A narrow-band tunable diode laser system with grating feedback, and a saturated absorption spectrometer for Cs and Rb

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Detailed instructions for the construction and operation of a diode laser system with optical feedback are presented. This system uses feedback from a diffraction grating to provide a narrow-band continuously tunable source of light at red or near-IR wavelengths. These instructions include machine drawings for the parts to be constructed, electronic circuit diagrams, and prices and vendors of the items to be purchased. It is also explained how to align the system and how to use it to observe saturated absorption spectra of atomic cesium or rubidium.

I. INTRODUCTION

Tunable diode lasers are widely used in atomic physics. This is primarily because they are reliable sources of narrow-band (<1 MHz) light and are vastly less expensive than dye or Ti:sapphire lasers. However, the frequency tuning characteristics of the light from an “off the shelf” laser diode is far from ideal, and this greatly limits its utility. In particular, the laser output is typically some tens of MHz wide and can be continuously tuned only over certain limited regions. These characteristics can be greatly improved by the use of optical feedback to control the laser frequency. Reference 1 gives a lengthy technical review of the characteristics of laser diodes, the use of optical feedback techniques to control them, and various applications in atomic physics. An earlier review by Camparo2 also gives much useful information, primarily relating to free-running diode lasers. The use of a wavelength-dispersive external cavity for diode laser tuning and mode selection was described by Ludeke and Harris,3 and the spectral characteristics of external-cavity stabilized diode lasers were investigated in detail by Fleming and Mooradian.4

During the past several years our laboratory has carried out a large number of experiments in optical cooling and trapping, and general laser spectroscopy of cesium and rubidium using diode lasers. In the course of this work, we have developed a simple inexpensive design for a diode laser system that uses optical feedback from a diffraction grating. This system produces over 10 mW of light with a bandwidth of well under 1 MHz and can be easily tuned over atomic resonance lines. We now have over a dozen such laser systems operating, including two in an undergraduate teaching lab, and the design has reached a reasonable level of refinement. There are many other designs for optical feedback systems5 and we make no claims for this one being superior. However, it is a reasonable compromise between several factors which are relevant to many laboratories: (1) low cost (about $400 not including labor), (2) ease of construction (several of these systems have been built by novice undergraduates), and (3) reliability. These lasers have achieved several notable successes in experiments on cooling and trapping cesium atoms, and the design has been successfully duplicated in a number of other laboratories. We prepared this article in response to a large number of requests for detailed instructions on how to build and operate such a system. This article provides a detailed and fully comprehensive recipe for construction of the system and its use to observe saturated absorption spectra in a rubidium or cesium vapor cell. We refer the reader to Ref. 1 and the references therein for information about the physics of laser diodes and the factors that motivated this design as well as design alternatives.

In this paper, we have attempted to respond to three frequent requests for information we receive. The first is from the undergraduate wanting to do high-resolution laser spectroscopy for a project, without expert local supervision. The second is from the faculty member who wants to construct a teaching laboratory experiment and wants instructions that can be given to a technician or undergraduate with favorable results. The third is from the research scientist who wants to use diode lasers in an experiment and would like to benefit from the accumulated practical experience in another laboratory.

We will first discuss the construction or purchase of the basic components and then explain how to put them together, align the laser system, and tune the frequency. Finally, we discuss how to observe saturated absorption spectra and how to use these spectra to evaluate the laser performance or to actively stabilize the laser frequency by locking it to narrow saturated absorption features.
II. SYNOPSIS OF COMPONENTS

As shown in Fig. 1, the laser system has three basic components, a commercial diode laser, a collimating lens, and a diffraction grating. These components are mounted on a baseplate. The laser and lens are mounted so that the lens can be carefully positioned relative to the laser to insure proper collimation. The diffraction grating is mounted in a Littrow configuration so that the light diffracted into the first order returns to the laser. As such, the grating serves as one end “mirror” of a laser cavity, with the back facet of the diode providing the second mirror. This means the grating must be carefully aligned and very stable. To achieve this we mount the grating on a standard commercial mirror mount which is attached to the baseplate. As with any laser, changes in the length of the cavity cause shifts in the laser frequency. Therefore, to obtain a stable output frequency, undesired changes in the length due to mechanical movement or thermal expansion must be avoided. To reduce movements due to vibration of the cavity we mount it on small soft rubber cushions. To avoid thermal changes, the baseplate is temperature controlled using heaters and/or thermoelectric coolers. In addition to controlling the temperature of the baseplate, we independently control the temperature of the laser diode. Finally, to avoid air currents interfering with the temperature control we enclose the entire laser system in a small insulated metal box. Of course, to finely tune the laser frequency one must have some way to change the length of the cavity in a carefully controlled manner. We do this using a piezoelectric transducer speaker disk which moves the grating in response to an applied voltage.

The laser system also requires a small amount of electronics. A stable low-noise current source is needed to run the laser, and temperature control circuits are used to stabilize the diode and baseplate temperatures. This electronics is readily available commercially. However, for those with more time than money, we provide circuit diagrams for the relatively simple circuits that we normally use.

This system contains both purchased and “homemade” components. Before discussing the construction aspects, we will provide some information concerning the purchasing of the commercial components. We purchase the diode laser itself, the collimating lens, the fine adjustment screw which controls the lens focus, the diffraction grating, the mirror mount which holds the grating, and the piezoelectric disks. The purchase of most of these items is straightforward. Fine adjustment screws and mirror mounts are available as standard items from most companies that sell optics hardware. Similarly, laser diode collimating lenses and diffraction gratings are available from numerous companies. For the convenience of the reader we list in the Appendix the exact products we use along with the prices and vendors. However, for these items our choice of vendors was primarily determined by expediency, and we have no reason to think that other vendors would not provide equal or superior products.

