

Factorial Invariance Walkthrough: A Replication of Horn & McArdle (1992)

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Study Set-up

Horn and McArdle demonstrate the use of factorial invariance concepts for measures of intelligence from the **Wechsler Adult Intelligence Scale – Revised**. A total of eight subscale measures were selected to identify two major constructs of intelligence, *Gf* fluid intelligence, and *Gc* crystallized intelligence. Additionally, a nationally representative sample was selected and age based cohorts were defined as:

- Young (Y) $N = 300$, Age $M = 19.00$, $sd = 2.28$, $range = 16 - 22$
- Adult (A) $N = 275$, Age $M = 34.05$, $sd = 4.99$, $range = 29.5 - 39.5$
- Middle (M) $N = 205$, Age $M = 53.40$, $sd = 4.89$, $range = 49.5 - 59.5$
- Older (O) $N = 160$, Age $M = 69.50$, $sd = 2.51$, $range = 67 - 72$

WAIS-R Data

Description of the WAIS-R subscales compliments of <http://www.iupui.edu/~flip/wechsler.html>

Verbal Scales

- **Information:** 28 items on a variety of information adults have presumably had opportunities to acquire in our culture. No specialized or academic information included; however, some of the items cover quite sophisticated information.
- **Comprehension:** 18 items that require examinee to explain what should be done in certain circumstances, the meaning of proverbs, why certain societal practices are followed, and so forth. The test measures practical judgement, common sense, and the ability to understand and adapt to social customs. Score on each item varies (0-2 pts) according to the degree to which the response describes the most pertinent aspects of the question.
- **Similarities:** 19 items requiring examinee to describe how two given things are alike. Score on each item varies according to the degree to which the response describes a general property primarily pertinent to both items in the pair. Measures concrete, functional, and abstract concept formation.
- **Vocabulary:** 66 words of increasing difficulty are presented orally and visually. Examinee required to define the words. Score (0-2) based on sophistication of definition. Measures verbal knowledge and concept formation.

Performance Scale

- **Picture Completion:** 25 cards, each containing a picture having a part missing. Examinee must identify the missing part. Measures ability to observe details and recognize specific features of the environment (i.e., whole to part discrimination). Also measures performance in deliberately focusing attention.
- **Block Design:** Perhaps the butt of more jokes than any other WAIS scale! Included in the test are nine red and white square blocks and a spiral booklet of cards showing different color designs that can be made with the blocks. The examinee must arrange the blocks to match the design formed by

examiner or shown on cards. In addition to being scored for accuracy, each item is scored for speed as well. Measures spatial problem-solving and manipulative abilities, and part to whole organization.

- **Picture Arrangement:** Eleven items. Each item consists of 3 to 6 cards containing pictures. The examinee must arrange the pictures from left to right to tell the intended story. Again, both accuracy and speed are scored. Partial credit is given for alternate, but less commonly given arrangements to some items. Measures nonverbal reasoning and sequencing skills, and grasp of social cause and effect (also known as social intelligence).
- **Object Assembly (Optional Test):** Four items, each item being a “cut up” object, like a puzzle. Examinee must correctly assemble the parts of the puzzle. Measures visual-motor problem-solving and organizational abilities, and visual anticipation skills.

```
# inf = information
# com = comprehension
# sim = similarities
# voc = vocabulary
# pcc = picture completion
# bkd = block design
# par = picture arrangement
# oas = object assembly
vnames = c("inf", "com", "sim", "voc", "pcc", "bkd", "par", "oas")

suppressMessages(library(lavaan))

y_n = 300
y_mu=c( 55.47,  57.06,  62.89,  54.93,  75.18,  58.90,  62.47,  73.00)
y_sd=c( 20.78,  17.46,  19.71,  20.83,  15.56,  20.50,  20.79,  15.96)
y_lwr='
1.0000
.6667 1.0000
.6931 .7117 1.0000
.8460 .7633 .7355 1.0000
.4561 .5265 .5023 .4865 1.0000
.4312 .4627 .5071 .5081 .5405 1.0000
.4833 .5319 .4936 .5346 .4558 .3726 1.0000
.3610 .3912 .4277 .4157 .5151 .5906 .3137 1.0000'
y_cov.mat = getCov(y_lwr, sd=y_sd, names=vnames)

a_n = 275
a_mu=c( 65.07,  64.98,  66.94,  67.24,  76.24,  57.67,  63.40,  74.42)
a_sd=c( 19.83,  18.85,  21.01,  21.37,  17.67,  22.12,  24.74,  15.79)
a_lwr='
1.0000
.6924 1.0000
.7091 .7041 1.0000
.8208 .7751 .7738 1.0000
.6173 .6133 .6365 .6594 1.0000
.6360 .5741 .6273 .6513 .6709 1.0000
.6451 .6076 .6282 .6700 .6078 .6225 1.0000
.4868 .4269 .5217 .5175 .6026 .7145 .4820 1.0000'
a_cov.mat = getCov(a_lwr, sd=a_sd, names=vnames)

m_n = 205
m_mu=c( 64.79,  63.60,  62.02,  64.50,  68.88,  48.70,  53.44,  66.27)
m_sd=c( 20.43,  20.16,  21.86,  20.86,  19.53,  20.20,  26.71,  17.34)
```

```

m_lwr='
1.0000
.7721 1.0000
.7532 .7784 1.0000
.8332 .7917 .7979 1.0000
.6493 .6722 .6599 .6531 1.0000
.5606 .5691 .5592 .5767 .5930 1.0000
.6389 .6111 .6474 .6396 .6443 .5863 1.0000
.4685 .5136 .5024 .4788 .5947 .6243 .5164 1.0000'
m_cov.mat = getCov(m_lwr, sd=m_sd, names=vnames)

o_n = 160
o_mu=c( 56.81, 55.29, 49.26, 58.51, 53.53, 35.25, 32.91, 57.01)
o_sd=c( 20.63, 20.50, 26.23, 22.89, 24.11, 18.75, 21.51, 19.16)
o_lwr='
1.0000
.7324 1.0000
.7397 .6933 1.0000
.8268 .7469 .7661 1.0000
.6792 .5891 .6684 .6615 1.0000
.6305 .5476 .6363 .5821 .6421 1.0000
.5393 .4764 .6047 .5485 .5982 .5179 1.0000
.5436 .4807 .5788 .5361 .6615 .6892 .5482 1.0000'
o_cov.mat = getCov(o_lwr, sd=o_sd, names=vnames)

grp.cov = list('young'=y_cov.mat,
               'adult'=a_cov.mat,
               'middl'=m_cov.mat,
               'older'=o_cov.mat)
grp.mu = list('young'=y_mu,
              'adult'=a_mu,
              'middl'=m_mu,
              'older'=o_mu)
grp.ns = list('young'=y_n,
              'adult'=a_n,
              'middl'=m_n,
              'older'=o_n)

print(grp.cov,digits=4)

```

```

$young
  inf  com  sim  voc  pcc  bkd  par  oas
inf 431.8 241.9 283.9 366.2 147.5 183.7 208.8 119.7
com 241.9 304.9 244.9 277.6 143.0 165.6 193.1 109.0
sim 283.9 244.9 388.5 302.0 154.0 204.9 202.3 134.5
voc 366.2 277.6 302.0 433.9 157.7 217.0 231.5 138.2
pcc 147.5 143.0 154.0 157.7 242.1 172.4 147.4 127.9
bkd 183.7 165.6 204.9 217.0 172.4 420.2 158.8 193.2
par 208.8 193.1 202.3 231.5 147.4 158.8 432.2 104.1
oas 119.7 109.0 134.5 138.2 127.9 193.2 104.1 254.7

