Two-Sided Learning in a Natural Rate Model

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Abstract

Cho, Williams and Sargent (2002) consider a natural rate model in which the central bank has imperfect control over inflation and is uncertain of the actual laws of motion of the economy. They show that if the central bank uses a misspecified approximating model to determine inflation there can be endogenous cycling (escape dynamics) between the time-consistent Nash equilibrium outcome and the optimal Ramsey outcome of Kydland and Prescott (1977). They obtain these escape dynamics assuming the central bank and the private sector have the same information and beliefs about the economy. In this paper we assume these two actors have different beliefs about the structure of the economy. The central bank and the private sector learn the economy with their own models separately. If the private sector learns the economy with a fully specified model instead of having rational expectations, escapes disappear and the economy converges to the Nash outcome. With a reverse robustness check we find that escapes can reappear if the private sector uses a misspecified model and the central bank uses a fully specified model. Thus escapes can arise in a model where the central bank is better informed than the private sector. Moreover under certain conditions the difference in beliefs in a two-sided learning model allows the central bank to exploit the expectations of the private sector to achieve an inflation rate lower than the Nash equilibrium outcome level of inflation.

Keywords: Central bank policies, Two sided learning, Commitment, Discretion, Escape dynamics, Inflation, Unemployment

JEL Clasification: E52, E58

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1 Introduction

Differences in people's perceptions play an important role in economics. Whenever we assume multiple agents, the possibility for disagreement in beliefs opens up the possibility of exploiting these differences. For example, agents with different views about the structure of the economy may derive different decision rules. Or agents may have different beliefs about the commitment technology of the government. In this paper, we study the effect of these differences in a natural rate model where the beliefs of the private sector affect the ability of the central bank to achieve its goals.

Kydland and Prescott (1977) use a natural rate model to argue that, if at each time policymakers select the best action given the current situation, the social objective function will typically not be maximized. Rather, they suggested that, economic performance can be improved by committing ahead of time to policy rules. The current decisions of economic agents depend on their expectations of future policy actions. If agents are rational and have the same information as policy makers, they can infer the actions the government will take. The resulting game dynamics can lead to suboptimal behavior that would not occur if the government sets its future policy independent of what other agents do in the meantime. The optimal policy maximizes the social objective function but it is not consistent due to the rationality of the agents. Specifically, Kydland and Prescott (1977) show that doing what is best given the current situation – i.e. a discretionary or time-consistent policy – results in an excessive level of inflation without any improvement in unemployment.

Sargent (1999) studied the post World War II American inflation under the assumption that policy makers learned to believe in natural unemployment rate hypotheses during this period. He relaxed the rational expectations hypothesis for the policy maker –but not the private sector– and assumed adaptive learning behavior in its place. This model exhibits recurrent escapes from the time-consistent outcome to the optimal outcome of Kydland and Prescott (1977). Cho, Williams and Sargent (2002) showed that the escapes from the time-consistent outcome occur via accidental experimentation induced by the government's adaptive algorithm and its misspecified model.

Assuming the private sector has rational expectations reduces the analysis to a single-agent decision problem. Barro and Gordon (1983) argued that this approach cannot deal with the game-theoretic situation that arises when decisions are made on an ongoing basis. Pursuing this idea, in this paper, neither the central bank nor the private sector know the true model but instead build independent approximating models that incorporate separate beliefs about how the economy works. Thus the model involves a dual-agent decision problem.

On one side, the central bank constructs a model with its own beliefs and commitment technology. It derives a policy rule as a function of its current information set. On the other side, the private sector constructs another model with the goal of predicting the policy of the central bank. Its expectation of the central bank's policy will be a function of its own current information set.

In this paper the central bank chooses the rate of price inflation and the private sector determines the rate of wage inflation (or the expected inflation rate) in a dynamic natural rate model. The two players can have different specifications for the laws of motion of the economy. They may also have different beliefs/knowledge about the commitment technology of the central bank. They update their information set every period as new data is generated.

Our results are as follows: 1) When the private sector learns the economy with a correctly specified model rather than having rational expectations, we observe the disappearance of the escapes of Cho, Williams and Sargent (2002) and convergence to the Nash equilibrium. The additional distortion from the learning model of the private sector makes it more difficult for an unusual sequence of shocks to deceive the central bank.

2) In a reverse robustness check we let the private sector have a misspecified approximating model while the central bank has a correctly specified approximating model. We observe escapes but this time the source of the fluctuations is the private sector rather than the central bank. This establishes that escapes can occur in a more plausible environment where the central bank is better informed than the private sector.

3) We observe that in some scenarios a difference in beliefs between the central bank and the private sector allows the central bank to exploit the private sector and achieve inflation lower than the Nash level. With this result we can explain the efforts of central banks to influence the private sector's expectations through announcements, release of more frequent policy forecasts, and fuller statements explaining interest-rate policy.

The structure of the paper is as follows: In sections 2 and 3 the model and the learning algorithm are introduced. In section 4 we review what happens when the private sector is rational as in Cho, Williams and Sargent (2002). We also show the convergence of the inflation rate to the central bank target when the central bank correctly specifies the economy. In section 5 we analyze what happens under different scenarios of two-sided learning. Finally in section 6 we talk about possibilities for further research.

2 The Model

The model we develop is a general model that encompasses the properties of the model used by Kydland and Prescott (1977) and Cho, Williams and Sargent (2002). The model describes the behavior of a central bank, which imprecisely chooses the rate of price inflation π_t, and the private sector, whose actions imprecisely determine the rate of wage inflation w_t. The private sector sets the rate of wage inflation aiming to set to equal to π_t. Thus w_t can also be viewed as the private sector's expectation of inflation.

The expectational Phillips curve determines the unemployment rate:

$$U_t = U^n - (\pi_t - w_t) + v_{1t}, \tag{1}$$

where

$$\pi_t = \mu_t + v_{2t}. \tag{2}$$

and

$$w_t = q_t + v_{3t}. \tag{3}$$

Here U^n is the constant natural rate of unemployment, U_t is the unemployment rate, μ_t is the central bank determined inflation rate or money growth rate, q_t is the private sector determined rate of wage inflation before its noise, and v_{1t}, v_{2t} and v_{3t} are normally distributed independent noises. The unemployment rate U_t is a convenient proxy for real activity in the economy. The slope of the Phillips curve is taken to be unity for convenience. Using another constant value will not change our results. In this model, surprise inflation lowers the unemployment rate but anticipated inflation does not. Equation (2) states that the central bank controls the money supply with some noise just as equation (3) states that the private sector determines the rate of wage inflation with some noise. The optimal choices of μ_t and q_t are explained in detail below.

The central bank's objective is summarized by the single-period return or payoff function, $Z_{cb,t}$, which depends on that period's values for the unemployment rate and inflation. Following the literature we assume a simple quadratic form:

$$Z_{cb,t} = -E\left\{\frac{1}{2}\pi_t^2 + \frac{b}{2}\left(U_t - (U^n - \alpha)\right)^2\right\} \tag{4}$$

The first term in this objective function captures the cost of inflation or, more precisely, penalizes deviations of the inflation rate π_t from the central bank's target of zero. Direct costs of changing prices would be a simple explanation for why inflation is costly. The nonnegative constant b is the weight that the central bank places on achieving its goal for unemployment, relative to its goal for inflation. The second term is the deviation from the targeted unemployment rate, which is α less than the natural unemployment rate, where α is a nonnegative constant. The natural rate of unemployment will tend to exceed the efficient level of unemployment in the presence of unemployment compensation and income taxation. The constant α captures this possibility. The central bank maximizes the single-period objective function (4) by choosing an inflation rate μ_t. The constraints on this maximization are explained below. The objective of the private sector is to maximize

$$Z_{ps,t} = -E\left\{\frac{1}{2}(\pi_t - w_t)^2\right\} \tag{5}$$

The private sector wants to set the wage inflation as close as possible to the central bank determined inflation rate.

The determination of the unemployment rate can be characterized as a game between the central bank and the private-sector. At period t, the central bank sets the inflation rate, μ_t, with the information set I_{t-1} and the belief set $\mathcal{B}_{cb}$. Private-sector agents set the wage inflation, q_t, with the same information set I_{t-1}, but with their own belief set $\mathcal{B}_{ps}$. We will define the belief sets $\mathcal{B}_{cb}$ and $\mathcal{B}_{ps}$ and the information set I_{t-1} below. We will consider cases where they choose their variables at the same time or sequentially with the private sector going first. The timing of decisions plays an important role and will be explained in detail below. It is also important to note that the belief sets of the agents are not time-dependent.

It should be stressed that in forming inflationary expectations, the private-sector knows that the choice of μ_t will emerge from the central bank's maximization function given in equation (4). After the random disturbances $v_t = (v_{1t}, v_{2t}, v_{3t})$ are realized, equations (1) - (3) determine the unemployment rate.