In contrast, in order to obtain satisfactory laser diodes and piezo disks we have tried and rejected a large number of different vendors. Piezo disks are widely sold as electronic speakers and are very inexpensive, but most models are not adequate for this application. The Appendix gives the only suitable product we have found. The purchasing of diode lasers can be filled with frustrations and pitfalls, and we refer the reader to Ref. 1 for full discussion of the subject. Here, we shall just give a brief summary of what must be specified, and our recommendations for suppliers. The basic requirement for a diode laser which is to be used in this system is that it have a high reflectivity coating on the back facet and a reduced reflectivity on the front, or output facet. Very inexpensive diodes which produce a few milliwatts of power have two uncoated facets, and will not work very well. We have used 20-mW lasers, but their performance is marginal. However, we have found that any laser we have tried that is specified to provide 30 mW or more single mode will have the necessary coatings and will work well. It will provide narrowband laser light that is tunable over 20–30 nm. If one wants this range to cover the 852-nm cesium or 780-nm rubidium resonance lines, the diode laser wavelength must be specified when purchasing. This greatly complicates the purchasing. We have tried numerous suppliers, but have now settled on STC as our supplier of lasers for 852 nm and Sharp as the supplier for 780 nm. The Sharp lasers are far less expensive and can usually be obtained rather quickly since 780 nm is near the center of the distribution of their normal mass-produced product. This is not the case for 852 nm, and thus the lasers must be produced as a custom run. STC has made several such custom runs and hence usually has 852-nm lasers available although they cost 3 to 4 times more than the Sharp lasers. The long wavelength edge of the distribution of Sharp lasers is at about 839 nm, and we have used such lasers to reach the cesium line by heating them. However, it can be difficult to obtain 839-nm lasers and to obtain reliable performance when tuning the laser this far from its free-running wavelength. The heating of the laser also degrades its lifetime.

The remaining components of the laser system are homemade. The key components are the laser mounting block which holds the actual diode laser, the holder for the collimating lens, and the baseplate onto which all the com-
components are fastened. In addition, we also make the box that encloses the system and a small jig that is useful for setting the position of the collimating lens. All these components have been designed so that they can be constructed by a novice machinist.

III. INSTRUCTIONS FOR CONSTRUCTION OF LASER COMPONENTS

Construction of the diode laser system begins in the machine shop and primarily requires a milling machine and drill press. Detailed machine drawings for the laser mounting block, baseplate, collimating-lens holder, and an alignment jig are given in Figs. 2 and 3. In addition, an enclosure should be fabricated, but its design is not critical. We provide dimensions for mounting the standard Sharp laser package. Small changes may be needed for lasers from other vendors. In view of the setup time required in machining, and the fact that many interesting experiments with diode lasers require more than one of them, it will probably be found economical to make two (or more) systems at once.

A. Laser mounting block

The laser diode is held firmly in a small aluminum block whose details are shown in Fig. 2. The critical dimensions are the 0.500-in. height of the laser center above the baseplate and the depths of holes that ensure that the 9-mm flange of the diode package is gripped by the mounting screws. For stability when the block is screwed down to the baseplate, the bottom surface of the block should be machined as shown with a 0.020-in. relief cut down the middle so that contact is along the edges of the block. This

Fig. 2. Laser mounting block, machine drawing. Dimensions are in inches. The hole sizes, spacings, and depths are correct for a Sharp LT025MO laser and may be modified for other types.

Fig. 3. Collimating lens mount and alignment jig machine drawing. Dimensions are in inches.

“bridge” design has been found to make a significant improvement on laser cavity stability. The 0.356-in. diam hole to receive the diode package may be made either by boring on a lathe fitted with a four-jaw chuck or, more easily, by a suitable end mill. Reground 3/8-in. end mills can often be found near this diameter. Some deburring or filing may be necessary to allow the diode to fit snugly into its recess but allow it to be rotated to its proper orientation in the initial step of alignment. A small hole whose diameter is selected to fit the thermistor should be drilled into the back side of the mounting block near the diode recess.

Fig. 4. Jig usage in collimation. LD=laser mounting block (Fig. 2), C=collimating lens mount (Fig. 3), J=alignment jig (Fig. 3). SP = spring or rubber pad to provide a restoring force against adjusting screw S2.
DIODE LASER BASEPLATE

SAW SLOT TO FORM HINGE.
WIDEN TO ALLOW CLEARANCE FOR TIP
OF DRILL AND REAMER.

2.094

0.938

4.678

4.40 CLEAR.

3/2

0.500

0.250

0.188

3/16

0.375

1/4 TAP

1/4 DIA

1.625

1.094

DRILL CLEARANCE
HOLE(S) FOR SCREW
TO ATTACH GRATING
MOUNT.

MATERIALS: ALUM. 1/8" THK.
DIMENSIONS IN INCHES

Fig. 5. Laser baseplate, machine drawing.

B. Collimator lens mount

Figure 3 shows the aluminum block that holds the flanged collimator lens. The placement of the lens axis at 0.500 in. above the base and the diameter of the hole are again the critical dimensions. The 30° bevel shown on the front side of the block allows clear passage of the output beam off the diffraction grating when very short cavities are used. The figure also shows dimensions of a suggested alignment jig that is used to allow transverse displacement of the lens holder without rotation or longitudinal movement. A 2-56 screw with rounded tip and a small piece of bent spring steel or resilient cushion should be prepared for use with the jig (as shown in Fig. 4).

C. Baseplate

The baseplate is shown in Fig. 5. We have found that aluminum is adequate for most purposes and is easy to machine. If greater thermal stability is required, however, the baseplate can be made of invar. The two pairs of 4-40 holes should be carefully positioned to match corresponding holes in the laser and collimator-lens blocks. The single 4-40 tapped hole is used to mount the alignment jig. The most obvious feature of the baseplate is its flex hinge design, which allows smooth variation of the spacing between diode and collimation lens by action of a commercial precision screw mounted to push against the hinge. The slot that forms the hinge can be cut by a bandsaw after all holes are laid out. The hole intended to receive the precision adjusting screw should be reamed to allow a fit without excess clearance. After all machining of the baseplate is complete the screw can be mounted in this hole with adhesive or by a set screw. The web that provides the flexible hinge should be left 1/16 in. or more in width: One can always remove material later if it proves too stiff. One or more holes should be drilled in the baseplate to mount the diffraction grating holder, but the exact position(s) depends on the dimensions of the holder and grating and on the desired cavity length. Several suitable holes drilled at this time will allow comparison of laser performance with different cavity lengths without complete disassembly of the collimated laser.

