draw the below picture to illustrate to you. To see the rainbow, the sun must be behind your back and be facing the rain.
3) **The Mathematics of the rainbows**

As many people say that the rainbow angle is fixed at 42 degrees. How can they know about this? I think that in order to understand about this we need to study about mathematics of rainbow. The Snell’s law, and law of reflection law that I learned in the optics class are applied to mathematics of rainbows. In order to calculate the rainbow angle we not only use the Snell’s law, law of reflection but also use the law of sine, geometry and derivative.

In this project, I only mention to the primary rainbow and tell you how a rainbow angle can be calculated. In order to calculate rainbow angle we need to draw some more extra lines in the circle that we image as a raindrop, such as the height, the radius, expand radius dash lines. The angle that we need to find is the angle (e).

Assume that raindrop is like a sphere.

\[ 360 = a + (180 - 2b) + (180 - 2b) + f \]
Let h is the height of the point where the ray hits the raindrop, and r is radius of the raindrop.

a is angle of incidence
b is angle of transmitted

Draw a dash line that parallel to the ray of light. We have the angle between the horizontal dash line and radius (r) is also a.

- **For refraction:**
  Index of refraction for air is 1
  Using Snell’s law: \( n_1 \sin (a) = n_2 \sin (b) \)
  Because the index of air equal 1, we can neglect \( n_1 \)
  Calculate angle (a) we need to look at the right triangle that formed by the height (h), dash line, and the radius (r).
  We have: \( \sin (a) = \frac{h}{r} \). I also named \( \sin (a) = M = \frac{h}{r} \) in order to easy for calculation.

- **For reflection:**
  Look at the triangle that forms by the ray light and two radii is isosceles triangle. It means that the angle at the base is equal. They are called b
  We have angle of incident equals angle of reflection. Therefore, the angle that makes with radius is also b.
  Snell’s law once again
  The incident angle is b and the angle that is made by out coming ray and extend radius is also a
  
  \( n_1 \sin (b) = \sin (a) \)
  
  • Sum of all angles at the center of circles is 360 degree
  
  \( 360 = a + (180 – 2b) + (180 – 2b) + f \)
  
  \( f = 4b – a \)
  
  \( f = e + a \)
  
  \( e = f – a \)
  
  \( e = 4b – 2a \)
  
  \( \sin (a) = \frac{h}{r} = M \)
  
  \( a = (\frac{1}{\sin})M \)
  
  \( n_1 \sin (b) = \sin (a) \)
  
  \( b = (\frac{1}{\sin})(M/n) \)
  
  • Finally we have final formula
  
  \( e = 4 \times (\frac{1}{\sin}) (M) – 2 \times (\frac{1}{\sin})(M) \quad (1) \)
  
  • Take derivative of e with respect to (M) and find where it has an extreme by setting the derivative equal 0 and solve for (M) we obtain:
  
  \( M = ((4-n^2)/3)^{1/2} \quad (a) \)
  
  Substitute (a) into equation (1) above we have
  
  \( e = 4\times (\frac{1}{\sin})(((4-n^2)/3)^{1/2})/n- 2\times (\frac{1}{\sin})(((4-n^2)/3)^{1/2}) \quad (b) \)
  
  n water= 1.33
  
  Substitute n = 1.33 into (a) and (b) we have
  
  \( M = 0.8608 \)
  
  \( e = 42.0 \) degrees
  
  4) **Types of Rainbow**
  
  Nature rainbow is the rainbow that is created by nature. We can see this type of rainbow at the falling rain, waterfalls with the sunlight shine.
Manmade rainbows: are the rainbows created by human. You can make your own rainbows in many different ways such as using a garden hose, a sprinkler, and a waterspout out.

Stern, an artist and visual arts professor creates the picture above; uses fire trucks, boats, pumps, and hoses to create rainbows at Germany.

You can also use a flashlight, a round container, and a large piece of cardboard. Below is my experiment.

I use a flashlight, a round container, water, and a piece of cardboard to make a rainbow.

- Cut a hole on the cardboard. Filled the container with water. Use the flashlight to shine the light through a hole of a cardboard and through the container. The light travel through the glass and water is refracted and split the light into spectrum colors on the wall that I called rainbow. However, this rainbow is different with the rainbow we see on the sky because it is only refracted not reflected. Please look at the picture below.
If you want to create a rainbow that forms the same rainbows on the sky, you can use a pan that fills with water, a mirror, and a flashlight. Put a mirror in the pan of water. Use a flashlight to shine the light into the mirror. The light will be refracted by changing index from air to water, and is reflected by the mirror, and it is refracted again when it goes out the water. Finally, the rainbow will appear on the ceiling.

IV) Conclusion: After I studied and researched information about the theory of the rainbow, I learned how a rainbow forms, what makes colors in the rainbow, mathematics of rainbow. Moreover, I also learned how refraction and reflection laws are applied in real life. The project is very helpful to me. It helped me to answer many questions that I concerned about the rainbow before. I hope that in the future I will have time to study more about other topics that relate to optics.

V) References
Books:
- Swartz, Clifford E. Teaching introductory Physics
Articles:
- http://www.fidcal.com/rainbows/
- http://brantacan.co.uk/rainbow.html
- http://www.batesville.k12.in.us/Physics/PhyNet/Optics/Refraction/Rainbows_2.html
- http://www.unidata.ucar.edu/staff/blynds/rnbw.html