Information and Belief Sets

The information set I_t includes all the data available up to and including time t. The data consists of all past values of the unemployment rate, inflation rate and wage inflation. The information set, I_t, is available both to the central bank and the private sector. Moreover the central bank and the private sector may have different beliefs about how the economy works. Each will learn the economy separately with its own approximating model based on its beliefs about the structure of the economy. We will talk more about the differences in the approximating models in the next section.

We also allow the two players to have different beliefs about the commitment technology of the central bank. Commitment technology is the ability of the central bank to credibly commit to a policy choice even if the optimal choice might be different in the following periods. Without the commitment technology the central bank makes policy under discretion. Thus the belief sets are defined as

$$\mathcal{B}_{cb}, \mathcal{B}_{ps} = \{structure\ of\ the\ economy, commitment\ technology\ of\ the\ central\ bank\}$$

Note that the central bank knows correctly and with certainty what its commitment technology, but the private sector may be misinformed about this.

Assuming that the central bank and the private sector have the same belief set means they believe in the same structure of the economy and the private sector knows the commitment technology of the central bank. For this section we assume that the central bank and the private-sector agents have the same belief sets, $\mathcal{B}_{cb} \equiv \mathcal{B}_{ps}$. This assumption makes it possible to assume rational expectations for the private sector. Later in the paper in section 5 we will look for the implications of having different belief sets.

Expectation Formation

In the formation of expectations, q_t, private-sector agents consider the central bank's maximization problem, which determines the choice of μ_t. Suppose that, given it's belief set, the private sector perceives this process as described by a strategy function, $F^e_{ps}(I_{t-1} \mid \mathcal{B}_{ps})$. Therefore inflationary expectations are given by

$$q_t = F^e_{ps}(I_{t-1} \mid \mathcal{B}_{ps}) \tag{6}$$

We also assume that the central bank understands that q_t is generated from equation (6).

Solutions to the Model

Substituting (2), (3) and (6) into (1) yields

$$U_t = U^n - (\mu_t - F^e_{ps}(I_{t-1} \mid \mathcal{B}_{ps}) + v_{2t} - v_{3t}) + v_{1t} \tag{7}$$

Assuming that the policymaker knows the true model, he selects μ_t that maximizes (4) with respect to the constraints, including equation (7). There are two possible timing protocols we could use, depending on the central bank's commitment technology. If the central bank cannot commit to a policy, it effectively makes its choice of μ_t after the private sector has embedded its expectations into a particular choice of q_t. Thus the central bank can take q_t as given, and maximize its objective function accordingly. Given its beliefs, the non-committed central bank has a strategy function that depends on its information set and q_t:

$$\mu_t^{nc} = F_{cb}^{nc}(I_{t-1}, q_t \mid \mathcal{B}_{cb})$$

In the second case, the central bank can does commit to a particular policy before the private sector institutionalizes its expectations. In this case, given its beliefs, the committed central bank has a strategy function that depends only on its information set, $\mu_t^c = F_{cb}^c(I_{t-1} \mid \mathcal{B}_{cb})$. In the following two definitions these policies are derived.

Definition 1 *Assume that the central bank is ether unwilling or unable to precommit to a policy and selects its policy choice μ_t after observing the private sector's expectations, q_t, given in (6). The solution to the problem*

$$\max_{\mu_t} Z_{cb,t} \text{ subject to (7)}$$

is called the Nash outcome since the solution is the best response to private sector expectations. Following the literature we also call this the policy of a non-committed central bank.

The strategy function of the non-committed central bank is

$$\mu_t^{nc} = F_{cb}^{nc}(I_{t-1}, q_t \mid \mathcal{B}_{cb}) = \frac{b}{1+b}\left(F_{ps}^e(I_{t-1} \mid \mathcal{B}_{ps}) + \alpha\right) \tag{8}$$

The property $E(v_t \mid I_{t-1}) = 0$ has been used in the computation of the strategy function. A private sector with the same information and belief sets with the central bank, $\mathcal{B}_{ps} \equiv \mathcal{B}_{cb}$, understands the optimization problem of the policymaker. In particular the private sector understands that the actual choice, μ_t^{nc} satisfies equation (8). Solving its maximization problem given in (5) and using equation (8), the private sector calculates $F_{ps}^e(I_{t-1} \mid \mathcal{B}_{ps})$ in equation (6). The private sector sets $F_{ps}^e(I_{t-1} \mid \mathcal{B}_{ps}) = \mu_t^{nc}$ which leads to the policy

$$\mu_t^{nc} = b\alpha$$

A non-committed central bank will be tempted to exploit the expectational Phillips curve in an effort to achieve its goal of pushing unemployment below the natural rate. The private sector understands the incentives of the central bank and knows the central bank faces this temptation to inflate. The private sector, therefore, builds these inflationary expectations into its wage-setting decisions so that unemployment remains at its natural rate.

Alternatively, we can assume the central bank is able to precommit to a choice for μ_t before the private sector embeds its expectations into a particular choice of q_t. This policy can be viewed as a once-and-for-all choice of a policy rule. The central bank will then view the condition $F^e_{ps}(I_{t-1} \mid \mathcal{B}_{ps}) = \mu_t$ as a constraint that links its choice of μ_t to a subsequent choice for q_t.

Definition 2 *Assume that the central bank can precommit to a choice for μ before the private sector embeds its expectations into a particular choice of q. Its problem is then*

$$\max_{\mu_t} Z_{cb,t} \text{ subject to } q_t = \mu_t, \text{ (7)}$$

and the solution thereof is called the Ramsey outcome. Following the literature we also call this the policy of a committed central bank.

The optimal monetary policy with commitment is

$$\mu^c_t = F^c_{cb}(I_{t-1} \mid \mathcal{B}_{cb}) = 0$$

When the central bank precommits to a choice for μ_t, it recognizes that it will lose ability it might otherwise have to surprise private-sector agents and thereby exploit the Phillips curve. Hence, under commitment, the central bank abandons any idea of pushing unemployment below the natural rate and, instead, focuses exclusively on achieving its goal of zero inflation.

A Ramsey outcome dominates a Nash outcome. Efforts to exploit the Phillips curve can lead only to a suboptimally high rate of inflation, $\mu^{nc}_t = b\alpha$, with no decrease in the unemployment rate.

3 Learning Dynamics

Now we assume that the central bank does not know (1) but believes that unemployment follows the process

$$U_t = \gamma_t z_t + \varepsilon_t$$

where γ is a vector of coefficients, z is a vector of regressors, and ε_t is a random variable orthogonal to z_t. The set of regressors will vary with the model that the central bank estimates. We assume two possible approximating models, a fully specified model;

$$U_t = \gamma_0 + \gamma_1 \pi_t + \gamma_2 w_t + \varepsilon_t \tag{9}$$

and a misspecified model;

$$U_t = \gamma_0 + \gamma_1 \pi_t + \varepsilon_t \tag{10}$$

Depending on their beliefs, the central bank and the private sector use either (9) or (10) to derive their policies. The second approximating model (10) is what Cho, Williams and Sargent (2002) used to explain the fluctuations in the US inflation rate. The omission of the private sector's expectation leads to a misperception of the shocks, which later leads to transitions between the Nash and Ramsey outcomes.

We suppose the central bank estimates γ by least squares regression of U on z in past data. Each period, the central bank updates its estimate of γ with the latest data and solves its optimization problem with the updated γ. In the standard least squares regression formula, the value of the coefficient vector γ is estimated by the formula

$$\gamma = \left(\sum_{1}^{T} z z' \right)^{-1} \left(\sum_{1}^{T} z U \right) \tag{11}$$

after T observations. This treats all data equally. More generally, γ can instead be computed using the formulas

$$\gamma_{t+1} = \gamma_t + a_t R_t^{-1} z_t \left(U_t - \gamma_t z_t \right) \tag{12}$$

$$R_{t+1} = R_t + a_t (z_t z_t' - R_t), \tag{13}$$

where a_t is a sequence of positive real numbers and R_t is an estimate of the moment matrix of z_t. Setting $a_t = 1/t$ gives back the standard least squares learning algorithm. Throughout this paper we will instead set $a_t = a$, employing what is known as a constant gain learning algorithm, which puts more weight on the recent observation and less weight on past observations. Constant gain learning is necassary to obtain the endogenous transitions between the Nash and Ramsey outcomes reported in Cho, Williams and Sargent (2002) and in section (4.1) of this paper. One justification for this constant gain algorithm is to formalize perpetual learning which is what we observe from policymakers.

With a constant gain algorithm the distribution of γ_t will not converge to a degenerate distribution since γ_t is nonnegligibly sensitive to random shocks even asymptotically. However, γ_t may converge to a limiting probability distribution. In the limit of small a, we can derive the limiting distribution.