D. Grating and grating mount

The baseplate design is intended for use with a 1200 line-per-mm grating. Suitable gratings are readily obtained with 500- and 750-nm blazes and dimensions 1 × 1 × 3/8 in. thick. When mounted, the grating has its rulings vertical and diffracts its first-order interference maximum back into the laser. The output beam is the zero-order beam or specular-reflection maximum, which passes horizontally beside the collimator block and out of the enclosure. The direction of the blaze is toward the output beam. A laser diode whose free-running wavelength is within about 3 nm of the desired wavelength requires less feedback for stabilized operation than a laser that must be pulled more severely. For this case, lower diffraction efficiency and thus a shorter blaze wavelength (500 nm) is suitable, and this allows more power to be brought out in the zero-order beam. If a laser must be pulled more severely, a longer blaze wavelength (750 nm) is used to provide stronger feedback at the price of lower output power.  

When a grating of suitable blaze has been selected it may be cut down to a small size since only about 0.3 in. parallel to the rulings and 0.5 in. perpendicular is required. Thus several gratings can be had for the price of one, and the others may be used to duplicate the diode laser system or for testing gratings properties outside the laser. The cutting may be safely done as follows. Apply a generous coating of clear acetate fingernail polish to the ruled face of the grating. Spread the fluid using a soft camel's hair brush, and avoid physical contact with the grating. After the coating is thoroughly dry, wax the back of the grating to a block of bakelite or phenolic to support the grating while it is sawed. Mark the coated surface of the grating into pieces of the desired size. A 1 × 1 in. grating will yield six suitable pieces. Saw the grating in an abrasive-wheel glass saw by holding the support block on its edge as the saw cuts directly into the face of the grating. Make sure the saw cuts penetrate completely through the grating into the support block without severing the block. Then melt off the cut segments. The nail polish can then be removed by submerging the grating segments in a small beaker of methanol and placing the beaker in an ultrasonic cleaner. Remove the gratings with tweezers, being very careful to avoid any contact with the now-exposed ruling surface, refill the beaker with fresh methanol, and repeat once or twice until the gratings, when drained and dried, appear completely clean. Harsher solvents may attack the plastic substrate of replica gratings, but methanol has been found to be safe and effective.

After cleaning, the grating is attached to the movable face of the grating mount in a location where the collimated laser beam will strike near the middle of the grating. Care should be taken to make the rulings vertical. A stiff but readily removable adhesive such as Duco cement is recommended for attaching the grating to the mount. The grating segment can be easily damaged when it is necessary to remove it or shift its position unless it can be detached with little physical force. If necessary, the efficiency of most inexpensive gratings at the 852-nm wavelength for Cs can be improved by 10%–20% by evaporating a gold coating onto the grating before it is installed.
Before the grating is attached to the mount, the mount should be modified, if necessary, so that it has the same "bridge" profile on its base as described earlier. In addition, the grating mount should be modified so that its adjustment screws can be turned by a ball-end wrench through holes in the temperature-control enclosure of the assembled laser. A good way to do this is to remove the heads from 1/4 in.-20 socket screws in a lathe and to attach them to the centers of the knobs of the adjustment screws with epoxy cement.

E. Piezoelectric disks

Piezoelectric (PZT) disks are inserted between the grating mount adjustment screw and the movable face of the mount in order to rotate the grating about a vertical axis and alter the cavity length with electrical control (Fig. 6). Each PZT element consists of a thin brass backing about 1 in. in diameter to which a thin smaller-diameter silver-plated piezoelectric slice is attached in the center with adhesive around its edge. When voltage is applied, the piezoelectric stress causes the backing to "dish" on the opposite side. Two such elements can be attached back to back, doubling the displacement of a single one, by lightly soldering the adjacent brass backings at four places around their circumference. If necessary for clearance in the grating mount, some of the excess brass can be clipped away without damaging the piezoelectric center. The double PZT is wired by lightly soldering one connection to the brass and the other to the two silver-plated piezo elements in parallel. For this and all other wiring of the laser, it is best to select a limp insulated wire that will not transmit vibration to the laser structure. Rubber-covered No. 24 test prod wire has been found suitable. After the PZT is assembled and wired, and the grating is glued to the mount, the PZT should be inserted between the mounting plate and the ball end of the adjusting screw as shown in Fig. 6. Small pieces of mylar should be inserted to electrically isolate the PZT from the mount. The PZT will provide about ±1 μm of displacement when ±15 V are applied.

F. Enclosure for the laser

An aluminum enclosure should be fabricated to hold the laser. It should have a sufficient thermal mass and conductivity to aid in temperature stabilization. Such a box can be made out of rectangular side plates screwed together, placed on a rectangular baseplate, and capped by a lid, or a single piece of hollow rectangular tubing may be selected to form the walls. Wall thickness should be 1/4 to 1/2 in. Inside dimensions about 3.5 in. wide, 5.5 in. long and 4 in. high are adequate. The floor of the enclosure should stand on some firm support, to bring the laser output beam to a desired height above the table. The lid of the enclosure should be easily removable to allow frequent access to the laser with minimal disruption of the thermal or mechanical stability. After the laser is assembled and satisfactorily aligned, drill holes in the box to allow access to the grating adjustment screws and drill an opening to allow exit of the laser beam. These steps should be delayed until one knows for certain where the holes should be placed. The output aperture is ultimately covered by a microscope slide, and the access holes should be plugged to limit air currents.

Tapped holes on opposite edges of the bottom plate of the enclosure allow the laser structure to be anchored onto the vibration isolation pads discussed below, for instance by stretching a rubber band over the laser baseplate and looping it over screw heads in the edges of bottom plate.

The bottom edge of one of the side walls of the enclosure should be provided with a notch or channel at both ends of the laser for egress of all wires. Soft rubber placed in the notches can serve to press the wires firmly against the bottom plate, and in this way the movement of wires outside the box will not transmit stress or vibration to the laser structure. Finally, one should make sure the enclosure is electrically grounded.

IV. TEMPERATURE CONTROL

Precise control of the temperature of both the baseplate and the diode laser itself is essential for the long term reliable operation of the laser at a particular wavelength. We control these temperatures using identical independent servosystems. The sensing element for the servo is a small thermistor, which is part of a bridge circuit. The amplified and filtered error signal drives a heater or thermoelectric cooler. In this area of thermal control we have made the largest compromises of potential performance in order to simplify the mechanical and electrical designs. Part of the reason we are willing to make this compromise is that we usually sense the output frequency of the laser and lock it directly to atomic transitions to insure long term stability at the sub-MHz level. This is discussed in Sec. X.

