Chromatic Aberration (CA)

6. Longitudinal CA
7. Lateral CA

Chromatic aberration results from the fact that

1. The focal length of a lens depends on the index of refraction $\eta$
   \[
   \frac{1}{f} = \frac{\eta_{\text{lens}} - \eta_{\text{air}}}{\eta_{\text{air}}} \left( \frac{1}{R_1} - \frac{1}{R_2} \right)
   \]

and

2. $\eta$ depends on the wavelength of wavelength $\lambda$ of the light
   \[
   \eta = \eta(\lambda)
   \]
   That is, different colors refract at different angles.

Thus,

focal length $f = f(\lambda)$
How to correct this chromatic aberration?
The previous ray-tracing diagrams suggest that chromatic aberration might be corrected by properly combining a converging lens with a diverging lens.

Indeed that is the case in the design of the achromatic doublet lens.

ACHROMATIC DOUBLET
Consisting of a crown equiconvex lens cemented to a negative flint glass lens

Let’s assume we want an “achromat doublet” of 15 cm focal length. Lets call it \( f_{\text{desired}} = 15 \text{ cm} \). Since, in general, the focal length depends on the wavelength, \( f_{\text{desired}} \) is conveniently specified as that associated with yellow light (the Fraunhofer wavelength \( \lambda_D = 587.6 \text{ nm} \)). Thus,

\[
f_{\text{desired}} = 15 \text{ cm}, \quad \text{at } \lambda = \lambda_D
\]

Considering a doublet as two thin lenses, its effective focal length in terms of its lens components can be expressed as,

\[
\frac{1}{f_{\text{doublet}}} = \frac{1}{f_1} + \frac{1}{f_2}
\]  \hspace{1cm} (1)

The focal length of the individual lenses is given by
\[ \frac{1}{f_1} = (n_1 - 1) \left( \frac{1}{r_{11}} - \frac{1}{r_{12}} \right) = (n_1 - 1) \rho_1 \]  
(2a)

\[ \frac{1}{f_2} = (n_2 - 1) \left( \frac{1}{r_{21}} - \frac{1}{r_{22}} \right) = (n_2 - 1) \rho_2 \]  
(2b)

The focal length of any doublet is then given by,

\[ \frac{1}{f_{\text{doublet}}} = (n_1 - 1) \rho_1 + (n_2 - 1) \rho_2 \]  
(3)

The value of \( 1/f_{\text{doublet}} \) in general will depend on \( \lambda \).

The condition of achromaticity, around the wavelength of the yellow light \( \lambda_D \), can be expressed as,

\[ \frac{d}{d\lambda} \left( \frac{1}{f_{\text{doublet}}} \right) = 0 , \quad \text{at } \lambda = \lambda_D \]

Or, equivalently, using expression (3),
\[ \rho_1 \frac{\text{dn}_1}{d\lambda} \bigg|_{\lambda = \lambda_D} + \rho_2 \frac{\text{dn}_2}{d\lambda} \bigg|_{\lambda = \lambda_D} = 0 \]  

(4)

The derivative of \( n \) at \( \lambda = \lambda_D \) can be approximated using the red and blue Fraunhofer wavelengths, \( \lambda_C = 656.3 \) nm and \( \lambda_F = 486.1 \) nm,

\[ \frac{\text{dn}}{d\lambda} \bigg|_{\lambda = \lambda_D} \approx \frac{n_F - n_C}{\lambda_F - \lambda_C} \]

In addition, using the reciprocal of the dispersion power \( \Delta \)

\[ V \equiv \frac{1}{\Delta} = \frac{n_D^{-1}}{n_F - n_C} \]

known as “Abbe number”,

we obtain,

\[ \rho_1 \frac{\text{dn}_1}{d\lambda} \bigg|_{\lambda = \lambda_D} = \rho_1 \frac{n_{1F} - n_{1C}}{\lambda_F - \lambda_C} \]

\[ = \rho_1 \frac{n_{1F} - n_{1C}}{\lambda_F - \lambda_C} \frac{n_{1D}^{-1}}{n_{1D}^{-1}} \]

\[ = \rho_1 \frac{1}{\lambda_F - \lambda_C} \frac{1}{V_1} \frac{n_{1D}^{-1}}{1} \]

and using (2a)

\[ \rho_1 \frac{\text{dn}_1}{d\lambda} \bigg|_{\lambda = \lambda_D} = \frac{1}{\lambda_F - \lambda_C} \frac{1}{V_1} \frac{1}{f_{1D}} \]
So, the condition for achromaticity, expression (4), becomes
\[
\frac{1}{V_1 f_{1D}} + \frac{1}{V_2 f_{2D}} = 0
\]  
(5)

(For two given materials, their corresponding Abbe number values are tabulated.)