4 Learning with the Same Belief Sets

First we assume that the central bank and the private sector have the same belief sets, $\mathcal{B}_{ps} \equiv \mathcal{B}_{cb}$. Later in the paper we assume the case where they have the same information set but different belief sets. The private sector wishes to forecast the decisions of the central bank. If they have the same information and belief sets they should find the same optimal behavior for the central bank. Depending on the shared belief set, there are three possible cases. In the first case, studied in section 4.1, the central bank misspecifies the economy, using the approximating model (10). In this approximating model the central bank ignores the expectations of the private sector. This will be very similar to what Cho, Williams and Sargent (2002) studied. Second, the central bank correctly incorporates the expectations of the private sector and uses (9) as its approximating model. In this case there are two possibilities. The central bank may move first and commit to a policy, section 4.2. Or the central bank may move after the private sector forms its expectations, section 4.3. This is the case where the central bank has no commitment technology and it is willing to exploit the expectations of the private sector.

The convergence analysis of least square learning depends on results from stochastic approximation theory. We will analyze the limiting behavior of the associated differential equations of the stochastic system. Similar work is done by Marcet and Sargent (1989) and Woodford (1990). Further details of the convergence results of each of the following sections are given in Appendix A.

4.1 Misspecified Central Bank Policy Rule

This section is a reproduction of Cho, Williams and Sargent (2002) with some minor differences. Their model has an unemployment target of 0 and it has equal weight on inflation and unemployment target in the objective function. But even with these minor differences the two models produce the same outcomes. Assume that the central bank does not know (1) but uses its own misspecified model

$$U_t = \gamma_0 + \gamma_1 \pi_t + \varepsilon_t \tag{14}$$

The commitment technology of the central bank is irrelevant since the central bank does not think the private sector matters. The central bank maximizes (4) with respect to (14) and (2). The resulting policy is

$$\mu_t = \frac{-b\gamma_1 (\gamma_0 - U^n + \alpha)}{1 + b\gamma_1^2} \tag{15}$$

With the misspecified model (14) the effects of expected inflation w_t are absorbed into the constant γ_0. Since μ_t and q_t are constant at the Nash equilibrium, the failure to include w_t as a regressor costs the central bank nothing in terms of statistical fit.

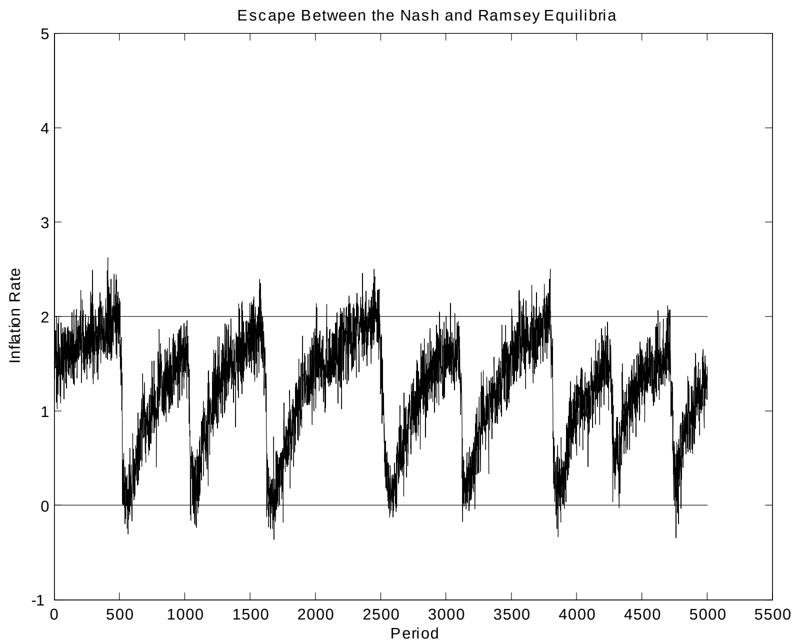

Figure 1: $U^n = 4, \alpha = 2, b = 1$. Nash Equilibrium is 2, Ramsey is 0.

With a misspecified learning model the inflation rate makes recurrent cycles between the time-consistent Nash outcome and the time-inconsistent Ramsey outcome. Figure 1 shows a simulation of the system. In this model the central bank fails to include the private sector's expectation into its regression equation, the misspecification. Referring to Cho, Williams and Sargent (2002) we call the endogenous movement of the inflation rate to the Ramsey outcome an escape. Escapes occur when the algorithm is driven by an unusual sequence of random shocks. By these particular unusual sequence of random variables, γ_1 in (15) increases. This steepens the estimated Phillips curve which leads the central bank to lower the inflation rate. Discounting past observations helps this process along. But the system cannot remain at the Ramsey outcome indefinitely since the Ramsey outcome is not a Nash equilibrium. Eventually the system will be drown back to the Nash equilibrium outcome.

4.2 A Committed Central Bank Learning the Economy with the Fully Specified Model

Learning with misspecified dynamics leads to escapes between the Nash outcome and the Ramsey outcome. It is of interest to see if the results change if the central bank considers the expectations of the private sector as a determinant of the unemployment rate. First let us suppose the central bank is committed. This adds one more condition to the maximization problem of the central bank: $q_t = \mu_t$. The central bank will maximize (4) with respect to (9), (2), (3) and $q_t = \mu_t$. The resulting policy rule is

$$\mu_t = \frac{-b\left(\gamma_1 + \gamma_2\right)\left(\gamma_0 - U^n + \alpha\right)}{1 + b\left(\gamma_1 + \gamma_2\right)^2} \tag{16}$$

Proposition 1 *When the central bank moves before the private sector and commits to a policy, the inflation rate, π_t, converges to a limiting probability distribution, a normal distribution with mean value equal to the Ramsey outcome.*

For the proof of this proposition refer to Appendix A. Figure 2 is a simulation of this economy.

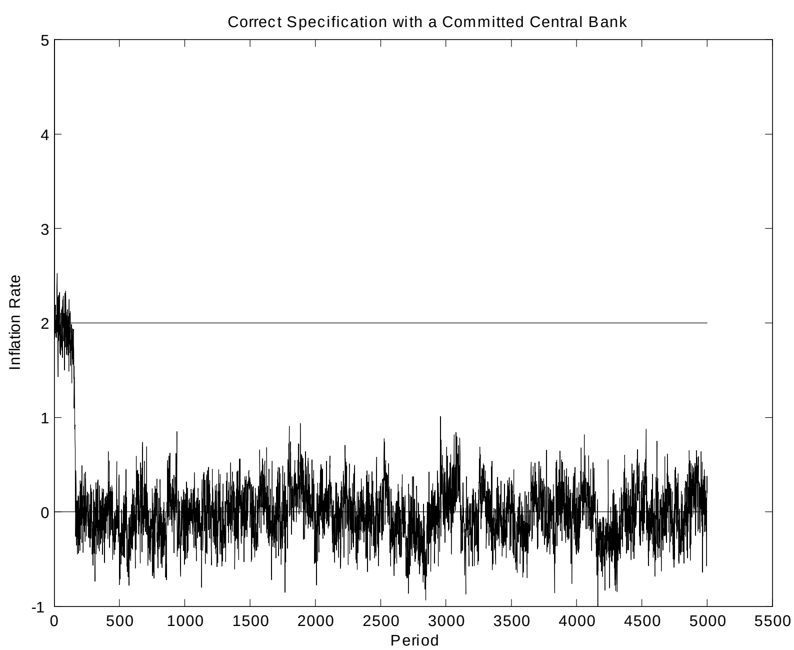

Figure 2: $U^n = 4, \alpha = 2, b = 1$. Nash equilibrium is 2, Ramsey is 0

This means that when the central bank plays the Ramsey plan every period, the inflation rate stays at the Ramsey equilibrium outcome even if this is not a Nash equilibrium outcome.

The associated differential equation, derived in Appendix A, has a unique steady state with, $\gamma = \begin{pmatrix} U^n & -1 & 1 \end{pmatrix}$. When substituted into the central bank's policy function we get the Ramsey equilibrium outcome, $\mu_t = 0$.

4.3 A Non-Committed Central Bank Learning the Economy with the Fully Specified Model

In the previous section, the central bank moves first and commits to a policy. What if the central bank moves second? First the private sector forms its expectation, w_t, about the central bank's policy. Then the central bank chooses its policy, the targeted inflation rate μ_t. Assume the central bank does not know (1) but uses its own model

$$U_t = \gamma_0 + \gamma_1 \pi_t + \gamma_2 w_t + \varepsilon_t \tag{17}$$

We call this model the fully specified model since the expectation of the private sector is not omitted. The central bank will maximize (4) with respect to (17), (2) and (3). The resulting policy rule is

$$\mu_t = \frac{-b\gamma_1(\gamma_0 - U^n + \alpha) - b\gamma_1\gamma_2 q_t}{1 + b\gamma_1^2} \tag{18}$$

Since the private sector can forecast this decision, it will set $q_t = \mu_t$. Then the policy rule of the central bank reduces to

$$\mu_t = \frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1 + b\gamma_1^2 + b\gamma_1\gamma_2} \tag{19}$$

Proposition 2 *When the central bank moves after observing the expectations of the private sector, the inflation rate, π_t, converges to a limiting probability distribution, a normal distribution with mean value equal to the Nash equilibrium inflation rate.*

Figure 3 is a simulation of the economy. The associated differential equation of this system has a unique steady state with $\gamma = \begin{pmatrix} U^n & -1 & 1 \end{pmatrix}$. When substituted into the central bank's policy function (19) we get the Nash equilibrium value, $\mu_t = \alpha b$. So at the equilibrium the central bank is playing the Nash equilibrium, the value that the inflation rate is converging. This shows us the Nash equilibrium is learnable if both agents believe the correct model specification.

