9.3 LIGHT-MATTER INTERACTION

Einstein's Law of Radiation

quantized energy levels \( E_m \) and \( E_n \).

\[ \Rightarrow \quad \wedge \quad \Rightarrow \]

\[ I(\omega) \quad \frac{m}{m} \quad \frac{n}{n} \quad \frac{\Delta E}{\Delta E} \]

When the incident light has the right frequency \( \hbar \omega = E_m - E_n \), three processes can occur:

**ABSORPTION**

An atom absorbs a photon, thus making a transition from state \( n \) to state \( m \).

The probability of this transition would depend on
- the intensity of the light \( I(\omega) \)
- the nature of the states \( m \) and \( n \)
let's assume the probability of this transition \( P_{nm} \) is proportional to \( I(w) \) per second \( P_{nm} = B_{nm} I(w) \) Absorption

**EMISSION**

Einstein suggested there should be two types of emission processes

**Spontaneous emission**

\[ m \]
\[ n \]

Even when there is no light present, there is a certain probability \( A_{mn} \) per second that the atom will transition from the excited state \( m \) to the lower state \( n \)

We further assume that \( A_{mn} \) is the same whether light is present or not

**Stimulated emission**

\[ m \]
\[ n \]

The emission probability is further influenced by the presence of light
Einstein assumed this probability per second to be proportional to \( I(w) \)

\[
\text{stimulated emission} = B_{mn} I(w)
\]

\[
\text{probability} \propto \text{const of proportionality}
\]

Accordingly, the total emission probability would be

\[
P_{mn} = A_{mn} + B_{mn} I(w)
\]

**Equilibrium conditions**

At temperature \( T \), \( N_m \) atoms will be in the state \( m \), and \( N_n \) in the state \( n \).

\[
R_{n\rightarrow m} = N_n B_{nm} I(w)
\]

Rate at which atoms transit from \( n \) to \( m \)

\[
R_{m\rightarrow n} = N_m (A_{mn} + B_{mn} I(w))
\]
At equilibrium, these two rates should be equal (so the number of atoms in each energy level remains constant.)

\[ N_n B_{nm} I(\omega) = N_m (A_{mn} + B_{mn} I(\omega)) \]

\[ I(\omega) = \frac{A_{mn}}{\frac{N_n}{N_m} B_{nm} - B_{mn}} \]

Since \( N_n \propto e^{-\frac{1}{kT}E_n} \)

\( N_m \propto e^{-\frac{1}{kT}E_m} \)

\[ \Rightarrow \frac{N_n}{N_m} = e^{-\frac{1}{kT}(E_n - E_m)} = e^{\frac{1}{kT}(E_m - E_n)} \]

But the frequency \( \omega \) that we are considering above is the one matching \( h\omega = E_m - E_n \).

Thus,

\[ \frac{N_n}{N_m} = e^{\frac{h\omega}{kT}} \]

\[ I(\omega) = \frac{A_{mn}}{B_{nm} e^{\frac{h\omega}{kT}} - B_{mn}} \]
But Planck's formula tells us that

\[ I(\omega) \propto \frac{k \omega^3}{\pi^2 c^2} \frac{1}{e^{\frac{k \omega}{k T}} - 1} \]

This implies

\[ B_{mn} = B_{nm} \]

Induced emission and absorption probabilities are equal.

and

\[ A_{mn} = \frac{k \omega^3}{\pi^2 c^2} B_{nm} \]

The cubic dependence of \( A_{mn} \) on \( \omega \) accounts for the principal difficulty in achieving laser action at x-ray frequencies.

At these high frequencies spontaneous emission occurs so rapidly that a sustained stimulated...
Emission is difficult to achieve. At lower frequencies (visible) this difficulty is, fortunately, no great.

Stimulated Emission (LASER)

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**Stimulated absorption**

**Spontaneous emission**

(1)  

(2)  

in phase!
The most striking feature in the stimulated emission process is that the photon resulting from the transition is in phase with the incident photon that provokes the transition.

Stimulated emission is a constructive interference process.

CAN WE HAVE A SUSTAINED STIMULATED EMISSION PROCESS?