Expression (5), together with the designer’s requirement that the doublet has a desired focal length of \( f_{\text{desired}} = 15 \) cm at \( \lambda = \lambda_D \), where
\[
\frac{1}{f_{\text{desired}}} = \frac{1}{f_{1D}} + \frac{1}{f_{2D}}
\]  
(6)
constitute the couple of equations from which \( f_{1D} \) and \( f_{2D} \) have to be determined.

The solution to (5) and (6) is:
\[
\frac{1}{f_{1D}} = -\frac{V_1}{V_2 - V_1} \frac{1}{f_{\text{desired}}}
\]  
and
\[
\frac{1}{f_{2D}} = \frac{V_2}{V_2 - V_1} \frac{1}{f_{\text{desired}}}
\]  
(7)
DESIGN of an ACHROMAT DOUBLET of 15 cm focal length

DOUBLET
(1) Equiconvex 520/636 crown glass
(2) 617/366 flint glass

<table>
<thead>
<tr>
<th>Catalog code</th>
<th>V</th>
<th>n_C</th>
<th>n_D</th>
<th>n_F</th>
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<tbody>
<tr>
<td></td>
<td>\frac{n_D - 1}{10V}</td>
<td>\frac{n_D - 1}{n_F - n_C}</td>
<td>656.5 nm</td>
<td>587.6 nm</td>
</tr>
<tr>
<td>(1) Borosilicate crown</td>
<td>520/636</td>
<td>63.59</td>
<td>1.51764</td>
<td>1.52015</td>
</tr>
<tr>
<td>(2) Dense flint</td>
<td>617/366</td>
<td>36.60</td>
<td>1.61218</td>
<td>1.61715</td>
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<table>
<thead>
<tr>
<th>f_{\text{desired}}</th>
<th>V_1</th>
<th>V_2</th>
<th>\frac{1}{f_{1D}}</th>
<th>\frac{1}{f_{2D}}</th>
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</thead>
<tbody>
<tr>
<td>15 cm</td>
<td>-V_1</td>
<td>\frac{V_2}{V_2 - V_1}</td>
<td>-V_1</td>
<td>\frac{1}{V_2 - V_1 \frac{f_{\text{desired}}}{f_{1D}}}</td>
</tr>
<tr>
<td></td>
<td>2.356</td>
<td>-1.356</td>
<td>0.157 cm(^{-1})</td>
<td>-0.0904 cm(^{-1})</td>
</tr>
</tbody>
</table>

\[ \frac{1}{f_{1D}} = (n_{1D} - 1)\left(\frac{1}{r_{11}} - \frac{1}{r_{12}}\right) = 0.52015\left(\frac{1}{r_{11}} - \frac{1}{r_{12}}\right) \]
\[
\left( \frac{1}{r_{11}} - \frac{1}{r_{12}} \right) = \frac{0.157}{0.52015} \text{cm}^{-1} = 0.3019 \text{cm}^{-1};
\]

by choosing a equiconvex lens \( r_{12} = -r_{11} < 0 \) gives \( \frac{2}{r_{11}} = 0.3019 \text{cm}^{-1} \). Thus, \( r_{11} = 6.624 \text{ cm} \)

Similarly
\[
\frac{1}{f_{2D}} = (n_{2D} - 1) \left( \frac{1}{r_{21}} - \frac{1}{r_{22}} \right) = (0.61715) \left( \frac{1}{r_{21}} - \frac{1}{r_{22}} \right)
\]
\[
\left( \frac{1}{r_{21}} - \frac{1}{r_{22}} \right) = \frac{-0.0904}{0.61715} \text{cm}^{-1} = -0.14649 \text{ cm}^{-1};
\]

by construction \( r_{21} = r_{12} \) (cemented lenses), which gives
\[
\left( \frac{1}{r_{12}} - \frac{1}{r_{22}} \right) = -0.14649 \text{ cm}^{-1}. \text{ Since } r_{12} = -r_{11}, \text{ we obtain } \left( -\frac{1}{r_{11}} - \frac{1}{r_{22}} \right) = -0.14649 \text{ cm}^{-1}. \text{ Thus, } r_{22} = -224.21 \text{ cm}
\]