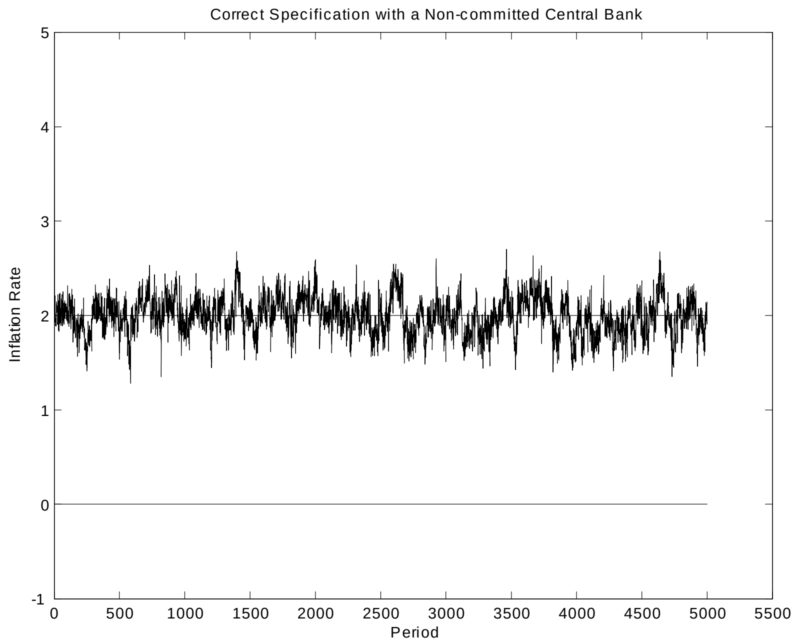

Figure 3: $U^n = 4, \alpha = 2, b = 1$. Nash Equilibrium is 2, Ramsey is 0.

5 Two-sided Learning

In section 4 we assumed that the central bank and the private sector have the same belief sets, $\mathcal{B}_{ps} \equiv \mathcal{B}_{cb}$. Therefore the problem was reduced to a single agent problem. Having the same information and belief sets, the private sector is able to correctly predict, up to a noise, what inflation will be. But we know this is not always the case. In reality the central bank and the private sector may often have different views about how the economy works. In this section we assume the central bank and the private sector have different belief sets.

5.1 A Robustness Check for Endogenous Fluctuations

In Cho, Williams and Sargent (2002), and in section 4.1 of this paper, the central bank learns the economy with a misspecified model, while the private sector has rational expectations. In this section we assume the private sector learns the economy with a fully specified model. It also correctly believes that the central bank is non-committed. The private sector believes the structure of the economy is described by

$$U_t = \eta_0 + \eta_1 \pi_t + \eta_2 w_t + \varepsilon_t \tag{20}$$

The vector of coefficients η will be used for the private sector while γ will continue to denote the vector of coefficients for the central bank. The problem the private sector thinks the central bank is solving is

$$\max_{\mu} Z_{cb,t} \text{ subject to (20), (2) and (3)}$$

The policy function the private sector forecasts is

$$F^e_{ps}(I_{t-1} \mid \mathcal{B}_{ps}) = \frac{-b\eta_1(\eta_0 - U^n + \alpha) - b\eta_1\eta_2 q_t}{1 + b\eta_1^2} \tag{21}$$

Given the information set and the belief set, $F^e_{ps}(I_{t-1} \mid \mathcal{B}_{ps})$ is what the private sector thinks the central bank's policy is. So the private sector will set wage inflation to

$$q_t = F^e_{ps}(I_{t-1} \mid \mathcal{B}_{ps}) = \frac{-b\eta_1(\eta_0 - U^n + \alpha)}{1 + b\eta_1^2 + b\eta_1\eta_2}$$

The central bank is the same central bank of section 4.1. The central bank uses the misspecified model, $U_t = \gamma_0 + \gamma_1\pi_t + \varepsilon_t$. The policy function of the central bank is given in (15).

Proposition 3 *When the central bank misspecifies the economy where as the private sector learns with the fully specified model assuming a non-committed central bank, the inflation rate, π_t, converges to a limiting probability distribution which is normal with mean equal to the Nash equilibrium value.*

A simulation of this economy is given in figure 4, and proof of this proposition is in Appendix A. When the private sector learns the economy with a correctly specified approximating model while the central bank has a misspecified model, we observe the convergence of the expectations of the private sector to the Nash equilibrium outcome and the inflation rate converges to the same mean also. This result is interesting in the sense that the escapes between the Nash and Ramsey outcomes of Cho, Williams and Sargent (2002) disappear. If, instead of assuming rational expectations for the private sector, we equip the private sector with a fully specified approximating model it can prevent the central bank from misinterpreting random shocks.

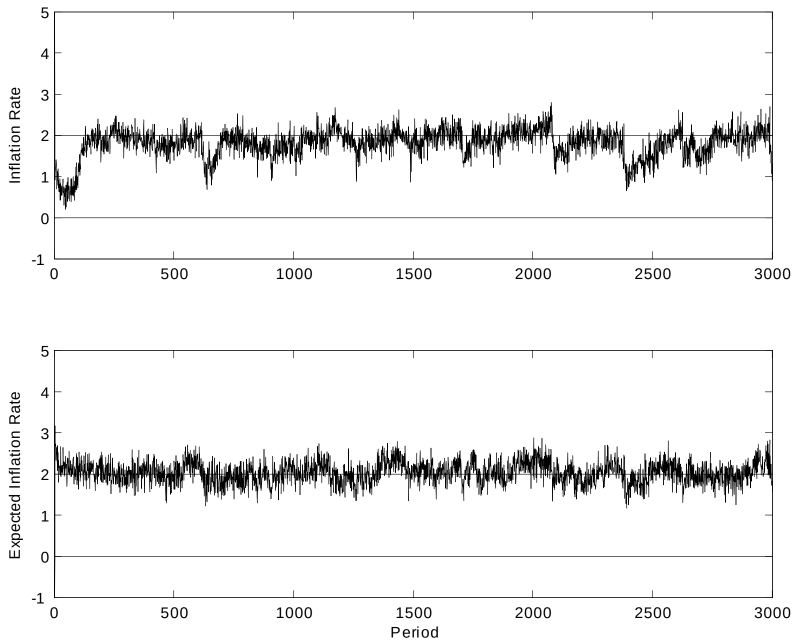

Figure 4: $U^n = 4, \alpha = 2, b = 1$. Nash equilibrium is 2, Ramsey is 0

5.2 Reverse Robustness Check

In the previous section we observed the disappearance of the endogenous fluctuations with a learning private sector even if the private sector learns the rational expectations policy. We would like to test the robustness of this result by considering the reverse case. Now a non-committed central bank learns the economy with a fully specified approximating model and the private sector learns the economy with a misspecified approximating model. This is a more plausible case since the central bank should be better informed than the private sector. The central bank's policy function is given in (18). Given the information set and the beliefs of the private sector, the policy function of the private sector will be similar to (15) but expressed in terms of the coefficient vector η:

$$q_t = F^e_{ps}(I_{t-1} \mid \mathcal{B}_{ps}) = \frac{-b\eta_1 \left(\eta_0 - U^n + \alpha\right)}{1 + b\eta_1^2} \tag{22}$$

Observing the wage inflation rate the central bank determines the inflation rate, up to a noise, using the policy function (18). The following proposition outlines the what happens.

