

# Logistic regression and Dose Response Data

Many thanks to Thaddeus Tarpey at Wright University Check out his cite for this and more  
<http://www.wright.edu/~thaddeus.tarpey/>

These data are a reproduction of data from C.I. Bliss (1935). The calculation of the dosage-mortality curve.  
*Annals of Applied Biology*, vol 22, Issue 1, 134-167.

## Data

Beetles were exposed to carbon disulphide at varying concentrations for 5 hours.

- dose = mf/L concentration of  $CS_2$
- nexpt = number of beetles exposed
- ndied = number of beetles killed
- prop = proportion of dead to exposed beetles

```
exp.dat = matrix(c(49.1,53,56.9,60.8,64.8,68.7,72.6,76.5,
                  59,60,62,56,63,59,62,60,
                  6,13,18,28,52,53,61,60,
                  .102,.217,.29,.5,.825,.898,.984,1),ncol=4)
colnames(exp.dat) = c('dose','nexpt','ndied','prop')
exp.dat = as.data.frame(exp.dat)
# compute how many lived
exp.dat$nalive = exp.dat$nexpt - exp.dat$ndied
```

## Logistic Model

Run a logistic regression of the proportion of dead to living beetles as a function of the dose of  $CS_2$  gas.

```
summary(glm(cbind(ndied,nalive) ~ dose,
              family=binomial,
              data=exp.dat)->exp.glm)
```

Call:

```
glm(formula = cbind(ndied, nalive) ~ dose, family = binomial,
     data = exp.dat)
```

Deviance Residuals:

Min	1Q	Median	3Q	Max
-1.2746	-0.4668	0.7688	0.9544	1.2990

Coefficients:

	Estimate	Std. Error	z value	Pr(> z )
(Intercept)	-14.82300	1.28959	-11.49	<2e-16 ***
dose	0.24942	0.02139	11.66	<2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 284.2024 on 7 degrees of freedom

Residual deviance: 7.3849 on 6 degrees of freedom  
AIC: 37.583

Number of Fisher Scoring iterations: 4

we may be interested in finding the concentration of  $CS_2$  gas that is lethal 50% of the time, the  $LD_{50}$  below is the function for this computation.

```
dose4prob = function(b0,b1,prob){  
  d = (-b0+log(-prob/(prob-1)))/b1  
  return(d)  
}  
dose4prob(coef(exp.glm)[[1]],coef(exp.glm)[[2]],.5)
```

```
[1] 59.43092
```

## Note that if we have a function with multiple predictors we

can solve for each variable using something similar. For example if

$$y \sim b_0 + b_1(x_1) + b_2(x_2) + b_3(x_3)$$

is the model. Then to find a specific value for one of the predictors ( $x_1, x_2, x_3$ ) that corresponds to a desired probability ( $y$ ).

- $x_1 = (-b_0 - b_2 - b_3 + \log\left(\frac{-y}{(y-1)}\right))/b_1$
- $x_2 = (-b_0 - b_1 - b_3 + \log\left(\frac{-y}{(y-1)}\right))/b_2$
- $x_3 = (-b_0 - b_1 - b_2 + \log\left(\frac{-y}{(y-1)}\right))/b_3$

```
# what is the range of doses  
range(exp.dat$dose)
```

```
[1] 49.1 76.5
```

```
# let's create our own range to predict with  
drange = seq(30,90,length=100)  
# predict probability of beetle death based on the model  
exp.pred = predict(exp.glm, newdata=data.frame(dose=drange))  
# Now we plot  
# first the predicted values  
plot(drange,  
      exp(exp.pred)/(1+exp(exp.pred)),  
      type='l',  
      xlab='dose',  
      ylab='probability')  
# add in the observed points  
points(prop ~ dose, data = exp.dat, col='red', pch=19)  
# compute our LD values  
ld50 = dose4prob(coef(exp.glm)[[1]],coef(exp.glm)[[2]],.50)  
ld25 = dose4prob(coef(exp.glm)[[1]],coef(exp.glm)[[2]],.25)  
# let's add these to our graph for visualization purposes  
abline(h=.5,lty='dashed',col='red')  
abline(h=.25,lty='dotted',col='blue')
```

```
legend('topleft',  
      c(expression(LD[50]),  
        expression(LD[25])),  
      col=c('red','blue'),lty=c('dashed','dotted'),  
      inset=.01)  
text(ld50,.52,sprintf("%.3f",ld50),pos=c(2),col='red')  
text(ld25,.27,sprintf("%.3f",ld25),pos=c(2),col='blue')
```