Two factors play against,

1) At equilibrium, there are only a few atoms in the excited state.

2) Out of the few excited atoms, the spontaneous emission process lowers even more the number of atoms available for stimulated emission.

\[
\begin{align*}
\text{a few} & \quad \rightarrow \quad \circ \circ \quad N_m \\
\text{many} & \quad \rightarrow \quad \circ \circ \circ \circ \circ \quad N_n
\end{align*}
\]

\[N_m < N_n\]
We cannot avoid the spontaneous emission. So, scientists had to figure out how to revert the condition $N_m < N_n$, which implies to consider situations out of equilibrium then. One strategy is shown in the figure below: \textbf{OPTICAL PUMPING}

\begin{itemize}
  \item \textit{pump light}
  \item \textit{metastable state (atom tends to stay long in this state)}
\end{itemize}

\begin{itemize}
  \item \textit{No pumping}
  \item \textit{Moderate pumping}
  \item \textit{Intense pumping}
\end{itemize}
For perfectly parallel mirrors, the horizontally collimated beam intensity is reinforced through many back and forth reflections, b) the accompanying stimulated emission they produced. The latter occurs provided the population inversion is maintained.
Absorption Coefficient and Population Inversion

Let's consider the variation in intensity of a collimated beam (photons of energy $h\nu$) advancing in a medium containing atoms that have energy levels $E_m$ and $E_n$ such that $E_n - E_m = h\nu$

![Diagram of collimated beam with atoms]

$N_n \rightarrow N_m$

$U(z) = \frac{I(z)}{c}$

$U$: energy per unit volume

$\frac{U}{h\nu} = \text{photons per unit volume} = \eta$

# of photons in that volume = $\frac{U(z)}{h\nu} A \Delta z = \eta(z) A \Delta z$
\[ N_n = N_n^0 A \Delta z \]
\[ N_m = N_m^0 A \Delta z \]

We assume there is a constant and uniform density of excited states \( N_n^0 \), \( N_m^0 \) (which will change, but only after the beam has passed).

\[ N_m B_{mn} I(x) \]
\[ \text{Probability per second an atom will be absorbed} \]

\[ N_n B_{nm} I(x) = \text{# of photons lost per second by the collimated beam.} \]

\[ (N_n - N_m) B_{nm} I(x) = \frac{\Delta n(x)}{\Delta t} \quad \text{Net gain of photon in the collimated beam} \]

The net gain of photons can also be expressed in terms of the net flux traversing the volume \( A \Delta z \):

\[ \frac{\Delta n}{\Delta t} = \frac{I(z + \Delta z)}{\text{kW}} A - \frac{I(z)}{\text{kW}} A \]
\[
\frac{\Delta n}{\Delta t} = \frac{A}{\hbar \omega} [I(z+\Delta z) - I(z)]
\]

\[
= \frac{A}{\hbar \omega} \frac{\Delta I}{\Delta z} \Delta z
\]

Equating (1) and (2)

\[
(N_n - N_m) B_{nm} \frac{I(x)}{dx} = \frac{dI}{dx} \frac{1}{\hbar \omega} \Delta z
\]

\[
N_n \Delta z
\]

\[
\Rightarrow \frac{dI}{dx} = \hbar \omega (N_n - N_m) B_{nm} I(x)
\]

If the gas of atoms is in equilibrium,

\[
N_n < N_m, \text{ so } \frac{dI}{dx} \text{ decreases.}
\]

\[
\frac{dI}{dz} = -\left( N_m - N_n \right) B_{nm} \hbar \omega \frac{I(z)}{\alpha}
\]

\[
I(z) = I(0) e^{-\alpha z} \quad \alpha = (N_m - N_n) B_{nm} \hbar \omega
\]

Absorption coefficient
If, however, we created a condition where

\[ N_m^0 > N_n^0 \]

\[ m \cdots \cdots \]

\[ n \cdots \cdots \]

the \( \alpha \) would become negative, and the intensity \( I \) would grow with distance

\[ I(z) = I(0) e^{\beta z} \]

\[ \beta = (N_n^0 - N_m^0) \]

called "small-signal gain coefficient"