```

```

$adult
  inf  com  sim  voc  pcc  bkd  par  oas

```

```

inf 393.2 258.8 295.4 347.8 216.3 279.0 316.5 152.4
com 258.8 355.3 278.9 312.2 204.3 239.4 283.4 127.1
sim 295.4 278.9 441.4 347.4 236.3 291.5 326.5 173.1
voc 347.8 312.2 347.4 456.7 249.0 307.9 354.2 174.6
pcc 216.3 204.3 236.3 249.0 312.2 262.2 265.7 168.1
bkd 279.0 239.4 291.5 307.9 262.2 489.3 340.7 249.6
par 316.5 283.4 326.5 354.2 265.7 340.7 612.1 188.3
oas 152.4 127.1 173.1 174.6 168.1 249.6 188.3 249.3

```

\$middl

```

      inf   com   sim   voc   pcc   bkd   par   oas
inf 417.4 318.0 336.4 355.1 259.1 231.4 348.6 166.0
com 318.0 406.4 343.0 332.9 264.7 231.8 329.1 179.5
sim 336.4 343.0 477.9 363.8 281.7 246.9 378.0 190.4
voc 355.1 332.9 363.8 435.1 266.1 243.0 356.4 173.2
pcc 259.1 264.7 281.7 266.1 381.4 233.9 336.1 201.4
bkd 231.4 231.8 246.9 243.0 233.9 408.0 316.3 218.7
par 348.6 329.1 378.0 356.4 336.1 316.3 713.4 239.2
oas 166.0 179.5 190.4 173.2 201.4 218.7 239.2 300.7

```

\$older

```

      inf   com   sim   voc   pcc   bkd   par   oas
inf 425.6 309.7 400.3 390.4 337.8 243.9 239.3 214.9
com 309.7 420.2 372.8 350.5 291.2 210.5 210.1 188.8
sim 400.3 372.8 688.0 460.0 422.7 312.9 341.2 290.9
voc 390.4 350.5 460.0 524.0 365.1 249.8 270.1 235.1
pcc 337.8 291.2 422.7 365.1 581.3 290.3 310.2 305.6
bkd 243.9 210.5 312.9 249.8 290.3 351.6 208.9 247.6
par 239.3 210.1 341.2 270.1 310.2 208.9 462.7 225.9
oas 214.9 188.8 290.9 235.1 305.6 247.6 225.9 367.1

```

```

config_model = '
# factor identification
gc =~ inf + com + sim + voc
gf =~ pcc + bkd + par + oas
# covariances
gc ~~ gf
# variances
gc ~~ gc
gf ~~ gf
'

config_fit = cfa(config_model,
                 sample.cov = grp.cov,
                 sample.nobs = grp.n,
                 sample.mean = grp.mus,
                 meanstructure = TRUE)

summary(config_fit, fit.measures=T, standardized=T)

```

Intelligence model

lavaan (0.5-20) converged normally after 389 iterations

Number of observations per group	
young	300
adult	275
middl	205
older	160
Estimator	ML
Minimum Function Test Statistic	216.732
Degrees of freedom	76
P-value (Chi-square)	0.000

Chi-square for each group:

young	90.717
adult	64.389
middl	31.888
older	29.738

Model test baseline model:

Minimum Function Test Statistic	5399.422
Degrees of freedom	112
P-value	0.000

User model versus baseline model:

Comparative Fit Index (CFI)	0.973
Tucker-Lewis Index (TLI)	0.961

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-30604.113
Loglikelihood unrestricted model (H1)	-30495.747
Number of free parameters	100
Akaike (AIC)	61408.226
Bayesian (BIC)	61892.814
Sample-size adjusted Bayesian (BIC)	61575.221

Root Mean Square Error of Approximation:

RMSEA	0.089
90 Percent Confidence Interval	0.075 0.103
P-value RMSEA <= 0.05	0.000

Standardized Root Mean Square Residual:

SRMR	0.039
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Parameter Estimates:

Information	Expected
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Standard Errors

Standard

Group 1 [young]:

Latent Variables:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc =~						
inf	1.000				18.146	0.875
com	0.782	0.043	18.343	0.000	14.194	0.814
sim	0.873	0.049	17.965	0.000	15.836	0.805
voc	1.079	0.045	23.953	0.000	19.587	0.942
gf =~						
pcc	1.000				11.493	0.740
bkd	1.326	0.115	11.486	0.000	15.237	0.744
par	1.074	0.115	9.340	0.000	12.344	0.595
oas	0.937	0.089	10.550	0.000	10.767	0.676

Covariances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc ~~						
gf	157.554	19.107	8.246	0.000	0.755	0.755

Intercepts:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
inf	55.470	1.198	46.311	0.000	55.470	2.674
com	57.060	1.006	56.697	0.000	57.060	3.273
sim	62.890	1.136	55.356	0.000	62.890	3.196
voc	54.930	1.201	45.750	0.000	54.930	2.641
pcc	75.180	0.897	83.821	0.000	75.180	4.839
bkd	58.900	1.182	49.846	0.000	58.900	2.878
par	62.470	1.198	52.131	0.000	62.470	3.010
oas	73.000	0.920	79.353	0.000	73.000	4.581
gc	0.000				0.000	0.000
gf	0.000				0.000	0.000

Variances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc	329.269	34.854	9.447	0.000	1.000	1.000
gf	132.085	19.259	6.858	0.000	1.000	1.000
inf	101.134	10.780	9.382	0.000	101.134	0.235
com	102.389	9.652	10.608	0.000	102.389	0.337
sim	136.430	12.723	10.723	0.000	136.430	0.352
voc	48.837	8.398	5.816	0.000	48.837	0.113
pcc	109.253	11.907	9.176	0.000	109.253	0.453
bkd	186.705	20.546	9.087	0.000	186.705	0.446
par	278.421	25.587	10.881	0.000	278.421	0.646
oas	137.955	13.618	10.131	0.000	137.955	0.543

Group 2 [adult]:

Latent Variables:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
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gc =~						
inf	1.000				17.138	0.866
com	0.903	0.051	17.675	0.000	15.482	0.823
sim	1.025	0.056	18.245	0.000	17.565	0.838
voc	1.161	0.052	22.324	0.000	19.891	0.933
gf =~						
pcc	1.000				14.287	0.811
bkd	1.313	0.083	15.872	0.000	18.761	0.850
par	1.311	0.096	13.711	0.000	18.736	0.759
oas	0.804	0.062	13.017	0.000	11.483	0.729

Covariances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc ~~						
gf	212.125	23.488	9.031	0.000	0.866	0.866

Intercepts:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
inf	65.070	1.193	54.555	0.000	65.070	3.290
com	64.980	1.134	57.297	0.000	64.980	3.455
sim	66.940	1.264	52.958	0.000	66.940	3.193
voc	67.240	1.286	52.305	0.000	67.240	3.154
pcc	76.240	1.063	71.730	0.000	76.240	4.325
bkd	57.670	1.331	43.335	0.000	57.670	2.613
par	63.400	1.489	42.591	0.000	63.400	2.568
oas	74.420	0.950	78.329	0.000	74.420	4.723
gc	0.000				0.000	0.000
gf	0.000				0.000	0.000

Variances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc	293.727	32.901	8.928	0.000	1.000	1.000
gf	204.107	25.877	7.888	0.000	1.000	1.000
inf	97.497	10.373	9.399	0.000	97.497	0.249
com	113.998	11.249	10.134	0.000	113.998	0.322
sim	130.852	13.176	9.931	0.000	130.852	0.298
voc	58.811	8.871	6.629	0.000	58.811	0.129
pcc	106.564	11.525	9.246	0.000	106.564	0.343
bkd	135.076	16.203	8.336	0.000	135.076	0.277
par	258.324	25.877	9.983	0.000	258.324	0.424
oas	116.372	11.322	10.278	0.000	116.372	0.469

