Pinhole Optics

I. The problem with imaging:
   A. Light rays leave an object in an infinite many Paths.
   B. In order to image an object anywhere there must be a selection process by which a given point on an object creates a point relative on the image zone.
   C. The most common (modern) method is of course the lens.

II. The pinhole
   A. Another option is the pinhole.
      i. The pinhole is in fact a simple machine
      ii. Its properties were first recorded by Mo Ti (China) c 400 b.c, and Later by Aristotle c 330B.C.
      iii. Alhazen (Egypt) was the first to classify the pinhole as a tool in Kitab al-Munazir published 1020 A.D.

III. Camera obscura
   A. “Dark Room” from the Latin
      i. It is a Dark vault with a pinhole as the only source of light.
      ii. Light enters the pinhole and images the outer world upside down and in complete focus.

IV. So what is happening?
   A. If we can imagine that light rays are leaving two points in an infinite many paths
Light rays leaving from these two points overlap.

B. If we place an opaque material, with a small pinhole, between the two points and the imaging zone. We see that only a select amount of light rays from either point will pass

C. In the case that there are many points (as in most real world cases) there will be an infinite amount of points whose light cones will be passing through the pinhole, as a one can tell these cones for near points will overlap on the imaging zone.
This leads us into a discussion of diffraction and Focal length.

V. Focal Length, Diffraction, and pinhole diameter.

A. There is an Optimum Focal length for a given pinhole Diameter.
   i. The pinhole actually allows a cone of light to pass for a given point

   ![Diagram of a cone passing through a pinhole]

   ii. In order for points to image these cones must not overlap (to an extent) to what extent I will talk about when I get to diffraction.

   iii. So when finding the focal length we are looking for the optimum length at which these cones overlap.

B. Diffraction.
   i. Here we must talk about diffraction, how light is actually a wave not just a straight line.

   ii. “It should be emphasized that optical instruments make use of only a portion of the complete incident wave front.” (Optics, Hecht, p. 393)

   iii. Huygens's principle tells us that light propagates spherically, and upon passing through a pinhole constructs a second spherical wave front.
It is the case then that an object would not be imaged because as you can see light from any given point would still strike a large area of the imaging zone. Because Huygen’s principle is actually generalizing how light travels after passing through a pinhole, it is unable to tell us how points are being imaged through a pinhole.

“Huygens’s principle by itself is unable to account for the details of the diffraction process.” (Hecht, p.393)

iv. “The corresponding Huygens-Fresnel principle states that ‘every unobstructed point of a wave front, at a given instant in time, serves as a source of spherical secondary wavelets (with the same frequency as that of the primary wave). The amplitude of the optical field at any point beyond is the superposition of all the wavelets (considering their amplitude and relative phases.’” (Hecht, p.393)

In other words, when the spherical propagation of a given point travels through a pinhole, every point on the arc, of the wave front, that passes, creates another spherical propagation (identical to the primary spherical wave front).

C. The image of the wave! How will these new wave fronts image?

i. Because imaging depends on whether $\lambda$ is bigger or smaller than the pinhole, we will deal with them separately. (It should also be noted that I am dealing with light and so when I refer to large and small pinholes I mean relative to the wave length of light waves $\lambda\approx 10\text{\&}-6$). First lets deal with the case where $\lambda\leq \bar{ab}$ (the
pinhole diameter). This we will call the geometrical optics. In this case the greater the pinhole size the greater focal length we will need. We will address later the case where $\lambda > \bar{a}b$, which is the physical optics (Imaging without Lenses or Mirrors, Matt Young).

ii. In the geometrical optics case the maximum optical path length of the secondary wave fronts $\Delta \text{max} = | \bar{a}p - \bar{b}p |$, but remember that $\Delta \text{max}$ cannot be larger than $\bar{a}b$ (pinhole diameter) by geometry it follows that the length of one side of a triangle subtracted from another must be equal or less than the third side.

iii. It so happens according to this model that the only space where $\lambda$ is greater than $\Delta \text{max}$ is in a direct line from the object point to the imaging zone, so this is the only place where the wavelets will interfere constructively. Beyond this zone deconstructive interference begins, and so does shadow.

iv. Therefore, in geometrical optics the best focal length for a given diameter will be where to touching object points are imaged so that the constructive interference cones are just touching edge on the imaging zone.

D. In the case of physical optics where $\lambda > \bar{a}b$, $\lambda$ will also be greater than $\Delta \text{max}$ “and since the waves were initially in phase, they must all interferes constructively (to vary
degrees) wherever P happens to be.” (Hecht, p.394) This will result in a blurred image.

VI. Focal length.

A. As we saw focal length really refers to the case where the pinhole diameter is greater than \( \lambda \), where it is less we get a blurred image.

B. Focal Length.

i. With that said, there are optimization equations for deriving the best focal length for a given pinhole. Let us just focus in on the case where \( \lambda = \bar{ab} \), and where \( \Delta_{\text{max}} = \bar{ab} \), meaning that p is on the image zone. We note then that \( \lambda = \bar{ab} = \Delta_{\text{max}} \) and in this case the image is the size of the pinhole. For a pinhole that is greater than \( \lambda \), then \( \lambda < \bar{ab} \), \( \Delta_{\text{max}} = \Delta_{\bar{ab}} \), therefore \( \lambda \) is only greater just past \( \Delta_{\text{max}} \) only when the image zone is just past the optimal path length, and so to the image will be larger than the pinhole.

ii. The natural pinhole focal length is \( f = s^2 / \lambda \) (Young) where \( s \) is the radius of the pinhole. So if \( s = \lambda \) the \( f = s \). and so on. \( D = 0.047 \sqrt{f} \) where \( D \) diameter and \( F \) are in mm.

VI. Vignette
A. If the material in which the pinhole is set is too thick Vignette will occur. Vignette is the effect of having differing amounts of light being imaged from different points.

VII The pinhole camera

A. Makes use of all of these things. There are many resources for making such a camera. They are easy and fun, please follow the resources links. However, here is one quick and easy experiment.

i. Materials: one toilet tissue roll, one sheet of black construction paper, a 2”x2” piece of aluminum foil paper, 2”x2” piece of wax paper, two rubber band, scotch tape, one pin.

ii. First Place the foil paper over one end of the Roll (using a rubber band) and the wax paper on the other end (“”).

Second roll the black construction paper length wise around the roll, so that the foil paper end is flush and the wax paper end is mid-way in the new construction tube.

Tape the tube in place.
Third make a small hole in the foil paper with the pin.

Fourth go out side and take a look, the outer world should be imaged onto the wax screen.