When a non-committed central bank learns the fully specified model where as the private

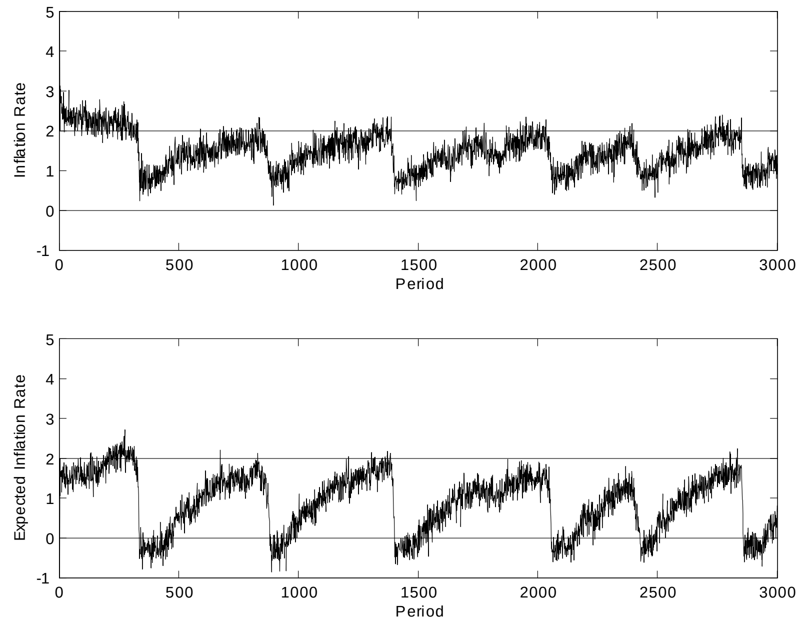

Figure 5: $U^n = 4, \alpha = 2, b = 1$. Nash equilibrium is 2, Ramsey is 0

sector learns the misspecified model, the inflation rate, π_t, endogenously fluctuates with sudden escapes from the Nash equilibrium outcome. A simulation of this economy is given in figure 5. As can be seen in this figure, the inflation rate fluctuates together with wage inflation. This leads to endogenous fluctuations as seen in section 4.1. Since the central bank follows the private sector in its policy we observe similar fluctuations in the central bank determined inflation rate. This provides an alternative explanation for fluctuations in the inflation rate where this time the private sector is the cause of the fluctuations. But fluctuations in the inflation rate are not as wide spread as they are for wage inflation rate. Since the central bank has the ability to exploit the expectations of the private sector it can achieve better results.

Comparing with the previous case we observe the endogenous fluctuations in a different environment. This is a reproduction of the escapes in a setup where the private sector does not have rational expectations and the central bank is better informed than the private sector. In this scenario the cause of the escapes of Cho, Williams and Sargent (2002) is the private sector rather than the central bank. A central bank without a commitment to a particular policy choice determines an inflation rate that fluctuates with the expectations of the private sector.

5.3 Exploiting the Difference in Beliefs

In a two-sided learning environment we allow the central bank and the private sector to have different beliefs about the economy. Assuming a difference in beliefs opens up the possibility of exploiting these differences. In a natural rate model, if the central bank can keep the beliefs of the private sector lower than its actual policy, it may take advantage of this difference to achieve a lower than Nash equilibrium outcome level of inflation. Assume the private sector thinks the central bank is committed to a policy using a fully specified model. The private sector solves the maximization problem of the central bank

$$\max_{\mu_t} Z_{cb,t} \text{ subject to } q_t = \mu_t,\, U_t = \eta_0 + \eta_1 \pi_t + \eta_2 w_t + \varepsilon_t,\, (2) \text{ and } (3)$$

The policy function of the private sector is

$$q_t = F^e_{ps}(I_{t-1} \mid \mathcal{B}_{ps}) = \frac{-b\,(\eta_1 + \eta_2)\,(\eta_0 - U^n + \alpha)}{1 + b\,(\eta_1 + \eta_2)^2}, \tag{23}$$

The private sector forms its expectations before the central bank determines the inflation rate. Observing the expected wage inflation, the non-committed central bank determines the inflation rate using the policy function (18). The following proposition outlines the case.

Proposition 4 *When the central bank is not committed to a policy where the private sector learns the economy assuming a committed central bank the inflation rate, π_t, converges to a limiting probability distribution which is normal with mean equal to a restricted perceptions equilibrium.*

A simulation of this economy is given in figure 6. The expectations of the private sector converge to the Ramsey equilibrium outcome where as the actual inflation rate converges to higher value. This is a restricted-perceptions equilibrium, an equilibrium that arises from the beliefs of agents rather than from the fundamentals of the model. This is an interesting result since if the difference in beliefs is maintained, the central bank attains a better result than the Nash equilibrium outcome, with an inflation rate between the Nash and the Ramsey outcomes.

It is well known that central banks make announcements to influence the private sector. The private sector pays attention to these announcements and they definitely have an important role in the formation of it expectations. According to rational expectations theory any kind of attempt to manipulate expectations should not work and the private sector should correctly predict the central bank determined inflation rate. But in reality the central bank

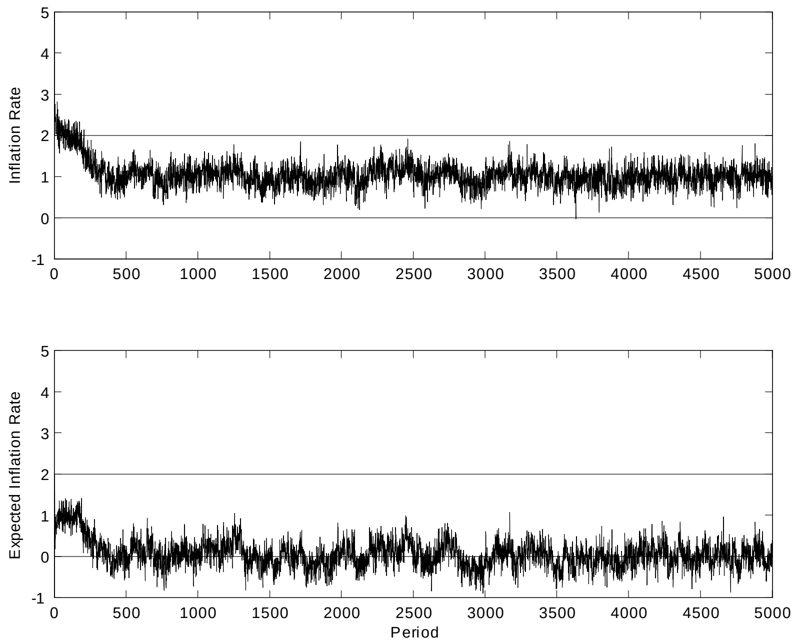

Figure 6: $U^n = 4, \alpha = 2, b = 1$. Nash equilibrium is 2, Ramsey is 0

does try to influence the beliefs of the private sector by making announcements, publishing more frequent policy forecasts and fuller statements explaining interest-rate policy are some examples to explain how the central bank is trying to influence the beliefs of the private sector to attain an advantage in determining the inflation rate. In this section we have seen these actions might work to the advantage of the central bank.

6 Conclusion

Expectations play an important role in the realization of the inflation rate, but different perceptions of the world lead to different expectations and policies. Whenever we consider models with multiple agents it is important to consider the implications of such differences. In this paper we test some previous results in the learning literature in a two-sided learning environment where two agents construct models and decision rules independently.

In his famous book "Conquest of American Inflation" Thomas Sargent analyzes the rise and fall of U.S. inflation after 1960. According to Sargent (1999) the role of expectations in economics was not well established before the 1970's. Policymakers of the time adopted methods derived from exploitation of the Phillips curve in the hope of lowering the inflation rate. As they learned from new data, they re-estimated their Phillips curve and adjusted their target inflation rate accordingly. But since they ignored the role of inflation expectations in the Phillips curve, fluctuations in the inflation rate resulted. First we show that with the inclusion of the expectations in a one-sided learning model the policymaker can achieve the results it targets.

In the second part of the paper we analyze the case where the central bank and the private sector have different views of the economy and they learn the economy with their own models. This allows us to test the robustness of the escapes of Cho, Williams and Sargent (2002) to two-sided learning. Our results show that the endogenous fluctuations of Cho, Williams and Sargent (2002) are not robust to a learning private sector. Even if the private sector learns the policy of the central bank we observe the disappearance of the fluctuations. But we show that it is possible to reproduce these endogenous fluctuations in a more plausible environment where the central bank uses a fully specified model and the private sector uses a misspecified model, so the central bank is better informed than the private sector. In this case the expectations of the private sector fluctuate, causing the inflation rate to fluctuate with it.

In another two-sided learning environment, the actual inflation rate and the expectations of the private sector converge to different values. Given its beliefs, the private sector is not capable of learning the policy of the central bank. The private sector updates its data set every period but this updating does not allow it to change its model specification. This is a weakness of the Evans-Honkapohja-Sargent learning mechanism. Since the unemployment rate decreases as much as the decrease in the expected rate of inflation, the regression coefficients do not respond to the divergence of the actual inflation rate from the expected inflation rate. The steady state that the inflation rate converges to is a restricted perceptions equilibrium, an equilibrium that arises from the beliefs of agents rather than from the fundamentals of the model. The existence of such a difference in beliefs lets the central bank achieve inflation lower than the Nash level.