Group 3 [middl]:

Latent Variables:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc =~						
inf	1.000				18.178	0.889
com	0.973	0.054	18.174	0.000	17.679	0.877
sim	1.050	0.058	18.011	0.000	19.089	0.873
voc	1.051	0.053	19.993	0.000	19.098	0.916
gf =~						
pcc	1.000				16.203	0.830

bkd	0.936	0.079	11.867	0.000	15.170	0.752
par	1.299	0.103	12.646	0.000	21.043	0.789
oas	0.744	0.069	10.737	0.000	12.058	0.696

Covariances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc ~~						
gf	256.200	32.049	7.994	0.000	0.870	0.870

Intercepts:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
inf	64.790	1.428	45.360	0.000	64.790	3.168
com	63.600	1.408	45.178	0.000	63.600	3.155
sim	62.020	1.526	40.630	0.000	62.020	2.838
voc	64.500	1.457	44.271	0.000	64.500	3.092
pcc	68.880	1.363	50.538	0.000	68.880	3.530
bkd	48.700	1.409	34.559	0.000	48.700	2.414
par	53.440	1.864	28.676	0.000	53.440	2.003
oas	66.270	1.209	54.793	0.000	66.270	3.827
gc	0.000				0.000	0.000
gf	0.000				0.000	0.000

Variances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc	330.443	41.004	8.059	0.000	1.000	1.000
gf	262.529	37.386	7.022	0.000	1.000	1.000
inf	87.800	11.186	7.849	0.000	87.800	0.210
com	93.728	11.549	8.116	0.000	93.728	0.231
sim	113.254	13.831	8.188	0.000	113.254	0.237
voc	70.426	10.076	6.989	0.000	70.426	0.162
pcc	118.275	15.998	7.393	0.000	118.275	0.311
bkd	176.957	20.705	8.547	0.000	176.957	0.435
par	269.145	33.166	8.115	0.000	269.145	0.378
oas	154.476	17.181	8.991	0.000	154.476	0.515

Group 4 [older]:

Latent Variables:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc =~						
inf	1.000				18.414	0.897
com	0.902	0.065	13.869	0.000	16.611	0.814
sim	1.205	0.080	15.099	0.000	22.180	0.850
voc	1.122	0.065	17.367	0.000	20.660	0.907
gf =~						
pcc	1.000				20.258	0.845
bkd	0.733	0.063	11.598	0.000	14.847	0.795
par	0.738	0.076	9.689	0.000	14.946	0.698
oas	0.737	0.065	11.332	0.000	14.923	0.782

Covariances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc ~~						

gf	318.817	45.013	7.083	0.000	0.855	0.855
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Intercepts:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
inf	56.810	1.623	35.013	0.000	56.810	2.768
com	55.290	1.613	34.270	0.000	55.290	2.709
sim	49.260	2.064	23.866	0.000	49.260	1.887
voc	58.510	1.801	32.490	0.000	58.510	2.569
pcc	53.530	1.895	28.243	0.000	53.530	2.233
bkd	35.250	1.476	23.889	0.000	35.250	1.889
par	32.910	1.693	19.435	0.000	32.910	1.536
oas	57.010	1.508	37.808	0.000	57.010	2.989
gc	0.000				0.000	0.000
gf	0.000				0.000	0.000

Variances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc	339.073	47.065	7.204	0.000	1.000	1.000
gf	410.395	64.250	6.387	0.000	1.000	1.000
inf	82.156	12.880	6.379	0.000	82.156	0.195
com	140.544	18.133	7.751	0.000	140.544	0.337
sim	189.704	25.792	7.355	0.000	189.704	0.278
voc	92.037	15.169	6.067	0.000	92.037	0.177
pcc	164.375	25.963	6.331	0.000	164.375	0.286
bkd	127.925	17.889	7.151	0.000	127.925	0.367
par	235.400	29.533	7.971	0.000	235.400	0.513
oas	141.087	19.312	7.305	0.000	141.087	0.388

```
# weak invariance (= loadings)
metric_fit = cfa(config_model,
  sample.cov = grp.cov,
  sample.nobs = grp.ns,
  sample.mean = grp.mus,
  meanstructure = TRUE,
  group.equal="loadings")

summary(metric_fit, fit.measures=T, standardized=T)
```

lavaan (0.5-20) converged normally after 350 iterations

Number of observations per group	
young	300
adult	275
middl	205
older	160

Estimator	ML
Minimum Function Test Statistic	302.650
Degrees of freedom	94
P-value (Chi-square)	0.000

Chi-square for each group:

young	108.315
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adult	77.542
middl	49.823
older	66.970

Model test baseline model:

Minimum Function Test Statistic	5399.422
Degrees of freedom	112
P-value	0.000

User model versus baseline model:

Comparative Fit Index (CFI)	0.961
Tucker-Lewis Index (TLI)	0.953

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-30647.072
Loglikelihood unrestricted model (H1)	-30495.747
Number of free parameters	82
Akaike (AIC)	61458.144
Bayesian (BIC)	61855.506
Sample-size adjusted Bayesian (BIC)	61595.080

Root Mean Square Error of Approximation:

RMSEA	0.097
90 Percent Confidence Interval	0.085 0.110
P-value RMSEA <= 0.05	0.000

Standardized Root Mean Square Residual:

SRMR	0.072
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Parameter Estimates:

Information	Expected
Standard Errors	Standard

Group 1 [young]:

Latent Variables:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc =~						
inf	1.000				17.031	0.853
com (.p2.)	0.887	0.026	34.111	0.000	15.109	0.838
sim (.p3.)	1.013	0.029	34.480	0.000	17.245	0.835
voc (.p4.)	1.107	0.027	41.725	0.000	18.860	0.927
gf =~						
pcc	1.000				12.166	0.765
bkd (.p6.)	1.094	0.044	24.953	0.000	13.306	0.677
par (.p7.)	1.149	0.051	22.462	0.000	13.975	0.652

oas	(.p8.)	0.811	0.036	22.325	0.000	9.871	0.632
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Covariances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc ~~						
gf	161.489	17.371	9.297	0.000	0.779	0.779

Intercepts:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
inf	55.470	1.153	48.111	0.000	55.470	2.778
com	57.060	1.041	54.805	0.000	57.060	3.164
sim	62.890	1.193	52.727	0.000	62.890	3.044
voc	54.930	1.175	46.748	0.000	54.930	2.699
pcc	75.180	0.918	81.891	0.000	75.180	4.728
bkd	58.900	1.134	51.927	0.000	58.900	2.998
par	62.470	1.237	50.482	0.000	62.470	2.915
oas	73.000	0.902	80.896	0.000	73.000	4.671
gc	0.000				0.000	0.000
gf	0.000				0.000	0.000

Variances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc	290.043	27.381	10.593	0.000	1.000	1.000
gf	148.013	16.754	8.835	0.000	1.000	1.000
inf	108.752	10.921	9.958	0.000	108.752	0.273
com	96.930	9.474	10.231	0.000	96.930	0.298
sim	129.381	12.581	10.284	0.000	129.381	0.303
voc	58.481	8.273	7.069	0.000	58.481	0.141
pcc	104.835	11.601	9.037	0.000	104.835	0.415
bkd	208.929	20.128	10.380	0.000	208.929	0.541
par	264.096	24.878	10.616	0.000	264.096	0.575
oas	146.867	13.609	10.792	0.000	146.867	0.601