The temperature of the diode laser mounting block is controlled only by heating, which means that it must be kept 1–2 °C hotter than the baseplate for proper temperature control. The heating is done by a small (0.3 × 1.5 in.) adhesive film heater which is attached to the top or side of the laser mounting block. The sensing thermistor rests in a small hole packed with heat sink compound in the mount-
Fig. 7. Temperature control circuit.

The laser should be run near or below room temperature. In this case, the baseplate must be cooled below room temperature using a thermoelectric cooler (TEC). This is somewhat more trouble, and the vibration isolation pads between the baseplate and the bottom of the enclosure are now replaced by a rigid TEC. The TEC is a square 1.5 in. on a side and fits between the baseplate and the aluminum plate which is the bottom of the enclosure. A thin layer of heat sink compound is applied on both sides of the TEC to insure good thermal contact. The bottom plate of the enclosure must have a large enough surface area or be in contact with a thermal reservoir so that it does not heat up enough to cause "thermal runaway" of the TEC. The baseplate temperature is monitored using a thermistor glued onto the middle of the baseplate. Since the thermal time constant for the baseplate is much longer, some adjustment (or removal) of capacitors C1, C2, and C3 from the temperature control circuit may be desirable to improve stability. If the laser is cooled below the dew point condensation may form. This may be avoided by flushing gently with dry N₂.

An alternative to controlling the baseplate temperature is to control the temperature of the entire enclosure. This is more effort because much more heating or cooling power is needed and the thermal time constant is very long. We find, however, that this technique gives better ultimate stability of the laser alignment. For most purposes, we have found that this is not worth the effort. However, the small additional effort required in putting insulation on the outside of the aluminum enclosure to attenuate room temperature fluctuations is worthwhile.
V. LASER DRIVE ELECTRONICS

The circuit diagram for the laser current controller is shown in Fig. 8. This is a stable low-noise current source. The output current can be modulated rapidly by sending a voltage into the “RF MOD IN” input. If such modulation is not needed and novices may be operating the laser, it is wise to disconnect or cover this input to minimize the possibility of accidentally damaging the laser. The output current of the supply is limited by potentiometer R36 to a value that cannot exceed the maximum allowed for the diode laser.

The primary concern when working with the current source is to avoid damaging the laser with an unwanted current or voltage spike. In Ref. 1, we discuss this danger at some length so here we will just provide a few helpful techniques. To avoid accidents we always carefully test a new power supply with resistors and light emitting diodes in place of the laser. We check that it produces the voltage and current desired, and that there are no significant transients when turning it on or off. It is also wise to check that all the appropriate grounding connections have been made so that turning on and off nearby electrical equipment or static discharges do not cause current or voltage spikes that exceed the maximum allowed by the laser diodes. When making these tests it is important to realize that lasers can be destroyed by spikes that last only a fraction of a microsecond. Only after the power supply has passed all these tests is it connected to the laser. The cables from the power supply to the laser should be shielded and there should be no possibility of them being accidentally disconnected.

One has to be fairly careful in handling the lasers to avoid static discharges, and it is a good idea to keep the leads shorted together as much as possible. Normally such lasers come with handling instructions that should be followed. These instructions will usually also mention that a fast, reverse-biased protection diode should be connected across the laser leads at the laser mounting block. This diode protects against voltages spikes which may exceed the few volts of back bias a diode laser can tolerate. We have found that the lifetime of diode lasers is substantially increased by also connecting several forward-biased diodes across the leads at the same point as shown in Fig. 9. These diodes have a large enough voltage drop that current does not flow through them under normal operation. However, if there is a large forward voltage, these diodes turn on allowing the current to flow through them instead of the laser diode. It may also be helpful to place a 10-Ω current limiting resistor in series with the supply right at the laser diode (at LD+ in Fig. 9) and, if modulation much above 1 MHz is not required, ferrite beads on the supply lead at this point. Switch S1 in Fig. 8 should be used to short the

![Fig. 8. Laser current control circuit.](image)

![Fig. 9. Protection diode wiring to laser.](image)
supply to ground before connecting the laser, and the current control R3 should be fully "off" whenever S1 is toggled.

VI. ASSEMBLY AND TESTING

A. Diode mounting

The laser diode, with its protection diodes already wired on and its leads temporarily shorted together for safe handling, is mounted in its recess in the laser mounting block with the screws only gently tightened at first. The desired orientation of the laser will produce a vertically polarized output beam and a widely diverging elliptical beam pattern whose major axis is horizontal. This corresponds to the rectangular output facet of the diode chip having its longer dimension vertical.

Next, the mounting block is attached to the baseplate by 4-40 screws extending from beneath. The baseplate should then be secured to some temporary stand so that the uncollimated laser beam can be easily observed after it has gone 1.5 m or more from the laser. The current supply is then set to a normal operating current. The output beam at 780 nm, when projected onto white paper attached to the wall, will hardly be visible with the naked eye, but will show up readily in an IR viewer. The 852-nm light can only be observed by the viewer or an IR sensitive card. At this time, interference rings or fringes may be apparent in the projected beam. These are normally caused by dust, fingerprints, etc., on the laser's output window. The window should be cleaned with an optical tissue dampened in methanol so that the beam pattern is uniform and clear. The orientation of the diode in its recess should be set either by noting the major axis direction or by checking the output polarization. Once the proper orientation is achieved, the mounting screws that hold the laser in its recess should be tightened. Make sure that connections to the diode, including the network of protection diodes, are insulated and arranged so that short circuits will not occur during routine handling. Note carefully the center position of the dispersed beam spot, both its height and lateral position and mark it on the wall. Despite the broad and undifferentiated beam spot, the center can be judged reliably within ±2°.

B. Collimation

The next step is collimation of the output beam and shining, if necessary, of the laser or lens mounting blocks to the correct height. The collimating lens should be firmly fastened into its mounting block with a set screw and the mounting block should be loosely screwed to the laser baseplate with the flat side of the lens toward the diode. The precision adjusting screw that pushes against the baseplate hinge should be advanced so that the hinge is opened enough to allow plus and minus 0.020 in. of motion without losing contact with the ball end of the screw. Next insert a clean microscope slide (about 1-mm thickness) between the lens flange and the front face of the diode laser mounting block. While holding the lens block, the slide, and the diode block together with finger pressure, observe the beam spot again with the IR viewer. It should be possible to slide the lens block back and forth to bring a more concentrated intensity maximum near to the original aiming point of the laser. Temporarily tighten the screws that hold the lens block in that position and remove the slide. Next, adjust the precision screw to bring the laser beam to a sharp focus on the wall. By a very slight adjustment of the screw the beam should then be brought to collimation in an oblong spot about 5 mm wide. It should be confirmed that no focus occurs between the laser and the wall. This constitutes a preliminary alignment.