We would like to see whether this result can be obtained in a less restrictive setting where the private sector learns via a mechanism (such as Bayesian learning) that does allow it to update its model specification. It is known that central banks make announcements or reveal information to affect the beliefs of the private sector. Is this because they actually can use their influence to manipulate the private sector's beliefs and achieve better than Nash outcome?

A Proofs

A.1 Proof of Proposition 1

We consider algorithms of the form

$$\theta_n = \theta_{n-1} + a\mathcal{H}\left(\theta_{n-1}, X_n\right) \tag{24}$$

$\theta_n \in \mathbb{R}^d, X_n \in \mathbb{R}^k$ with a starting point for θ_0. X_n is the vector of state variables. $\mathcal{H}(\cdot)$ is the functions describing the learning rule. Here n denotes discrete time so that we can use t below for continuous time.

We use Theorem 7.9 of Evans and Honkapohja (2001). Provided that the necessary assumptions of the theorem are satisfied the distribution of θ_n can be approximated, for small a and large n, by

$$\theta_t \backsim N(\theta^*, aC)$$

where θ^* is a globally asymptotically stable equilibrium point of the ODE $d\theta/dt = h(\theta)$, $h(\cdot)$ will be derived in a moment, and

$$C = \int_0^\infty e^{sB}\mathcal{R}\left(\theta^*\right) e^{sB'} ds$$

where $B = D_\theta h(\theta^*)$, $\mathcal{R}^{ij}(\theta) = \sum\limits_{k=-\infty}^{\infty} cov[\mathcal{H}^i(\theta, X_k^\theta), \mathcal{H}^j(\theta, X_0^\theta)]$.

We should derive the ordinary differential equation $d\theta/d\tau = h(\theta)$ first. The algorithm for updating γ_t is

$$\gamma_t = \gamma_{t-1} + aR_{t-1}^{-1} z_{t-1}(U_{t-1} - \gamma_{t-1} z_{t-1}) \tag{25}$$

$$R_t = R_{t-1} + a(z_{t-1} z_{t-1}' - R_{t-1}) \tag{26}$$

where $U_t = U^n - (\pi_t - w_t) + v_{1t}$, $z_t = \begin{pmatrix} 1 & \pi_t & w_t \end{pmatrix}'$ and $\gamma_t = \begin{pmatrix} \gamma_{0t} & \gamma_{1t} & \gamma_{2t} \end{pmatrix}'$, $\pi_t = \mu_t + v_{2t}$, $w_t = q_t + v_{3t}$, $\mu_t = \frac{-b(\gamma_1+\gamma_2)(\gamma_0 - U^n + \alpha)}{1+b(\gamma_1+\gamma_2)^2}$, $q_t = \mu_t$.

The algorithm given in (25) and (26) is in the standard form of (24) when we define $\theta_t = \binom{\gamma_t}{vec(R_t)}$ and $X_t = \binom{z_{t-1}}{v_{1t}}$. The appropriate $\mathcal{H}$ function can be derived from (25) and (26). We can rewrite the algorithm in the following form

$$\gamma_t = \gamma_{t-1} + aR_{t-1}^{-1} z_{t-1}(U^n - (\pi_{t-1} - w_{t-1}) + v_{1t} - \gamma_0 - \gamma_1 \pi_{t-1} - \gamma_2 w_{t-1})$$

$$R_t = R_{t-1} + a(z_{t-1} z_{t-1}' - R_{t-1})$$

or

$$\gamma_t = \gamma_{t-1} + aR_{t-1}^{-1} z_{\gamma t-1} \left(\begin{array}{r} U^n - \gamma_0 - (\gamma_1 + \gamma_2) \frac{-b(\gamma_1+\gamma_2)(\gamma_0 - U^n + \alpha)}{1+b(\gamma_1+\gamma_2)^2} + v_{1t-1} .. \\ .. - (1+\gamma_1) v_{2t-1} + (1-\gamma_2) v_{3t-1} \end{array} \right)$$

$$R_t = R_{t-1} + a(z_{t-1} z_{t-1}' - R_{t-1})$$

$$\gamma_t = \gamma_{t-1} + aR^{-1} \begin{bmatrix} 1 \\ \mu_{t-1} + v_{2t-1} \\ q_{t-1} + v_{3t-1} \end{bmatrix} \left[\begin{array}{r} U^n - \gamma_0 - (\gamma_1 + \gamma_2) \frac{-b(\gamma_1+\gamma_2)(\gamma_0 - U^n + \alpha)}{1+b(\gamma_1+\gamma_2)^2} + v_{1t-1} .. \\ .. - (1+\gamma_1) v_{2t-1} + (1-\gamma_2) v_{3t-1} \end{array} \right]$$

$$\dot{\gamma} = R^{-1} \begin{bmatrix} U^n - \gamma_0 - (\gamma_1 + \gamma_2) \frac{-b(\gamma_1+\gamma_2)(\gamma_0 - U^n + \alpha)}{1+b(\gamma_1+\gamma_2)^2} \\ \mu_{t-1} \left(U^n - \gamma_0 - (\gamma_1 + \gamma_2) \frac{-b(\gamma_1+\gamma_2)(\gamma_0 - U^n + \alpha)}{1+b(\gamma_1+\gamma_2)^2} \right) - (1+\gamma_1)\sigma_2^2 \\ q_{t-1} \left(U^n - \gamma_0 - (\gamma_1 + \gamma_2) \frac{-b(\gamma_1+\gamma_2)(\gamma_0 - U^n + \alpha)}{1+b(\gamma_1+\gamma_2)^2} \right) + (1-\gamma_2)\sigma_3^2 \end{bmatrix} \quad (27)$$

There is a unique steady state of the differential equation (27) which is $\gamma^* = \begin{pmatrix} U^n & -1 & 1 \end{pmatrix}$. It is trivial to derive $h_R(\gamma, R)$ and its steady state. The steady state of $h(\theta)$ is a globally asymptotically stable equilibrium point since the eigenvalues of the 6×6 matrix $D_\theta h(\theta^*)$ have strictly negative real parts. Now we need to show that the assumptions of the theorem hold for our case.

Let $D = \{(\gamma, R) \mid \gamma \in \mathbb{R}^3, R \in (\zeta, \infty)^3\}$ for some fixed arbitrarily small $\zeta > 0$. Assume that z_t has support on some closed set and let $m^z = E(z_{t-1} z_{t-1}')$ be PSD. The polynomial bounds and Lipschitz conditions (A.2), (A.3) on $\mathcal{H}(\cdot)$ and $\partial\mathcal{H}/\partial X$ are met for compact sets $Q \subset D$. Conditions (M.1)-(M.5) follow immediately from the assumptions that z_t and v_t are iid exogenous processes with bounded support. From the theorem of Coddington (1961, p. 248), it follows that $D_\theta h(\theta^*)$ is Lipschitz on D. The eigenvalues of $D_\theta h(\theta^*)$ are all negative which implies that θ^* is a globally asymptotically stable equilibrium point of the ODE. Hence assumptions (H.1)-(H.3) are met, the theorem applies to this case.

A.2 Proof of Proposition 2

We should derive the ordinary differential equation $d\theta/d\tau = h(\theta)$ and show that the unique steady state of this equation is globally asymptotically stable. The conditions of the theorem are similar to the first case.

For this case the adaptive system can be written in the form

$$\gamma_t = \gamma_{t-1} + aR_{t-1}^{-1} z_{t-1}(U_{t-1} - \gamma_{t-1} z_{t-1})$$

$$R_t = R_{t-1} + a(z_{t-1}z_{t-1}' - R_{t-1})$$

where $U_t = U^n - (\pi_t - w_t) + v_{1t}$, $z_t = \begin{pmatrix} 1 & \pi_t & w_t \end{pmatrix}'$ and $\gamma_t = \begin{pmatrix} \gamma_{0t} & \gamma_{1t} & \gamma_{2t} \end{pmatrix}'$, $\pi_t = \mu_t + v_{2t}$, $w_t = q_t + v_{3t}$, $\mu_t = \frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1 + b\gamma_1^2 + b\gamma_1\gamma_2}$, $q_t = \mu_t$.