Group 2 [adult]:

Latent Variables:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc =~						
inf	1.000				17.633	0.873
com (.p2.)	0.887	0.026	34.111	0.000	15.643	0.826
sim (.p3.)	1.013	0.029	34.480	0.000	17.855	0.843
voc (.p4.)	1.107	0.027	41.725	0.000	19.527	0.928
gf =~						
pcc	1.000				15.424	0.838
bkd (.p6.)	1.094	0.044	24.953	0.000	16.870	0.810
par (.p7.)	1.149	0.051	22.462	0.000	17.718	0.737
oas (.p8.)	0.811	0.036	22.325	0.000	12.514	0.757

Covariances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc ~~						
gf	236.303	24.003	9.845	0.000	0.869	0.869

Intercepts:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
inf	65.070	1.217	53.447	0.000	65.070	3.223
com	64.980	1.141	56.929	0.000	64.980	3.433
sim	66.940	1.277	52.424	0.000	66.940	3.161
voc	67.240	1.269	52.980	0.000	67.240	3.195
pcc	76.240	1.110	68.693	0.000	76.240	4.142
bkd	57.670	1.257	45.897	0.000	57.670	2.768
par	63.400	1.449	43.742	0.000	63.400	2.638
oas	74.420	0.997	74.675	0.000	74.420	4.503
gc	0.000				0.000	0.000
gf	0.000				0.000	0.000

Variances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc	310.908	30.263	10.273	0.000	1.000	1.000
gf	237.905	25.616	9.287	0.000	1.000	1.000
inf	96.704	10.340	9.353	0.000	96.704	0.237
com	113.587	11.177	10.162	0.000	113.587	0.317
sim	129.586	13.051	9.929	0.000	129.586	0.289
voc	61.652	8.578	7.188	0.000	61.652	0.139
pcc	100.837	11.499	8.769	0.000	100.837	0.298
bkd	149.598	15.976	9.364	0.000	149.598	0.345
par	263.805	25.684	10.271	0.000	263.805	0.457
oas	116.524	11.570	10.071	0.000	116.524	0.427

Group 3 [middl]:

Latent Variables:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc =~						
inf	1.000				18.246	0.891
com (.p2.)	0.887	0.026	34.111	0.000	16.186	0.849
sim (.p3.)	1.013	0.029	34.480	0.000	18.476	0.863
voc (.p4.)	1.107	0.027	41.725	0.000	20.206	0.928
gf =~						
pcc	1.000				15.708	0.818
bkd (.p6.)	1.094	0.044	24.953	0.000	17.180	0.798
par (.p7.)	1.149	0.051	22.462	0.000	18.044	0.723
oas (.p8.)	0.811	0.036	22.325	0.000	12.744	0.722

Covariances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc ~~						
gf	246.792	29.007	8.508	0.000	0.861	0.861

Intercepts:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
inf	64.790	1.430	45.302	0.000	64.790	3.164
com	63.600	1.331	47.777	0.000	63.600	3.337
sim	62.020	1.495	41.478	0.000	62.020	2.897
voc	64.500	1.521	42.412	0.000	64.500	2.962
pcc	68.880	1.342	51.345	0.000	68.880	3.586

bkd	48.700	1.503	32.403	0.000	48.700	2.263
par	53.440	1.742	30.675	0.000	53.440	2.142
oas	66.270	1.233	53.748	0.000	66.270	3.754
gc	0.000				0.000	0.000
gf	0.000				0.000	0.000

Variances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc	332.900	36.662	9.080	0.000	1.000	1.000
gf	246.735	30.636	8.054	0.000	1.000	1.000
inf	86.416	10.987	7.865	0.000	86.416	0.206
com	101.268	11.735	8.630	0.000	101.268	0.279
sim	116.987	13.877	8.430	0.000	116.987	0.255
voc	65.864	10.123	6.506	0.000	65.864	0.139
pcc	122.192	15.783	7.742	0.000	122.192	0.331
bkd	167.920	20.824	8.064	0.000	167.920	0.363
par	296.615	33.481	8.859	0.000	296.615	0.477
oas	149.237	16.825	8.870	0.000	149.237	0.479

Group 4 [older]:

Latent Variables:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc =~						
inf	1.000				19.013	0.907
com (.p2.)	0.887	0.026	34.111	0.000	16.867	0.819
sim (.p3.)	1.013	0.029	34.480	0.000	19.253	0.799
voc (.p4.)	1.107	0.027	41.725	0.000	21.055	0.912
gf =~						
pcc	1.000				15.485	0.725
bkd (.p6.)	1.094	0.044	24.953	0.000	16.936	0.840
par (.p7.)	1.149	0.051	22.462	0.000	17.788	0.758
oas (.p8.)	0.811	0.036	22.325	0.000	12.563	0.711

Covariances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc ~~						
gf	251.133	33.699	7.452	0.000	0.853	0.853

Intercepts:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
inf	56.810	1.658	34.270	0.000	56.810	2.709
com	55.290	1.629	33.948	0.000	55.290	2.684
sim	49.260	1.906	25.846	0.000	49.260	2.043
voc	58.510	1.825	32.061	0.000	58.510	2.535
pcc	53.530	1.689	31.684	0.000	53.530	2.505
bkd	35.250	1.594	22.121	0.000	35.250	1.749
par	32.910	1.855	17.741	0.000	32.910	1.403
oas	57.010	1.396	40.836	0.000	57.010	3.228
gc	0.000				0.000	0.000
gf	0.000				0.000	0.000

Variances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc	361.486	44.691	8.089	0.000	1.000	1.000
gf	239.784	34.113	7.029	0.000	1.000	1.000
inf	78.187	12.636	6.188	0.000	78.187	0.178
com	139.906	18.012	7.768	0.000	139.906	0.330
sim	210.529	26.546	7.931	0.000	210.529	0.362
voc	89.551	14.911	6.006	0.000	89.551	0.168
pcc	216.914	27.829	7.794	0.000	216.914	0.475
bkd	119.462	18.806	6.352	0.000	119.462	0.294
par	234.173	31.153	7.517	0.000	234.173	0.425
oas	154.008	19.557	7.875	0.000	154.008	0.494

```
# test of model comparison
anova(config_fit,metric_fit)
```

Chi Square Difference Test

```

      Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
config_fit 76 61408 61893 216.73
metric_fit 94 61458 61856 302.65      85.918      18 7.742e-11 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# strong invariance (= loadings + intercepts)
scalar_fit = cfa(config_model,
  sample.cov = grp.cov,
  sample.nobs = grp.ns,
  sample.mean = grp.mus,
  meanstructure = TRUE,
  group.equal=c("loadings","intercepts"))

summary(scalar_fit, fit.measures=T, standardized=T)
```

lavaan (0.5-20) converged normally after 485 iterations

```
Number of observations per group
young          300
adult          275
middl          205
older          160
```

```
Estimator          ML
Minimum Function Test Statistic    464.712
Degrees of freedom          112
P-value (Chi-square)          0.000
```

Chi-square for each group:

```
young          175.167
adult          82.148
middl          69.999
older          137.398
```

Model test baseline model:

Minimum Function Test Statistic	5399.422
Degrees of freedom	112
P-value	0.000

User model versus baseline model:

Comparative Fit Index (CFI)	0.933
Tucker-Lewis Index (TLI)	0.933

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-30728.103
Loglikelihood unrestricted model (H1)	-30495.747
Number of free parameters	64
Akaike (AIC)	61584.206
Bayesian (BIC)	61894.342
Sample-size adjusted Bayesian (BIC)	61691.083

Root Mean Square Error of Approximation:

RMSEA	0.116
90 Percent Confidence Interval	0.105 0.127
P-value RMSEA <= 0.05	0.000

Standardized Root Mean Square Residual:

SRMR	0.080
------	-------

Parameter Estimates:

Information	Expected
Standard Errors	Standard

Group 1 [young]:

Latent Variables:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc =~						
inf	1.000				17.197	0.856
com (.p2.)	0.884	0.025	35.129	0.000	15.208	0.838
sim (.p3.)	0.979	0.029	33.463	0.000	16.842	0.797
voc (.p4.)	1.110	0.026	43.015	0.000	19.089	0.929
gf =~						
pcc	1.000				12.026	0.759
bkd (.p6.)	1.077	0.038	28.513	0.000	12.953	0.662
par (.p7.)	1.214	0.045	26.721	0.000	14.596	0.671
oas (.p8.)	0.801	0.032	25.141	0.000	9.633	0.619

Covariances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
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gc ~~							
gf	161.723	17.310	9.343	0.000	0.782	0.782	

Intercepts:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
inf	(.20.)	56.314	1.073	52.507	0.000	56.314	2.804
com	(.21.)	56.758	0.964	58.875	0.000	56.758	3.128
sim	(.22.)	57.828	1.084	53.329	0.000	57.828	2.737
voc	(.23.)	56.249	1.159	48.533	0.000	56.249	2.738
pcc	(.24.)	75.659	0.834	90.713	0.000	75.659	4.777
bkd	(.25.)	57.903	0.941	61.507	0.000	57.903	2.960
par	(.26.)	62.057	1.071	57.968	0.000	62.057	2.852
oas	(.27.)	73.451	0.729	100.707	0.000	73.451	4.718
gc		0.000				0.000	0.000
gf		0.000				0.000	0.000

Variances:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc		295.733	27.815	10.632	0.000	1.000	1.000
gf		144.632	16.197	8.930	0.000	1.000	1.000
inf		107.477	10.945	9.819	0.000	107.477	0.267
com		97.967	9.626	10.178	0.000	97.967	0.298
sim		162.807	15.148	10.748	0.000	162.807	0.365
voc		57.805	8.514	6.790	0.000	57.805	0.137
pcc		106.242	11.622	9.141	0.000	106.242	0.423
bkd		214.955	20.417	10.528	0.000	214.955	0.562
par		260.349	24.956	10.432	0.000	260.349	0.550
oas		149.544	13.739	10.885	0.000	149.544	0.617

Group 2 [adult]:

Latent Variables:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc =~							
inf		1.000				17.666	0.873
com	(.p2.)	0.884	0.025	35.129	0.000	15.623	0.826
sim	(.p3.)	0.979	0.029	33.463	0.000	17.302	0.834
voc	(.p4.)	1.110	0.026	43.015	0.000	19.610	0.929
gf =~							
pcc		1.000				15.314	0.836
bkd	(.p6.)	1.077	0.038	28.513	0.000	16.494	0.798
par	(.p7.)	1.214	0.045	26.721	0.000	18.586	0.757
oas	(.p8.)	0.801	0.032	25.141	0.000	12.267	0.748

Covariances:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc ~~							
gf		235.838	23.784	9.916	0.000	0.872	0.872

Intercepts:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
inf	(.20.)	56.314	1.073	52.507	0.000	56.314	2.784
com	(.21.)	56.758	0.964	58.875	0.000	56.758	3.001

sim	(.22.)	57.828	1.084	53.329	0.000	57.828	2.787
voc	(.23.)	56.249	1.159	48.533	0.000	56.249	2.665
pcc	(.24.)	75.659	0.834	90.713	0.000	75.659	4.131
bkd	(.25.)	57.903	0.941	61.507	0.000	57.903	2.801
par	(.26.)	62.057	1.071	57.968	0.000	62.057	2.527
oas	(.27.)	73.451	0.729	100.707	0.000	73.451	4.479
gc		9.446	1.517	6.226	0.000	0.535	0.535
gf		0.597	1.264	0.472	0.637	0.039	0.039

Variances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc	312.100	30.298	10.301	0.000	1.000	1.000
gf	234.516	24.813	9.451	0.000	1.000	1.000
inf	97.098	10.392	9.343	0.000	97.098	0.237
com	113.546	11.175	10.161	0.000	113.546	0.317
sim	131.135	13.043	10.054	0.000	131.135	0.305
voc	61.106	8.612	7.096	0.000	61.106	0.137
pcc	100.952	11.442	8.823	0.000	100.952	0.301
bkd	155.153	16.221	9.565	0.000	155.153	0.363
par	257.428	25.522	10.087	0.000	257.428	0.427
oas	118.493	11.650	10.171	0.000	118.493	0.441

Group 3 [middl]:

Latent Variables:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc =~						
inf	1.000				18.295	0.890
com (.p2.)	0.884	0.025	35.129	0.000	16.179	0.848
sim (.p3.)	0.979	0.029	33.463	0.000	17.917	0.843
voc (.p4.)	1.110	0.026	43.015	0.000	20.307	0.930
gf =~						
pcc	1.000				15.675	0.815
bkd (.p6.)	1.077	0.038	28.513	0.000	16.883	0.792
par (.p7.)	1.214	0.045	26.721	0.000	19.023	0.747
oas (.p8.)	0.801	0.032	25.141	0.000	12.555	0.714

Covariances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc ~~						
gf	247.305	28.926	8.550	0.000	0.862	0.862

Intercepts:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
inf (.20.)	56.314	1.073	52.507	0.000	56.314	2.740
com (.21.)	56.758	0.964	58.875	0.000	56.758	2.976
sim (.22.)	57.828	1.084	53.329	0.000	57.828	2.721
voc (.23.)	56.249	1.159	48.533	0.000	56.249	2.575
pcc (.24.)	75.659	0.834	90.713	0.000	75.659	3.936
bkd (.25.)	57.903	0.941	61.507	0.000	57.903	2.715
par (.26.)	62.057	1.071	57.968	0.000	62.057	2.437
oas (.27.)	73.451	0.729	100.707	0.000	73.451	4.176
gc	7.238	1.678	4.314	0.000	0.396	0.396

gf	-7.724	1.428	-5.410	0.000	-0.493	-0.493
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Variances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc	334.699	36.814	9.092	0.000	1.000	1.000
gf	245.691	30.080	8.168	0.000	1.000	1.000
inf	87.564	11.178	7.833	0.000	87.564	0.207
com	101.875	11.832	8.610	0.000	101.875	0.280
sim	130.772	15.070	8.678	0.000	130.772	0.289
voc	64.725	10.216	6.336	0.000	64.725	0.136
pcc	123.795	15.865	7.803	0.000	123.795	0.335
bkd	169.901	20.771	8.180	0.000	169.901	0.373
par	286.778	33.052	8.677	0.000	286.778	0.442
oas	151.771	16.979	8.939	0.000	151.771	0.491

Group 4 [older]:

Latent Variables:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc =~						
inf	1.000				18.934	0.910
com (.p2.)	0.884	0.025	35.129	0.000	16.744	0.815
sim (.p3.)	0.979	0.029	33.463	0.000	18.543	0.731
voc (.p4.)	1.110	0.026	43.015	0.000	21.017	0.903
gf =~						
pcc	1.000				15.376	0.721
bkd (.p6.)	1.077	0.038	28.513	0.000	16.562	0.833
par (.p7.)	1.214	0.045	26.721	0.000	18.661	0.766
oas (.p8.)	0.801	0.032	25.141	0.000	12.317	0.700

Covariances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc ~~						
gf	248.390	33.234	7.474	0.000	0.853	0.853