The beam spot will very likely fall 2° or more from the aiming spot. Horizontal corrections can be made smoothly by use of the alignment jig later, but vertical corrections require shining first. Note the vertical displacement of the spot from the aiming point. A low spot will require raising the lens mount by about 0.0025 in. per degree of misalignment and a high spot will require raising the diode block by the same amount. Layers of aluminum foil (avoiding crinkles) or shim stock should be selected to shim the preliminary alignment beam height to within 1° of the aiming spot.

The alignment jig is installed next by screwing it to the baseplate using its oversize hole and a large washer (or stack of washers) so that it snugly touches the lens mount as shown in Fig. 4. It should be positioned with its 2-64 screw and a spring or elastic cushion so that when the screws of the lens mount are released, the mount can be pushed in both directions without losing contact. With the lens-mount screws now loosened the mount may be displaced smoothly to bring the collimated spot horizontally to the aiming point. A rubber band or finger pressure should be used to hold the loose lens mount against the jig.

A properly aligned laser will exhibit a symmetrical and elliptical beam spot. The effects of aberration can be observed by purposely misaligning the lens to one side or the other with the jig, and a symmetrical behavior allows one to confirm that the designated aiming spot was initially correct. After a satisfactory alignment and collimation has been obtained, the lens mount screws should be firmly tightened and the jig removed. After the lens mount is tightened in place, the fine adjustment screw should again be adjusted to precisely collimate the beam. Positioning the lens without the jig is also possible for those users with a steady hand, but it is very difficult to avoid random rotations and displacement along the beam when only a transverse adjustment is desired.

C. Power output and threshold current measurements

After the laser has been aligned and collimated and before the grating is installed, the power output and threshold characteristics should be recorded and compared with the specifications. The threshold current depends on laser temperature, so it may be desirable to stabilize the temperature of the diode mount at this time. The output power can be measured as a function of drive current by illuminating the face of a wide-aperture photodiode.

D. Mounting and adjusting the diffraction grating

The mounting of the diffraction grating has been described earlier. The laser cavity length is determined by the distance from the back of the laser chip to the illuminated spot on the grating and can be as short as about 20 mm in this design. The grating mount should be screwed to the laser baseplate to form a cavity of the desired length so that the collimated beam illuminates the center of the grating at
approximately the Littrow angle. Best results have been obtained with the shortest possible cavities, apparently because the corresponding mode spacing (about 8 GHz) avoids excitation of adjacent cavity modes by the inherent relaxation noise of the diode at around 3 GHz from line center. It is possible, with a carefully aligned 780-nm laser having 20-mm cavity length to tune electrically over 7 GHz without a mode hop using only the PZT.

The following procedure is used to align the diffraction grating. A small card cut from stiff white paper or a file folder, about 2 x 1/4 in., is useful as a probe to see that the beam diffracted from the grating returns approximately to the center of the lens. The beam spot at 780 nm is readily visible to the eye on the card, but at 852 nm the IR viewer is required. Before screwing the grating mount firmly to the baseplate in this coarse alignment make sure that the adjustment screws are in midrange. The card should be used next to make a more careful alignment of the grating. If the return beam is, for example, too high, as the card is lowered vertically in front of the lens the outward face of the card will be illuminated along a narrow region at its edge until the beam is completely cut off. The width of this narrow region indicates the degree of vertical misalignment. When an edge of the card is raised from below to cut off the beam, no such region of direct illumination will be visible in this example, although direct light from the lens may weakly filter through the card. Probing from all four directions into the collimated beam will indicate both the horizontal and vertical misalignment of the return beam, and the objective is to adjust the screws of the grating mount so that the width of the illuminated region on the card edge is brought exactly to zero for each direction of approach.

A precise vertical alignment of the return beam is made by reducing diode current to just above threshold. Then observe the intensity of the output beam while adjusting the tilt of the grating around a horizontal axis. If the preliminary Littrow alignment was adequate, the output beam should significantly brighten at the exact vertical position that optimizes feedback into the diode. After completing this adjustment the threshold current will be lower than the value recorded earlier for the diode. The laser should now be operating with grating controlled feedback near its free-running wavelength.

If more than one vertical setting of the grating appears to enhance the laser output near threshold, or if the output beam projected on a distant surface consists of more than a single collimated spot, the fault may lie with imperfections (chips, scratches, dirt) on the grating, laser window, or lens surfaces.

VII. TUNING THE LASER FREQUENCY

A low-resolution (<1 nm) grating spectrometer is useful to assess the tuning characteristics of the laser discussed below. After initial alignment of the grating, the output wavelength of the laser will be within about 2 nm of the wavelength specified by the manufacturer, and near the center of the tuning range. Small adjustments of the grating rotation screw (vertical axis) should smoothly shift the laser wavelength. A region of the grating angle adjustment should be identified over which the laser can be tuned. As one nears the end of the tuning range the laser output will be seen to hop back and forth or share power between two very different frequencies. One is the fixed “free-running” frequency at which the laser will operate if there is too little or no feedback from the grating, and the other is the angle dependent frequency set by the grating feedback. At a given temperature, tilting the grating should tune the output wavelength over a range 10 to 30 nm, depending on the particular laser and the amount of feedback. Changing the diode temperature shifts the entire range by 0.25 nm/°C. If the grating is misaligned, the output wavelength will either be insensitive to small changes of the grating angle or will move only a small amount and then jump backwards.

Although this tuning may appear continuous when observed on a low or medium resolution spectrometer, there can actually be small gaps. These occur because the wavelength-dependent feedback of the grating dominates but does not always totally overwhelm feedback off the AR coated output facet of the chip. If it proves impossible to excite some desired atomic absorption line by tilting the grating, it is necessary to operate at a different temperature and/or current. This is best assessed by means of an atomic absorption cell (discussed below) since the gaps in tuning can be narrow and vary randomly from one laser to another. It is helpful to record the tuning rate vs grating rotation (about 14 nm/turn with an 80 thread-per-in. screw pitch), because one can easily mistrue the grating grossly, requiring a retreat to earlier steps in the alignment process. After the grating rotation has been set to produce approximately the correct wavelength, the vertical alignment should be rechecked using the threshold current technique.