**Observation**

Although the condition \( \frac{d}{d\lambda} \left( \frac{1/f}{\text{doublet}} \right) = 0 \), at \( \lambda = \lambda_{D} \), is quite appealing to use it as a requirement for obtaining a achromatic doublet, such a condition may does not necessarily ensure that \( 1/f \) is independent of the wavelength. The graph below is an example of what may happen in some cases.
That is, $1/f$ may still be different for different colors, despite the fact that the derivative of $1/f$ is zero at $\lambda = \lambda_D$.

A stronger condition is to require that, for example, that the focal length of the doublet for the red (C) and blue (F) light (Fraunhofer lines) to be equal,

$$\frac{1}{f_{\text{doublet}, C}} = \frac{1}{f_{\text{doublet}, F}}$$  \hspace{1cm} \text{requirement for achromaticity}  \hspace{1cm} (8)

Using (1) and (2) we obtain,

$$\frac{1}{f_{\text{doublet}, C}} = (n_{1C} - 1) \, \rho_1 + (n_{2C} - 1) \, \rho_2$$

$$\frac{1}{f_{\text{doublet}, F}} = (n_{1F} - 1) \, \rho_1 + (n_{2F} - 1) \, \rho_2$$

So, the condition (8) is equivalent to

$$(n_{1F} - n_{1C}) \, \rho_1 + (n_{2F} - n_{2C}) \, \rho_2 = 0,$$

This expression is nothing but expression (4).
Indeed, the resemblance can be more direct if we divide by the difference $\lambda_F - \lambda_C$.

$$
\rho_1 \frac{n_{1F} - n_{1C}}{\lambda_F - \lambda_C} + (n_{2F} - n_{2C}) \rho_2 \frac{n_{2F} - n_{2C}}{\lambda_F - \lambda_C} = 0
$$

where the fractions are the approximate values of the derivative of $n$ with respect to $\lambda$.

**SUMMARY**

The procedure above ensures that the achromat doublet will have equal focal length for at least the red and blue colors.
CLASIFICATION of OBJECTIVE LENSES

1) Achromats. 2) Fluorites. 3) Apochromats.

Added **Plan** designation to lenses with wide flat field (low curvature of field and distortion).

<table>
<thead>
<tr>
<th>Type</th>
<th>Spherical</th>
<th>Chromatic</th>
<th>Flatness correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Achromat</td>
<td>*</td>
<td>2λ</td>
<td>No</td>
</tr>
<tr>
<td>F-Achromat</td>
<td>*</td>
<td>2λ</td>
<td>Improved</td>
</tr>
<tr>
<td>Neofluar</td>
<td>3λ</td>
<td>&lt;3λ</td>
<td>No</td>
</tr>
<tr>
<td>Plan Neofluar</td>
<td>3λ</td>
<td>&lt;3λ</td>
<td>Yes</td>
</tr>
<tr>
<td>Plan Apochromat</td>
<td>4λ</td>
<td>&gt;4λ</td>
<td>Yes</td>
</tr>
</tbody>
</table>

* Corrected for two wavelengths
2λ Corrected for **blue** and **red**
3λ Corrected for **blue, green** and **red**
4λ Corrected for **dark blue, blue, green** and **red**

**Fluorites objectives.**

- Considerably **better** corrected than **achromats**, but not quite as well corrected as the apochromats.

- Uses **fluorite crystals** (which have lower dispersion) in place of some of the glass elements.

- Correct for spherical aberrations in three wavelengths at considerably **lower cost** than the apochromats.
Apochromatic objectives.

- Provides the same focal length for three wavelengths, and free of spherical aberration for two wavelengths.
- Magnification still vary with wavelength (a compensating eyepiece is used to cancel the colored fringes).

60x Plan Apochromat Objective

http://www.microscopyu.com/articles/optics/objectivespecs.html