We can write these equations as

$$\gamma_t = \gamma_{t-1} + aR_{t-1}^{-1} z_{t-1}(U^n - (\pi_{t-1} - w_{t-1}) + v_{1t} - \gamma_0 - \gamma_1 \pi_{t-1} - \gamma_2 w_{t-1})$$

$$R_t = R_{t-1} + a(z_{t-1}z_{t-1}' - R_{t-1})$$

or

$$\gamma_t = \gamma_{t-1} + aR_{t-1}^{-1} z_{\gamma t-1} \begin{pmatrix} U^n - \gamma_0 - (\gamma_1 + \gamma_2)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1 + b\gamma_1^2 + b\gamma_1\gamma_2} + v_{1t-1}.. \\ .. - (1 + \gamma_1)v_{2t-1} + (1 - \gamma_2)v_{3t-1} \end{pmatrix}$$

$$R_t = R_{t-1} + a(z_{t-1}z_{t-1}' - R_{t-1})$$

$$\gamma_t = \gamma_{t-1} + aR^{-1} \begin{bmatrix} 1 \\ \mu_{t-1} + v_{2t-1} \\ q_{t-1} + v_{3t-1} \end{bmatrix} \begin{bmatrix} U^n - \gamma_0 - (\gamma_1 + \gamma_2)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1 + b\gamma_1^2 + b\gamma_1\gamma_2} + v_{1t-1}.. \\ .. - (1 + \gamma_1)v_{2t-1} + (1 - \gamma_2)v_{3t-1} \end{bmatrix}$$

$$\dot{\gamma} = R^{-1} \begin{bmatrix} U^n - \gamma_0 - (\gamma_1 + \gamma_2)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1 + b\gamma_1^2 + b\gamma_1\gamma_2} \\ \mu_{t-1}\left(U^n - \gamma_0 - (\gamma_1 + \gamma_2)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1 + b\gamma_1^2 + b\gamma_1\gamma_2}\right) - (1 + \gamma_1)\sigma_2^2 \\ q_{t-1}\left(U^n - \gamma_0 - (\gamma_1 + \gamma_2)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1 + b\gamma_1^2 + b\gamma_1\gamma_2}\right) + (1 - \gamma_2)\sigma_3^2 \end{bmatrix}$$

There is a unique steady state of this differential equation which is $\gamma = \begin{pmatrix} U^n & -1 & 1 \end{pmatrix}$. This steady state is a globally asymptotically stable equilibrium point of $h(\gamma, R)$ since the eigenvalues of $D_\theta h(\theta^*)$ have strictly negative real parts.

A.3 Proof of Proposition 3

We will show that the equilibrium point of the ordinary differential equation $d\theta/d\tau = h(\theta)$ is globally asymptotically stable. The other required conditions can be easily shown to be satisfied. The central bank is using 2 parameters, the private sector is using 3 parameters. Together with the adjustment matrices the system is represented by a 10×10 matrix. The eigenvalues of the matrix $B = D_\theta h(\theta^*)$ should have all negative real parts.

$$\gamma_t = \gamma_{t-1} + aR^{-1}_{\gamma t-1} z_{\gamma t-1}(U_{t-1} - \gamma_{t-1} z_{\gamma t-1})$$

$$R_{\gamma t} = R_{\gamma t-1} + a(z_{\gamma t-1} z'_{\gamma t-1} - R_{\gamma t-1})$$

$$\eta_t = \eta_{t-1} + aR^{-1}_{\eta t-1} z_{\eta t-1}(U_{t-1} - \eta_{t-1} z_{\eta t-1})$$

$$R_{\eta t} = R_{\eta t-1} + a(z_{\eta t-1} z'_{\eta t-1} - R_{\eta t-1})$$

where $U_t = U^n - (\pi_t - w_t) + v_{1t}$, $z_{\gamma t} = \begin{pmatrix} 1 & \pi_t \end{pmatrix}'$ and $\gamma_t = \begin{pmatrix} \gamma_{0t} & \gamma_{1t} \end{pmatrix}'$,$z_{\eta t} = \begin{pmatrix} 1 & \pi_t & w_t \end{pmatrix}'$ and $\eta_t = \begin{pmatrix} \eta_{0t} & \eta_{1t} & \eta_{2t} \end{pmatrix}'$, $\pi_t = \mu_t + v_{2t}$, $w_t = q_t + v_{3t}$, $\mu_t = \frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1+b\gamma_1^2}$, $q_t = \frac{-b\eta_1(\eta_0 - U^n + \alpha)}{1+b\eta_1^2 + b\eta_1\eta_2}$.

We can write these equations as

$$\gamma_t = \gamma_{t-1} + aR^{-1}_{\gamma t-1} z_{\gamma t-1}(U^n - (\pi_{t-1} - w_{t-1}) + v_{1t-1} - \gamma_0 - \gamma_1 \pi_{t-1})$$

$$R_{\gamma t} = R_{\gamma t-1} + a(z_{\gamma t-1} z'_{\gamma t-1} - R_{\gamma t-1})$$

$$\gamma_t = \gamma_{t-1} + aR^{-1}_{\gamma t-1} z_{\gamma t-1} \begin{pmatrix} U^n - \gamma_0 - (1+\gamma_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1+b\gamma_1^2} .. \\ .. + \frac{-b\eta_1(\eta_0 - U^n + \alpha)}{1+b\eta_1^2 + b\eta_1\eta_2} + v_{1t-1} - (1+\gamma_1)v_{2t-1} + v_{3t-1} \end{pmatrix}$$

$$R_{\gamma t} = R_{\gamma t-1} + a(z_{\gamma t-1} z'_{\gamma t-1} - R_{\gamma t-1})$$

$$\gamma_t = \gamma_{t-1} + a R_\gamma^{-1} \begin{bmatrix} 1 \\ \mu_{t-1} + v_{2t-1} \end{bmatrix} \begin{bmatrix} U^n - \gamma_0 - (1+\gamma_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1+b\gamma_1^2} .. \\ .. + \frac{-b\eta_1(\eta_0 - U^n + \alpha)}{1+b\eta_1^2 + b\eta_1\eta_2} + v_{1t-1} - (1+\gamma_1) v_{2t-1} + v_{3t-1} \end{bmatrix}$$

$$\dot{\gamma} = R_\gamma^{-1} \begin{bmatrix} U^n - \gamma_0 - (1+\gamma_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1+b\gamma_1^2} + \frac{-b\eta_1(\eta_0 - U^n + \alpha)}{1+b\eta_1^2 + b\eta_1\eta_2} \\ \mu_{t-1}\left(U^n - \gamma_0 - (1+\gamma_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1+b\gamma_1^2} + \frac{-b\eta_1(\eta_0 - U^n + \alpha)}{1+b\eta_1^2 + b\eta_1\eta_2}\right) - (1+\gamma_1)\sigma_2^2 \end{bmatrix}$$

For the private sector:

$$\eta_t = \eta_{t-1} + a R_{\eta t-1}^{-1} z_{\eta t-1} \left(U^n - (\pi_{t-1} - w_{t-1}) + v_{1t-1} - \eta_0 - \eta_1 \pi_{t-1} - \eta_2 w_{t-1}\right)$$

$$R_{\eta t} = R_{\eta t-1} + a(z_{\eta t-1} z'_{\eta t-1} - R_{\eta t-1})$$

$$\eta_t = \eta_{t-1} + a R_{\eta t-1}^{-1} z_{\eta t-1} \begin{pmatrix} U^n - \eta_0 - (1+\eta_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1+b\gamma_1^2} .. \\ .. + (1-\eta_2)\frac{-b\eta_1(\eta_0 - U^n + \alpha)}{1+b\eta_1^2 + b\eta_1\eta_2} + v_{1t-1} - (1+\eta_1) v_{2t-1} + (1-\eta_2) v_{3t-1} \end{pmatrix}$$

$$\eta_t = \eta_{t-1} + a R_\eta^{-1} \begin{bmatrix} 1 \\ \mu_{t-1} + v_{2t-1} \\ q_{t-1} + v_{3t-1} \end{bmatrix} \begin{bmatrix} U^n - \eta_0 - (1+\eta_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1+b\gamma_1^2} + (1-\eta_2)\frac{-b\eta_1(\eta_0 - U^n + \alpha)}{1+b\eta_1^2 + b\eta_1\eta_2} .. \\ .. + v_{1t-1} - (1+\eta_1) v_{2t-1} + (1-\eta_2) v_{3t-1} \end{bmatrix}$$

$$\dot{\eta} = R_\eta^{-1} \begin{bmatrix} U^n - \eta_0 - (1+\eta_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1+b\gamma_1^2} + (1-\eta_2)\frac{-b\eta_1(\eta_0 - U^n + \alpha)}{1+b\eta_1^2 + b\eta_1\eta_2} \\ \mu_{t-1}\left(U^n - \eta_0 - (1+\eta_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1+b\gamma_1^2} + (1-\eta_2)\frac{-b\eta_1(\eta_0 - U^n + \alpha)}{1+b\eta_1^2 + b\eta_1\eta_2}\right) - (1+\eta_1)\sigma_2^2 \\ q_{t-1}\left(U^n - \eta_0 - (1+\eta_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1+b\gamma_1^2} + (1-\eta_2)\frac{-b\eta_1(\eta_0 - U^n + \alpha)}{1+b\eta_1^2 + b\eta_1\eta_2}\right) + (1-\eta_2)\sigma_3^2 \end{bmatrix}$$

There is a unique steady state of this differential equation, $\gamma = \begin{pmatrix} U^n + \alpha b & -1 \end{pmatrix}$, $\eta =$

$\begin{pmatrix} U^n & -1 & 1 \end{pmatrix}$. The eigenvalues of $B = D_\theta h(\theta^*)$ for this steady state are all negative.