The simple laser design described here suffers from a defect that may be annoying in wideband usage: Its output beam is deflected horizontally as the wavelength is scanned, approximately at the angular rate,

\[
\frac{d\theta_{\text{BEAM}}}{d\lambda} = \left( d^2 - \frac{(\lambda/2)^2}{2} \right)^{1/2} \approx 0.08 \text{ deg/nm},
\]

for grating constant \( d \). This is normally of no consequence, however, for saturated absorption or neutral atom trapping, e.g., in Rb where the \( 3s_1/2 - 5p_{3/2} \) hyperfine multiplets of the two naturally occurring isotopes span a total of less than 0.014 nm. If it is necessary to avoid beam deflection, the simplest technique is to take the output beam off a beam splitter inserted between the collimating lens and the grating.

VIII. ENCLOSURE AND VIBRATION ISOLATION

After all preceding steps of alignment have been completed, the laser should be thermally and vibrationally isolated in its enclosure. We have achieved an adequate degree of mechanical isolation by supporting the laser inside the enclosure on three rubber pads that form a tripod under the solid part of the baseplate (avoiding the hinge). Soft “sorbothane” rubber, 1/8 in. thick, cut into 1/2-in. squares and stacked to a height 3/8-in. forms a springy but well damped support that isolates from vibrations above about 100 Hz. For extra isolation, additional rubber may be placed under the support which holds the enclosure at the desired height. The wires from heaters, thermistors, the diode, and the PZT should be taped down to the laser and/or the enclosure baseplates to decouple them mechanically from the laser cavity. The suggested enclosure design offers additional decoupling by pressing the wires firmly against the bottom plate where they exit from the box.
Holes should now be drilled in the sidewalls of the box to allow the beam to exit and to allow manual grating adjustments without having to open the enclosure. Because of the very nonrigid support of the laser, mechanical adjustments, although not often necessary after stabilization, require a delicate touch. The laser structure takes one or more hours to fully stabilize inside its box with temperature-control electronics active. However, preliminary output tests can proceed immediately if steady frequency drift is not an obstacle.

Depending on the degree of stability required and the environment, the laser may be operated on anything from an ordinary laboratory bench, to a fully isolated optical table. A room location near a load-bearing wall or in a basement laboratory can often be worth the price of an expensive optical table. Since the laser itself is one of the best vibration detectors obtainable, experience will be the best guide.

IX. SATURATED ABSORPTION SPECTROMETER

The simplest spectroscopy one can perform with these lasers is to observe the absorption and Doppler-free saturated absorption spectra of rubidium or cesium. This can easily be done in small glass vapor cells which are at room temperature. Such spectroscopy experiments also provide the simplest way to determine the short and long term frequency stability and tuning behavior of the laser frequency.

A. Vapor cells

Rubidium and cesium vapor cells can be obtained commercially, but they are usually rather expensive. However, they can be prepared quite easily, if one has a vacuum pump and some basic glassblowing skills. We use pyrex or quartz tubing, typically 1 in. in diameter and 2 to 4 in. long, and fuse windows onto the ends. The optical quality of the windows is unimportant. The glass cell is connected to a vacuum system through a glass tube about 1/4 in. diameter so that the cell can be evacuated to between $10^{-5}$ and $10^{-6}$ Torr. After the cell is filled, it will be "tipped off" by heating this connecting tube until it collapses in on itself. A few grams of alkali metal in a glass ampule are placed in a separate arm on the vacuum system. The system should be pumped down and the cell outgassed briefly by heating it for several minutes with a torch. At the point where it will be tipped off, the glass connecting arm should be repeatedly heated until it just starts to soften but does not collapse in. After the outgassing is completed, the ampule should be broken to release the alkali metal into the system. Often when one purchases ampules of alkali metal they come packed with an inert gas. In this case there will be a burst of gas also released into the system which must be pumped away. The alkali metal can be moved into the cell simply by heating the glass around it and thus distilling it down the glass tubing into the cell. It is only necessary to have a few very small droplets in the cell so one ampule is sufficient to fill many cells. It is desirable to put much less than 1 g of metal into the cell to reduce the tendency of the metal to coat the windows. Once the alkali is in the cell, the sidearm is tipped off and the cell is ready for use.

B. Optical setup

Figure 10 illustrates a typical layout of beams for a simple saturated absorption apparatus. Initially only a single beam passing through the cell is required, which should be the full laser intensity for maximum sensitivity. In this step one tunes the laser to an atomic transition and finds the optimum laser temperature, current, and mechanical arrangement for stable operation. When the cell is viewed through an IR viewer, or a CCD television camera, a strong track of fluorescence should become visible as the laser is tuned within the Doppler profile of an absorption line by mechanically rotating the grating. It is helpful to ramp the PZT at a frequency of 20 Hz over a 15-V range during this search. The diode current should be arbitrarily set between about 75% and 90% of $I_{op}$. If no fluorescence is apparent at any grating angle with the known tuning range, the temptation to turn the grating farther or to adjust the vertical alignment of the grating should be resisted. Most likely the laser has a tuning discontinuity that encompasses the desired wavelength. The current should be changed several mA and the procedure repeated. If this still fails, the temperature should be changed up or down 0.5 °C to 1 °C and the search for the absorption line should be repeated. If this process is iterated several times without success, it may be desirable to look once again with the grating spectrometer to confirm that the laser is still tuning in the desired range and that the grating has not been grossly misaligned by a random walk. When one finds a grating position which produces fluorescence, the current can be adjusted to maximize the fluorescence.
Once a proper temperature has been set it should not be necessary to change it. However, when the laser is turned on in the morning it is not uncommon to find that the proper drive current has changed by up to 1 mA, or, at a fixed current, minute readjustment of the grating angle is needed, to hit the absorption line again. This drift may be caused by environmental changes, hysteresis in the electrical tuning characteristics, aging of the diode, or mechanical creep of laser cavity components.