Intercepts:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
inf (.20.)	56.314	1.073	52.507	0.000	56.314	2.706
com (.21.)	56.758	0.964	58.875	0.000	56.758	2.762
sim (.22.)	57.828	1.084	53.329	0.000	57.828	2.280
voc (.23.)	56.249	1.159	48.533	0.000	56.249	2.416
pcc (.24.)	75.659	0.834	90.713	0.000	75.659	3.549
bkd (.25.)	57.903	0.941	61.507	0.000	57.903	2.913
par (.26.)	62.057	1.071	57.968	0.000	62.057	2.546
oas (.27.)	73.451	0.729	100.707	0.000	73.451	4.175
gc	-0.162	1.868	-0.087	0.931	-0.009	-0.009
gf	-21.892	1.617	-13.539	0.000	-1.424	-1.424

Variances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc	358.502	44.397	8.075	0.000	1.000	1.000
gf	236.431	33.183	7.125	0.000	1.000	1.000
inf	74.686	12.720	5.871	0.000	74.686	0.172

com	141.923	18.393	7.716	0.000	141.923	0.336
sim	299.460	36.336	8.241	0.000	299.460	0.465
voc	100.519	16.400	6.129	0.000	100.519	0.185
pcc	218.127	27.975	7.797	0.000	218.127	0.480
bkd	120.929	18.713	6.462	0.000	120.929	0.306
par	245.840	33.113	7.424	0.000	245.840	0.414
oas	157.879	19.916	7.927	0.000	157.879	0.510

```
# test of model comparisons
anova(config_fit, metric_fit, scalar_fit)
```

Chi Square Difference Test

	Df	AIC	BIC	Chisq	Chisq diff	Df diff	Pr(>Chisq)
config_fit	76	61408	61893	216.73			
metric_fit	94	61458	61856	302.65	85.918	18	7.742e-11 ***
scalar_fit	112	61584	61894	464.71	162.062	18	< 2.2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
# strict invariance (= loadings + intercepts + uniquenesses)
strict_fit = cfa(config_model,
  sample.cov = grp.cov,
  sample.nobs = grp.ns,
  sample.mean = grp.mus,
  meanstructure = TRUE,
  group.equal=c("loadings","intercepts","residuals"))

summary(strict_fit, fit.measures=T, standardized=T)
```

lavaan (0.5-20) converged normally after 351 iterations

Number of observations per group	
young	300
adult	275
middl	205
older	160

Estimator	ML
Minimum Function Test Statistic	557.234
Degrees of freedom	136
P-value (Chi-square)	0.000

Chi-square for each group:

young	202.950
adult	97.843
middl	73.452
older	182.989

Model test baseline model:

Minimum Function Test Statistic	5399.422
---------------------------------	----------

Degrees of freedom	112
P-value	0.000

User model versus baseline model:

Comparative Fit Index (CFI)	0.920
Tucker-Lewis Index (TLI)	0.934

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-30774.364
Loglikelihood unrestricted model (H1)	-30495.747

Number of free parameters	40
Akaike (AIC)	61628.728
Bayesian (BIC)	61822.563
Sample-size adjusted Bayesian (BIC)	61695.526

Root Mean Square Error of Approximation:

RMSEA	0.115
90 Percent Confidence Interval	0.105 0.125
P-value RMSEA <= 0.05	0.000

Standardized Root Mean Square Residual:

SRMR	0.091
------	-------

Parameter Estimates:

Information	Expected
Standard Errors	Standard

Group 1 [young]:

Latent Variables:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc =~						
inf	1.000				17.187	0.870
com (.p2.)	0.883	0.025	34.966	0.000	15.183	0.822
sim (.p3.)	1.000	0.030	33.314	0.000	17.185	0.799
voc (.p4.)	1.103	0.026	42.728	0.000	18.953	0.917
gf =~						
pcc	1.000				12.386	0.745
bkd (.p6.)	1.054	0.036	28.947	0.000	13.052	0.703
par (.p7.)	1.179	0.043	27.506	0.000	14.605	0.668
oas (.p8.)	0.783	0.030	25.959	0.000	9.697	0.632

Covariances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc ~~						
gf	165.720	17.766	9.328	0.000	0.778	0.778

Intercepts:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
inf	(.20.)	56.312	1.070	52.621	0.000	56.312	2.849
com	(.21.)	56.768	0.967	58.711	0.000	56.768	3.072
sim	(.22.)	57.339	1.107	51.785	0.000	57.339	2.666
voc	(.23.)	56.566	1.157	48.892	0.000	56.566	2.738
pcc	(.24.)	75.805	0.864	87.730	0.000	75.805	4.559
bkd	(.25.)	57.953	0.935	62.000	0.000	57.953	3.121
par	(.26.)	62.078	1.071	57.940	0.000	62.078	2.841
oas	(.27.)	73.433	0.732	100.321	0.000	73.433	4.783
gc		0.000				0.000	0.000
gf		0.000				0.000	0.000

Variances:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc		295.401	27.714	10.659	0.000	1.000	1.000
gf		153.415	16.982	9.034	0.000	1.000	1.000
inf	(.12.)	95.207	5.780	16.472	0.000	95.207	0.244
com	(.13.)	110.977	6.075	18.267	0.000	110.977	0.325
sim	(.14.)	167.392	8.904	18.800	0.000	167.392	0.362
voc	(.15.)	67.733	5.241	12.923	0.000	67.733	0.159
pcc	(.16.)	123.084	7.518	16.371	0.000	123.084	0.445
bkd	(.17.)	174.414	9.939	17.549	0.000	174.414	0.506
par	(.18.)	264.134	14.447	18.283	0.000	264.134	0.553
oas	(.19.)	141.675	7.495	18.901	0.000	141.675	0.601

Group 2 [adult]:

Latent Variables:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc =~							
inf		1.000				17.616	0.875
com	(.p2.)	0.883	0.025	34.966	0.000	15.562	0.828
sim	(.p3.)	1.000	0.030	33.314	0.000	17.614	0.806
voc	(.p4.)	1.103	0.026	42.728	0.000	19.426	0.921
gf =~							
pcc		1.000				15.429	0.812
bkd	(.p6.)	1.054	0.036	28.947	0.000	16.259	0.776
par	(.p7.)	1.179	0.043	27.506	0.000	18.193	0.746
oas	(.p8.)	0.783	0.030	25.959	0.000	12.079	0.712

Covariances:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc ~~							
gf		240.810	24.201	9.950	0.000	0.886	0.886

Intercepts:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
inf	(.20.)	56.312	1.070	52.621	0.000	56.312	2.796
com	(.21.)	56.768	0.967	58.711	0.000	56.768	3.021
sim	(.22.)	57.339	1.107	51.785	0.000	57.339	2.624
voc	(.23.)	56.566	1.157	48.892	0.000	56.566	2.681
pcc	(.24.)	75.805	0.864	87.730	0.000	75.805	3.989

bkd	(.25.)	57.953	0.935	62.000	0.000	57.953	2.767
par	(.26.)	62.078	1.071	57.940	0.000	62.078	2.545
oas	(.27.)	73.433	0.732	100.321	0.000	73.433	4.330
gc		9.370	1.518	6.174	0.000	0.532	0.532
gf		0.547	1.291	0.424	0.672	0.035	0.035

Variances:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc		310.323	30.165	10.287	0.000	1.000	1.000
gf		238.060	25.499	9.336	0.000	1.000	1.000
inf	(.12.)	95.207	5.780	16.472	0.000	95.207	0.235
com	(.13.)	110.977	6.075	18.267	0.000	110.977	0.314
sim	(.14.)	167.392	8.904	18.800	0.000	167.392	0.350
voc	(.15.)	67.733	5.241	12.923	0.000	67.733	0.152
pcc	(.16.)	123.084	7.518	16.371	0.000	123.084	0.341
bkd	(.17.)	174.414	9.939	17.549	0.000	174.414	0.397
par	(.18.)	264.134	14.447	18.283	0.000	264.134	0.444
oas	(.19.)	141.675	7.495	18.901	0.000	141.675	0.493

Group 3 [middl]:

Latent Variables:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc =~							
inf		1.000				18.149	0.881
com	(.p2.)	0.883	0.025	34.966	0.000	16.033	0.836
sim	(.p3.)	1.000	0.030	33.314	0.000	18.147	0.814
voc	(.p4.)	1.103	0.026	42.728	0.000	20.014	0.925
gf =~							
pcc		1.000				15.945	0.821
bkd	(.p6.)	1.054	0.036	28.947	0.000	16.803	0.786
par	(.p7.)	1.179	0.043	27.506	0.000	18.802	0.757
oas	(.p8.)	0.783	0.030	25.959	0.000	12.483	0.724

Covariances:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc ~~							
gf		249.318	29.165	8.549	0.000	0.862	0.862

Intercepts:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
inf	(.20.)	56.312	1.070	52.621	0.000	56.312	2.733
com	(.21.)	56.768	0.967	58.711	0.000	56.768	2.959
sim	(.22.)	57.339	1.107	51.785	0.000	57.339	2.573
voc	(.23.)	56.566	1.157	48.892	0.000	56.566	2.614
pcc	(.24.)	75.805	0.864	87.730	0.000	75.805	3.902
bkd	(.25.)	57.953	0.935	62.000	0.000	57.953	2.712
par	(.26.)	62.078	1.071	57.940	0.000	62.078	2.498
oas	(.27.)	73.433	0.732	100.321	0.000	73.433	4.257
gc		7.249	1.674	4.330	0.000	0.399	0.399
gf		-7.903	1.454	-5.435	0.000	-0.496	-0.496

Variances:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc		329.401	36.441	9.039	0.000	1.000	1.000
gf		254.254	30.764	8.265	0.000	1.000	1.000
inf	(.12.)	95.207	5.780	16.472	0.000	95.207	0.224
com	(.13.)	110.977	6.075	18.267	0.000	110.977	0.302
sim	(.14.)	167.392	8.904	18.800	0.000	167.392	0.337
voc	(.15.)	67.733	5.241	12.923	0.000	67.733	0.145
pcc	(.16.)	123.084	7.518	16.371	0.000	123.084	0.326
bkd	(.17.)	174.414	9.939	17.549	0.000	174.414	0.382
par	(.18.)	264.134	14.447	18.283	0.000	264.134	0.428
oas	(.19.)	141.675	7.495	18.901	0.000	141.675	0.476

Group 4 [older]:

Latent Variables:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc =~							
inf		1.000				19.333	0.893
com	(.p2.)	0.883	0.025	34.966	0.000	17.079	0.851
sim	(.p3.)	1.000	0.030	33.314	0.000	19.331	0.831
voc	(.p4.)	1.103	0.026	42.728	0.000	21.320	0.933
gf =~							
pcc		1.000				16.804	0.835
bkd	(.p6.)	1.054	0.036	28.947	0.000	17.708	0.802
par	(.p7.)	1.179	0.043	27.506	0.000	19.814	0.773
oas	(.p8.)	0.783	0.030	25.959	0.000	13.155	0.742

Covariances:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc ~~							
gf		275.569	36.410	7.568	0.000	0.848	0.848

Intercepts:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
inf	(.20.)	56.312	1.070	52.621	0.000	56.312	2.600
com	(.21.)	56.768	0.967	58.711	0.000	56.768	2.829
sim	(.22.)	57.339	1.107	51.785	0.000	57.339	2.465
voc	(.23.)	56.566	1.157	48.892	0.000	56.566	2.475
pcc	(.24.)	75.805	0.864	87.730	0.000	75.805	3.765
bkd	(.25.)	57.953	0.935	62.000	0.000	57.953	2.623
par	(.26.)	62.078	1.071	57.940	0.000	62.078	2.422
oas	(.27.)	73.433	0.732	100.321	0.000	73.433	4.139
gc		-0.558	1.885	-0.296	0.767	-0.029	-0.029
gf		-22.387	1.688	-13.264	0.000	-1.332	-1.332

Variances:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc		373.783	46.033	8.120	0.000	1.000	1.000
gf		282.381	37.756	7.479	0.000	1.000	1.000
inf	(.12.)	95.207	5.780	16.472	0.000	95.207	0.203
com	(.13.)	110.977	6.075	18.267	0.000	110.977	0.276
sim	(.14.)	167.392	8.904	18.800	0.000	167.392	0.309
voc	(.15.)	67.733	5.241	12.923	0.000	67.733	0.130

pcc	(.16.)	123.084	7.518	16.371	0.000	123.084	0.304
bkd	(.17.)	174.414	9.939	17.549	0.000	174.414	0.357
par	(.18.)	264.134	14.447	18.283	0.000	264.134	0.402
oas	(.19.)	141.675	7.495	18.901	0.000	141.675	0.450

