An Analogy between electronic excitation in an atom and the motion of a mechanically forced oscillator.

\[ \text{me} = 9.1 \times 10^{-31} \text{kg} \]

But, how to choose \( K \)?

**Resonant Absorption**

Transition between Energy levels

\[ \Delta E = kw_0 \]

\[ w_{rcs} = \sqrt{\frac{K}{m_e}} \]

\[ K \equiv m_e w_0^2 \]
If \( w_f = w_0 \) the incident photon is absorbed by the atom

Resonant absorption

Atom driven by the external radiation of frequency \( w_f \)
The emission of electromagnetic waves by the accelerated charge, can be considered as a channel of "dissipative" energy.

Light-atom interaction by a resonance absorption process, can therefore be modelled by an equation similar to a harmonically forced damped oscillator.

\[ m \frac{d^2 x}{dt^2} + b \frac{dx}{dt} + kx = F_0 \cos(w_0 t) \]  

- \( x \) is the "position" of the electron
- \( m \) is the electron's mass
- \( k = m w_0^2 \)
- \( w_0 \) is one of the atom's discrete resonant absorption frequencies.
- \( F_0 = E E_0 \)
- \( F_0 \) is the external electric field amplitude
- \( b = b \left( w_0, m, \ldots \right) \)
- \( b \) is related to the rate at which the accelerated electron re-emits light.
- We'll give later a specific expression for \( b \).
Solving Eq (1) using complex variable. 

First, we apply a trick:

Let \( y = y(t) \) be the solution of

\[
m \frac{d^2 y}{dt^2} + b \frac{dy}{dt} + ky = F_0 \sin(\omega_f t)
\]

(2)

Multiply (2) by \( i \)

\[
m \frac{d^2 (iy)}{dt^2} + b \frac{d(iy)}{dt} + k(iy) = F_0 \left[ i \sin(\omega_f t) \right]
\]

(3)

(1) + (3) gives

\[
m \frac{d^2 z}{dt^2} + b \frac{dz}{dt} + k z = F_0 e^{i \omega_f t}
\]

(4)

where \( z(t) \equiv x(t) + iy(t) \)

If we find a solution in (4), let's say \( z(t) \)
then a solution to Eq (1) can be obtained by taking \( x(t) = \text{Re} \{ z(t) \} \)
Solving Eq. 4

Since the driving force is harmonic of frequency \( \omega_f \), we guess the displacement \( z(t) \) will also be harmonic of frequency \( \omega_f \). Potentially, there may be phase difference between \( F_0 e^{i \omega_f t} \) and \( z(t) \).

Thus, we propose a solution of the form,

\[
z(t) = A e^{i(\omega_f t + \phi)}
\]

where \( A \) and \( \phi \) can depend on \( \omega_f \) and other parameters.

In Eq. 4 gives,

\[
(-\omega_f^2 + \omega_0^2 + i \frac{b}{m} \omega_f) A e^{i \phi} = \frac{F_0}{m}
\]

\[
\Rightarrow A e^{i \phi} = \frac{\frac{F_0}{m}}{-\omega_f^2 + \omega_0^2 + i \frac{b}{m} \omega_f} = \frac{F_0}{m} \frac{1}{\left(-\omega_f^2 + \omega_0^2\right) - i \frac{b}{m} \omega_f} = \frac{F_0}{m} \frac{1}{\left(-\omega_f^2 + \omega_0^2\right)^2 + \left(\frac{b}{m} \omega_f\right)^2}
\]
Notice, the complex number in the numerator can be expressed as,

\[ (-\omega_f^2 + \omega_0^2) + i \frac{b}{m} \omega_f = \left[ (-\omega_f^2 + \omega_0^2)^2 + \left( \frac{b}{m} \omega_f \right)^2 \right]^{1/2} e^{i \tan^{-1} \frac{-\frac{b}{m} \omega_f}{-\omega_f^2 + \omega_0^2}} \]

\[ \Rightarrow \]

\[ A e^{i \Phi} = \frac{F_0/m}{\left[ (-\omega_f^2 + \omega_0^2)^2 + \left( \frac{b}{m} \omega_f \right)^2 \right]^{1/2}} e^{i \tan^{-1} \frac{-\frac{b}{m} \omega_f}{-\omega_f^2 + \omega_0^2}} \]

\[ A(\omega_f) = \frac{F_0/m}{\left[ (-\omega_f^2 + \omega_0^2)^2 + \left( \frac{b}{m} \omega_f \right)^2 \right]^{1/2}} \]

\[ \Phi(\omega_f) = \tan^{-1} \frac{-\frac{b}{m} \omega_f}{-\omega_f^2 + \omega_0^2} \]
Summary,

\[ m \frac{d^2 z}{dt^2} + b \frac{dz}{dt} + k z = F_0 e^{i \omega_f t} \]

This equation admits solutions of the form

\[ z(t) = A e^{i (\omega_f t + \phi)} \]

where \( A = A(\omega_f) \) and \( \phi = \phi(\omega_f) \) are given in expression (5).

Accordingly,

The solution to \( z(t) \)

\[ m \frac{d^2 x}{dt^2} + b \frac{dx}{dt} + k x = F_0 \cos(\omega_f t) \]

is given by

\[ x(t) = \text{Real} \{ z(t) \} \]

\[ = A(\omega_f) \cos(\omega_f t + \phi) \]
**Graphic Analysis**

Phasor force: \( F_0 e^{i \omega_f t} \)

Phasor position: \( A e^{i (\omega_f t + \phi)} \)

Phasor velocity: \( A \omega_f e^{i (\omega_f t + \phi + \pi/2)} \)

**Notice:**

Since \( \phi \) is always negative, the position phasor always lags the force phasor.
\[ A_{\text{max}} = \frac{F_0 / K}{1 - \frac{1}{4Q^2}} \]

For \( Q \gg 1 \)

\[ A_{\text{max}} \propto Q \frac{F_0}{\kappa} \]

\[ \omega_0' = \sqrt{\omega_0^2 - \frac{1}{2} \left( \frac{b}{m} \right)^2} \]

\[ = \omega_0 \sqrt{1 - \frac{1}{2Q^2}} \quad (Q = \frac{m}{b} \omega_0) \]

\[ \omega_0' \approx \omega_0 \text{ for high } Q \]