C. Piezoelectric scanning

After the laser is mechanically tuned onto an absorption line as observed in the IR viewer, the transmitted (probe) beam should be attenuated so that the intensity is less than 3 mW/cm² and directed into a photodiode. A preliminary assessment of mechanical, electrical, and thermal stability may be made merely by observing the single-beam absorption line. The photodiode output is converted to a voltage by an I/V amplifier, whose circuit is shown in Fig. 11, and the resulting signal is displayed on an oscilloscope. Make sure the I/V offset is not set to an extreme value that saturates the amplifier at either the positive or negative supply voltage. Next the piezoelectric element should be driven by a triangle wave from an ordinary function generator at 15 to 30 Hz, with peak-to-peak amplitude up to 30 V. The photodiode signal should vary by 5%–50% (depending on the particular cell) as the PZT scans the laser across the absorption line. It is helpful to trigger the scope from the function-generator sync pulse or TTL output, or operate the scope in X–Y mode in order to obtain a stable display as the PZT drive is adjusted. When electrical tuning of the laser over the absorption line has been obtained, it is a good time to reexamine mechanical adjustments of the grating angle and diode drive currents. An absorption line or its neighbors, corresponding to different hyperfine levels of the ground state or different isotopes, recurs several times for nearby currents or grating angles. Also discontinuous steps of photodiode output occur across the oscilloscope trace. These steps correspond to transitions from one longitudinal external cavity mode to another. These mode hops may be as far as 8 GHz apart but will exhibit somewhat random spacings as well as hysteresis.

Fig. 12. Single beam saturated absorption in $^{85}\text{Rb}$, $F=3 \rightarrow F'$.

Fig. 13. Saturated absorption curves for (a) Rb and (b) Cs. The $^{87}\text{Rb}$ $F=2 \rightarrow F'$ peaks are broader than the others because they were made with a different setup. The widths and relative heights are affected by beam alignment, beam intensities, electronic damping constants, and absorption cell pressure. These are only representative results.
D. Observing the saturated absorption

The full saturated-absorption setup of Fig. 10 is required for a more detailed test of stability and tuning rates and for locking of the laser output frequency at the level of 1 MHz or better. When first observing a saturated absorption signal it is useful to block the nonoverlapped probe beam. The counter propagating saturating beam can easily be aligned to overlap the probe beam at a 1° intersection angle or smaller. The intensities of the beams are not important for initial adjustments, but typically only a small fraction of the laser output, less than a few percent should be used for the saturated absorption. Reflection from a microscope slide provides an ample intensity that will allow further attenuation by neutral density filters or exposed photographic film. When adequate pump and probe beam overlap has been obtained, small saturated absorption dips should become evident near the center of the absorption line (Fig. 12). They may be recognized unambiguously by their disappearance from the Doppler profile if the saturating beam is blocked. The height of the narrow dips may be maximized by adjusting the alignment. The width can be reduced by reducing the angle of intersection of the overlapped beams and by attenuating either or both beams to avoid power broadening. The triangle wave amplitude and dc offset can be adjusted to zoom in on a particular region of the scan.

For more detailed observations it is helpful to unblock the second probe beam. This second probe beam is directed into a photodiode identical to the first and wired in parallel with reversed polarity. The two probe beams can easily be obtained by utilizing the reflections off both front and rear surfaces of a piece of 3/8-in.-thick transparent plastic or glass. When the two photodiodes are properly positioned, the differential output signal cancels the large and featureless Doppler profile of the absorption line and allows saturated absorption features from the first probe beam to appear on a nearly flat background. If the Doppler broadened absorption is observed but the saturated absorption peaks cannot be seen, it often means that there is too much background gas in the vapor cell.

E. Saturated absorption patterns in Rb and Cs

After saturated absorption peaks have been observed, one can compare the patterns to known hyperfine structures of the ground and excited states to assess the electrical tuning range possible without hopping external cavity modes and to establish the tuning direction. The widths and resolution of the saturated absorption peaks for a given resonance line will depend on electronic time constants, the triangle-wave frequency, and possibly on diode current, in addition to alignment and intensity factors noted above. Figure 13 shows several saturated absorption patterns in Rb (780 nm) and Cs (852 nm) vapors photographed from an oscilloscope. These may aid new users in finding their way. Note that the patterns contain both true Doppler-free peaks and crossover peaks, which occur at frequencies \((v_1 + v_2)/2\) for each pair of true peaks at frequency \(v_1\) and \(v_2\). The crossovers are often more intense than the true peaks.

F. Typical tuning rates observed by saturated absorption

Tuning rates for the grating-feedback laser, operated with a single longitudinal mode of the external cavity, depend on geometrical, thermal, and electrical properties of the laser components. In particular, tilting the grating changes both the wavelength of light diffracted back to the diode and the length of the cavity. These two effects interact in determining the change of output frequency. Typical tuning rates for a 780-nm laser having a 20-mm cavity on an aluminum baseplate are: (1) diode drive current: 200 MHz/ma, (2) temperature change of diode: 4 GHz/°C, (3) temperature change of baseplate (cavity length): 7 GHz/°C, (4) grating angle change (80 pitch screw): \(5 \times 10^6\) MHz/turn, and (5) piezoelectric tuning: 1 GHz/V.
X. SERVOLOCKED OPERATION OF THE DIODE LASER

For stabilized operation of the laser, it may be locked to either side of any of the sufficiently well-resolved saturated absorption peaks such as those shown in Fig. 12. A simple servolock circuit is given in Fig. 14. Figure 15 indicates how the lock box is connected to the other components. Locking is not difficult after a little practice, provided that the saturated absorption signals are not too noisy and the laser frequency jitter caused by environmental or electrical backgrounds is less than the saturated absorption linewidths.

First, the laser is tuned to the desired hyperfine multiplet of saturated absorption peaks and the ramp gain and ramp offset are adjusted, both on the ramp generator and on the lock box, so that one can zoom in to the desired side of a particular peak simply by turning down the ramp gain on the lock box to zero. With feedback and output gain controls set at minimum and the laser tuned to the side of a peak, the error offset is adjusted to a value near 0 V, as observed on an oscilloscope. The feedback and output gains are then gradually increased until the circuit corrects for deviations from the desired lock point and thus holds the frequency on the side of the peak. If the servolock seems to “repel” the saturated absorption peak, the input invert switch is reversed to select the opposite slope. When the laser is properly locked, it should be possible to turn the feedback fully on and the output gain up to a point where the PZT begins to oscillate at about 1 kHz. The best operating point is just below the onset of oscillation. Locking is confirmed by noting that the setpoint, indicated by the level of the now flat saturated absorption signal on the oscilloscope, can be varied by the error offset control within a range from about 10% to 90% of the height of the selected peak without a noticeable change in the monitored error output. Independently, the error output can be varied over a wide range by the ramp offset control without affecting the locked level of the saturated absorption signal.