```
# test of model comparisons
anova(config_fit, metric_fit, scalar_fit, strict_fit)
```

Chi Square Difference Test

	Df	AIC	BIC	Chisq	Chisq diff	Df diff	Pr(>Chisq)
config_fit	76	61408	61893	216.73			
metric_fit	94	61458	61856	302.65	85.918	18	7.742e-11 ***
scalar_fit	112	61584	61894	464.71	162.062	18	< 2.2e-16 ***
strict_fit	136	61629	61823	557.23	92.522	24	5.491e-10 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Explicit Group Constraints for Strict Invariance

```
grp_fmus_model = '
# Needed for weak (metric) invariance
# factor identification
gc =~ c(1,1,1,1)*inf
      + c(11,11,11,11)*com
      + c(12,12,12,12)*sim
      + c(13,13,13,13)*voc
gf =~ c(1,1,1,1)*pcc
      + c(14,14,14,14)*bkd
      + c(15,15,15,15)*par
      + c(16,16,16,16)*oas

# factor covariances
gc ~~ gf
# factor variances
gc ~~ gc
gf ~~ gf

# factor means
# estimated differences in factor means
# cross-sectional differences.
gc ~ c(0,NA,NA,NA)*1
gf ~ c(0,NA,NA,NA)*1

# Needed for strong (scalar) invariance
# manifest intercepts
inf ~ c(t1,t1,t1,t1)*1
com ~ c(t2,t2,t2,t2)*1
sim ~ c(t3,t3,t3,t3)*1
voc ~ c(t4,t4,t4,t4)*1
pcc ~ c(t5,t5,t5,t5)*1
bkd ~ c(t6,t6,t6,t6)*1
```



```

par ~ c(t7,t7,t7,t7)*1
oas ~ c(t8,t8,t8,t8)*1

# Needed for strict invariance
# manifest residual variances
inf ~~ c(u1,u1,u1,u1)*inf
com ~~ c(u2,u2,u2,u2)*com
sim ~~ c(u3,u3,u3,u3)*sim
voc ~~ c(u4,u4,u4,u4)*voc
pcc ~~ c(u5,u5,u5,u5)*pcc
bkd ~~ c(u6,u6,u6,u6)*bkd
par ~~ c(u7,u7,u7,u7)*par
oas ~~ c(u8,u8,u8,u8)*oas