A.4 Proof of Proposition 4

We will show that the equilibrium point of the ordinary differential equation $d\theta/d\tau = h(\theta)$ is globally asymptotically stable. The other required conditions can be easily shown to be satisfied. The central bank and the private sector are using 3 parameters. Together with the adjustment matrices the system will be represented by a 12×12 matrix. The eigenvalues of the matrix $B = D_\theta h(\theta^*)$ should have all negative real parts.

$$\gamma_t = \gamma_{t-1} + aR^{-1}_{\gamma t-1} z_{\gamma t-1}(U_{t-1} - \gamma_{t-1} z_{\gamma t-1})$$

$$R_{\gamma t} = R_{\gamma t-1} + a(z_{\gamma t-1} z'_{\gamma t-1} - R_{\gamma t-1})$$

$$\eta_t = \eta_{t-1} + aR^{-1}_{\eta t-1} z_{\eta t-1}(U_{t-1} - \eta_{t-1} z_{\eta t-1})$$

$$R_{\eta t} = R_{\eta t-1} + a(z_{\eta t-1} z'_{\eta t-1} - R_{\eta t-1})$$

where $U_t = U^n - (\pi_t - w_t) + v_{1t}$, $z_{\gamma t} = \begin{pmatrix} 1 & \pi_t & w_t \end{pmatrix}'$ and $\gamma_t = \begin{pmatrix} \gamma_{0t} & \gamma_{1t} & \gamma_{2t} \end{pmatrix}'$, $z_{\eta t} = \begin{pmatrix} 1 & \pi_t & w_t \end{pmatrix}'$ and $\eta_t = \begin{pmatrix} \eta_{0t} & \eta_{1t} & \eta_{2t} \end{pmatrix}'$, $\pi_t = \mu_t + v_{2t}$, $w_t = q_t + v_{3t}$, $\mu_t = \frac{-b\gamma_1(\gamma_0 - U^n + \alpha) - b\gamma_1\gamma_2 q_t}{1 + b\gamma_1^2}$, $q_t = \frac{-b(\eta_1+\eta_2)(\eta_0 - U^n + \alpha)}{1 + b(\eta_1+\eta_2)^2}$.

We can write these equations as

$$\gamma_t = \gamma_{t-1} + aR^{-1}_{\gamma t-1} z_{\gamma t-1}(U^n - (\pi_{t-1} - w_{t-1}) + v_{1t-1} - \gamma_0 - \gamma_1\pi_{t-1} - \gamma_2 w_{t-1})$$

$$R_{\gamma t} = R_{\gamma t-1} + a(z_{\gamma t-1} z'_{\gamma t-1} - R_{\gamma t-1})$$

$$\gamma_t = \gamma_{t-1} + aR^{-1}_{\gamma t-1} z_{\gamma t-1} \begin{pmatrix} U^n - \gamma_0 - (1+\gamma_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha) - b\gamma_1\gamma_2 q_{t-1}}{1 + b\gamma_1^2}.. \\ .. + (1-\gamma_2)\,\frac{-b(\eta_1+\eta_2)(\eta_0 - U^n + \alpha)}{1 + b(\eta_1+\eta_2)^2} + v_{1t-1} - (1+\gamma_1)v_{2t-1} + (1-\gamma_2)\,v_{3t-1} \end{pmatrix}$$

$$R_{\gamma t} = R_{\gamma t-1} + a(z_{\gamma t-1} z'_{\gamma t-1} - R_{\gamma t-1})$$

$$\gamma_t = \gamma_{t-1} + aR_\gamma^{-1} \begin{bmatrix} 1 \\ \mu_{t-1} + v_{2t-1} \\ q_{t-1} + v_{3t-1} \end{bmatrix} \begin{bmatrix} U^n - \gamma_0 - (1+\gamma_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha) - b\gamma_1\gamma_2 q_{t-1}}{1+b\gamma_1^2} + (1-\gamma_2) .. \\ .. \frac{-b(\eta_1+\eta_2)(\eta_0 - U^n + \alpha)}{1+b(\eta_1+\eta_2)^2} + v_{1t-1} - (1+\gamma_1)v_{2t-1} + (1-\gamma_2)\, v_{3t-1} \end{bmatrix}$$

$$\dot{\gamma} = R_\gamma^{-1} \begin{bmatrix} U^n - \gamma_0 - (1+\gamma_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha) - b\gamma_1\gamma_2 q_{t-1}}{1+b\gamma_1^2} + (1-\gamma_2)\, \frac{-b(\eta_1+\eta_2)(\eta_0 - U^n + \alpha)}{1+b(\eta_1+\eta_2)^2} \\ \mu_{t-1} \left(U^n - \gamma_0 - (1+\gamma_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha) - b\gamma_1\gamma_2 q_{t-1}}{1+b\gamma_1^2} + (1-\gamma_2)\, \frac{-b(\eta_1+\eta_2)(\eta_0 - U^n + \alpha)}{1+b(\eta_1+\eta_2)^2} \right) - (1+\gamma_1)\sigma_2^2 \\ q_{t-1} \left(U^n - \gamma_0 - (1+\gamma_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha) - b\gamma_1\gamma_2 q_{t-1}}{1+b\gamma_1^2} + (1-\gamma_2)\, \frac{-b(\eta_1+\eta_2)(\eta_0 - U^n + \alpha)}{1+b(\eta_1+\eta_2)^2} \right) + (1-\gamma_2)\sigma_3^2 \end{bmatrix}$$

For the private sector:

$$\eta_t = \eta_{t-1} + aR_{\eta t-1}^{-1} z_{\eta t-1} \left(U^n - (\pi_{t-1} - w_{t-1}) + v_{1t-1} - \eta_0 - \eta_1 \pi_{t-1} - \eta_2 w_{t-1} \right)$$

$$R_{\eta t} = R_{\eta t-1} + a(z_{\eta t-1} z'_{\eta t-1} - R_{\eta t-1})$$

$$\eta_t = \eta_{t-1} + aR_{\eta t-1}^{-1} z_{\eta t-1} \begin{pmatrix} U^n - \eta_0 - (1+\eta_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha) - b\gamma_1\gamma_2 q_{t-1}}{1+b\gamma_1^2} .. \\ .. + (1-\eta_2)\frac{-b(\eta_1+\eta_2)(\eta_0 - U^n + \alpha)}{1+b(\eta_1+\eta_2)^2} + v_{1t-1} - (1+\eta_1)v_{2t-1} + (1-\eta_2)v_{3t-1} \end{pmatrix}$$

$$\eta_t = \eta_{t-1} + aR_\eta^{-1} \begin{bmatrix} 1 \\ \mu_{t-1} + v_{2t-1} \\ q_{t-1} + v_{3t-1} \end{bmatrix} \begin{bmatrix} U^n - \eta_0 - (1+\eta_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha) - b\gamma_1\gamma_2 q_{t-1}}{1+b\gamma_1^2} + (1-\eta_2) .. \\ .. \frac{-b(\eta_1+\eta_2)(\eta_0 - U^n + \alpha)}{1+b(\eta_1+\eta_2)^2} + v_{1t-1} - (1+\eta_1)v_{2t-1} + (1-\eta_2)v_{3t-1} \end{bmatrix}$$

$$\dot{\eta} = R_\eta^{-1} \begin{bmatrix} U^n - \eta_0 - (1+\eta_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha) - b\gamma_1\gamma_2 q_{t-1}}{1+b\gamma_1^2} + (1-\eta_2)\frac{-b(\eta_1+\eta_2)(\eta_0 - U^n + \alpha)}{1+b(\eta_1+\eta_2)^2} \\ \mu_{t-1} \left(U^n - \eta_0 - (1+\eta_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha) - b\gamma_1\gamma_2 q_{t-1}}{1+b\gamma_1^2} + (1-\eta_2)\frac{-b(\eta_1+\eta_2)(\eta_0 - U^n + \alpha)}{1+b(\eta_1+\eta_2)^2} \right) - (1+\eta_1)\sigma_2^2 \\ q_{t-1} \left(U^n - \eta_0 - (1+\eta_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha) - b\gamma_1\gamma_2 q_{t-1}}{1+b\gamma_1^2} + (1-\eta_2)\frac{-b(\eta_1+\eta_2)(\eta_0 - U^n + \alpha)}{1+b(\eta_1+\eta_2)^2} \right) + (1-\eta_2)\sigma_3^2 \end{bmatrix}$$

There is a unique steady state of this differential equation, $\gamma = \begin{pmatrix} U^n & -1 & 1 \end{pmatrix}$, $\eta = \begin{pmatrix} U^n & -1 & 1 \end{pmatrix}$. The eigenvalues of $B = D_\theta h(\theta^*)$ for this steady state are all negative.

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