When the laser is locked, environmental noise appears on the error output and error signal monitor instead of on the saturated absorption signal because the error output compensates for laser frequency variations that would otherwise occur. The error signal monitor thus becomes an excellent indicator of the magnitude and spectral characteristics of the compensated noise, out to the bandwidth of the servolock circuit.

The drift rate of the unlocked laser is normally under 5 MHz/min when the system is properly stabilized, and this slow drift is eliminated by locking. The short-term jitter amplitude of the unlocked laser frequency is typically ±3 MHz on a 1-s time scale if the laser is on a reasonably stable lab table. The short-term intensity variations are much smaller than 1%. When locked, the laser frequency is stabilized to 1 MHz or better.

The locked diode laser described in this paper is well suited for studies of neutral-atom cooling and trapping, for which some elaborations of the servolock circuitry are desirable. A future paper will describe trapping of Rb and Cs atoms from a vapor cell in a user-manual style similar to that used here.

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APPENDIX: PARTS AND SUPPLIERS

1. Collimating lens #1403-108, $75.00, f=5 mm, numerical aperture 0.5, Rodenstock Precision Optics Inc., 4845 Colt Road, Rockford, IL 61109, (815) 874-8300
2. Sorbothane Pad P/N: C37,000, $49.95, Edmund Scientific, 101 E. Gloucester Pike, Barrington, NJ 08007-1380, (609) 573-6250
3. Photodiode Pin-10D (1 cm² active area), $55.25, United Detector Technology—Sensors, 12525 Chadron Avenue, Hawthorne, CA 90250, (213) 978-1150 x360
4. Kodak IR detection card R11-236, $49.50, Edmund Scientific, (address as above)
5. Hand held infrared viewer P/N 84499, $1195.00, FJW Optical Systems, Inc., 629 S. Vermont Street, Palatine, IL 60067-6949, (708) 358-2500
A less expensive alternative is to use a CCD surveillance camera. These can be purchased from many sources including home and office security companies, and discount department stores. For an adequate model, prices for a camera, lens, and monitor will range from $500 to over $1000.
6. Sharp Diode Laser LT025MDO, $170.85, wavelength 780 nm, Added Value Electronic Distributors, Inc. (local Sharp distributor), 4090 Youngfield Street, Wheatridge, CO 80033, (303) 422-1701
7. STC LT50A-034 laser diodes (STC was recently purchased by Northern Telecom), wavelength 852 nm. We have purchased these lasers for ~$650 from a German distributor: Laser 200 GMBH, Argelsrieder Feld 14, D-8031 Welring, Germany
8. Minco Thermfoil Kapton Heater, Minco 8941 P/N HK5207R12.5L12A, $23.50, #10 PSA (Pressure sensitive
adhesive) sheet, $4.00, Minco Products, Inc., 7300 Commerce Lane, Minneapolis, MN 55432, (612) 571-3121 x3177
8. Diffraction grating 1200 l/mm, 500 nm blaze: P/N C43,005, $72.85, 750 nm blaze: P/N C43,210, $72.85, Edmund Scientific (address as above)
9. Thermistor, P/N 121-503IAJ-Q01, $8.25, Fenwall Electronics, 450 Fortune Blvd., Milford, MA 01757 (also available from electronics distributors)
10. PZT disk, P/N PE-8, $0.75 (Murata/Erie # 7BB-27-4), All Electronics Corp., P.O. Box 567, Van Nuys, CA 91408, (818) 904-0524
11. Kinematic Mirror Mount Mod. MML, $52.00, Thorlabs, Inc., P.O. Box 366, Newton, NJ 07860, (201) 579-7227
12. Fine Adjustment Screw Mod., AJS-0.5, $30.00, Newport Corp., P.O. Box 8020, 18235 Mt. Baldy Circle, Fountain Valley, CA 92728-8020, (714) 963-9811
13. Thermolectric cooler, 30×30 mm, #CP1.4-71-045L, $19.00, MELCOR, 990 Spruce St., Trenton, NJ 08648, (609) 393-4178
14. Cesium and rubidium vapor cells. We have never used these cells, but this company has announced that they will sell low cost vapor cells to educational institutions. Environmental Optical Sensors, Inc., 3704 N. 26th St., Boulder, CO 80302, (303) 440-7786

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Railgun recoil, ampere tension, and the laws of electrodynamics

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There has recently been a revival of an old controversy over whether Ampere's original law of force between current elements is to be preferred over the more familiar and universally used Biot–Savart–Lorentz law. Although it is agreed that the two laws give identical results when used to calculate the force between two circuits, it has been claimed that if Ampere's law is applied to the action of a circuit upon part of itself it predicts internal longitudinal forces that are excluded by Biot–Savart–Lorentz. The existence of longitudinal forces has been inferred from observations of the buckling of the rails in a railgun, attributed to compressive stress, and the fracture of long wires subjected to pulsed currents, attributed to "Ampere tension." An experiment has been performed that should provide an unambiguous demonstration of this longitudinal force, if it exists. The apparatus consisted of a rigid coaxial circuit in which an unsymmetrical section of the center conductor was free to move in the axial direction under the influence of the longitudinal force. Pulsed currents of up to 100 kA were passed through the circuit but no evidence of a longitudinal force was found. It is shown that this result is consistent with Ampere's force law, and that claims that the law predicts longitudinal forces are based on the incomplete application of the law.

I. INTRODUCTION

The original law of force between electric circuits was formulated by Ampere who considered individual current elements to act upon each other as if they were Newtonian particles. He derived an empirical relationship for the force between current elements which, expressed vectorially, is

\[ d^2 \mathbf{F}_2 = -i i_2 \left[ 2 \mathbf{ds}_1 \times \mathbf{ds}_2 - 3 \mathbf{ds}_1 \mathbf{r}_{12} \right] \times \left( \mathbf{ds}_2 \mathbf{r}_{12} / r_{12}^2 \right) r_{12} / \mathbf{r}_{12}^2. \]