'

grp_fmus_fit = cfa(grp_fmus_model,
                  sample.cov = grp.cov,
                  sample.nobs = grp.ns,
                  sample.mean = grp.mus)

summary(grp_fmus_fit, fit.measures=T, standardized=T)

```

lavaan (0.5-20) converged normally after 291 iterations

Number of observations per group	
young	300
adult	275
middl	205
older	160

Estimator	ML
Minimum Function Test Statistic	557.234
Degrees of freedom	136
P-value (Chi-square)	0.000

Chi-square for each group:

young	202.950
adult	97.843
middl	73.452
older	182.989

Model test baseline model:

Minimum Function Test Statistic	5399.422
Degrees of freedom	112
P-value	0.000

User model versus baseline model:

Comparative Fit Index (CFI)	0.920
Tucker-Lewis Index (TLI)	0.934

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-30774.364
Loglikelihood unrestricted model (H1)	-30495.747
Number of free parameters	40
Akaike (AIC)	61628.728
Bayesian (BIC)	61822.563
Sample-size adjusted Bayesian (BIC)	61695.526

Root Mean Square Error of Approximation:

RMSEA	0.115
90 Percent Confidence Interval	0.105 0.125
P-value RMSEA <= 0.05	0.000

Standardized Root Mean Square Residual:

SRMR	0.091
------	-------

Parameter Estimates:

Information	Expected
Standard Errors	Standard

Group 1 [young]:

Latent Variables:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc =~						
inf	1.000				17.187	0.870
com (11)	0.883	0.025	34.966	0.000	15.183	0.822
sim (12)	1.000	0.030	33.314	0.000	17.185	0.799
voc (13)	1.103	0.026	42.728	0.000	18.953	0.917
gf =~						
pcc	1.000				12.386	0.745
bkd (14)	1.054	0.036	28.947	0.000	13.052	0.703
par (15)	1.179	0.043	27.506	0.000	14.605	0.668
oas (16)	0.783	0.030	25.959	0.000	9.697	0.632

Covariances:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc ~~						
gf	165.720	17.766	9.328	0.000	0.778	0.778

Intercepts:

	Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc	0.000				0.000	0.000
gf	0.000				0.000	0.000
inf (t1)	56.312	1.070	52.621	0.000	56.312	2.849
com (t2)	56.768	0.967	58.711	0.000	56.768	3.072
sim (t3)	57.339	1.107	51.785	0.000	57.339	2.666

voc	(t4)	56.566	1.157	48.892	0.000	56.566	2.738
pcc	(t5)	75.805	0.864	87.730	0.000	75.805	4.559
bkd	(t6)	57.953	0.935	62.000	0.000	57.953	3.121
par	(t7)	62.078	1.071	57.940	0.000	62.078	2.841
oas	(t8)	73.433	0.732	100.321	0.000	73.433	4.783

Variances:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc		295.401	27.714	10.659	0.000	1.000	1.000
gf		153.414	16.982	9.034	0.000	1.000	1.000
inf	(u1)	95.207	5.780	16.472	0.000	95.207	0.244
com	(u2)	110.977	6.075	18.267	0.000	110.977	0.325
sim	(u3)	167.392	8.904	18.800	0.000	167.392	0.362
voc	(u4)	67.733	5.241	12.923	0.000	67.733	0.159
pcc	(u5)	123.084	7.518	16.371	0.000	123.084	0.445
bkd	(u6)	174.414	9.939	17.549	0.000	174.414	0.506
par	(u7)	264.134	14.447	18.283	0.000	264.134	0.553
oas	(u8)	141.675	7.495	18.901	0.000	141.675	0.601

Group 2 [adult]:

Latent Variables:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc =~							
inf		1.000				17.616	0.875
com	(11)	0.883	0.025	34.966	0.000	15.562	0.828
sim	(12)	1.000	0.030	33.314	0.000	17.614	0.806
voc	(13)	1.103	0.026	42.728	0.000	19.426	0.921
gf =~							
pcc		1.000				15.429	0.812
bkd	(14)	1.054	0.036	28.947	0.000	16.259	0.776
par	(15)	1.179	0.043	27.506	0.000	18.193	0.746
oas	(16)	0.783	0.030	25.959	0.000	12.079	0.712

Covariances:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc ~~							
gf		240.810	24.201	9.950	0.000	0.886	0.886

Intercepts:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc		9.370	1.518	6.174	0.000	0.532	0.532
gf		0.547	1.291	0.424	0.672	0.035	0.035
inf	(t1)	56.312	1.070	52.621	0.000	56.312	2.796
com	(t2)	56.768	0.967	58.711	0.000	56.768	3.021
sim	(t3)	57.339	1.107	51.785	0.000	57.339	2.624
voc	(t4)	56.566	1.157	48.892	0.000	56.566	2.681
pcc	(t5)	75.805	0.864	87.730	0.000	75.805	3.989
bkd	(t6)	57.953	0.935	62.000	0.000	57.953	2.767
par	(t7)	62.078	1.071	57.940	0.000	62.078	2.545
oas	(t8)	73.433	0.732	100.321	0.000	73.433	4.330

Variances:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc		310.323	30.165	10.287	0.000	1.000	1.000
gf		238.059	25.499	9.336	0.000	1.000	1.000
inf	(u1)	95.207	5.780	16.472	0.000	95.207	0.235
com	(u2)	110.977	6.075	18.267	0.000	110.977	0.314
sim	(u3)	167.392	8.904	18.800	0.000	167.392	0.350
voc	(u4)	67.733	5.241	12.923	0.000	67.733	0.152
pcc	(u5)	123.084	7.518	16.371	0.000	123.084	0.341
bkd	(u6)	174.414	9.939	17.549	0.000	174.414	0.397
par	(u7)	264.134	14.447	18.283	0.000	264.134	0.444
oas	(u8)	141.675	7.495	18.901	0.000	141.675	0.493

Group 3 [middl]:

Latent Variables:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc =~							
inf		1.000				18.149	0.881
com	(11)	0.883	0.025	34.966	0.000	16.033	0.836
sim	(12)	1.000	0.030	33.314	0.000	18.147	0.814
voc	(13)	1.103	0.026	42.728	0.000	20.014	0.925
gf =~							
pcc		1.000				15.945	0.821
bkd	(14)	1.054	0.036	28.947	0.000	16.803	0.786
par	(15)	1.179	0.043	27.506	0.000	18.802	0.757
oas	(16)	0.783	0.030	25.959	0.000	12.483	0.724

Covariances:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc ~~							
gf		249.318	29.165	8.549	0.000	0.862	0.862

Intercepts:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc		7.249	1.674	4.330	0.000	0.399	0.399
gf		-7.903	1.454	-5.435	0.000	-0.496	-0.496
inf	(t1)	56.312	1.070	52.621	0.000	56.312	2.733
com	(t2)	56.768	0.967	58.711	0.000	56.768	2.959
sim	(t3)	57.339	1.107	51.785	0.000	57.339	2.573
voc	(t4)	56.566	1.157	48.892	0.000	56.566	2.614
pcc	(t5)	75.805	0.864	87.730	0.000	75.805	3.902
bkd	(t6)	57.953	0.935	62.000	0.000	57.953	2.712
par	(t7)	62.078	1.071	57.940	0.000	62.078	2.498
oas	(t8)	73.433	0.732	100.321	0.000	73.433	4.257

Variances:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc		329.401	36.441	9.039	0.000	1.000	1.000
gf		254.254	30.764	8.265	0.000	1.000	1.000
inf	(u1)	95.207	5.780	16.472	0.000	95.207	0.224
com	(u2)	110.977	6.075	18.267	0.000	110.977	0.302
sim	(u3)	167.392	8.904	18.800	0.000	167.392	0.337
voc	(u4)	67.733	5.241	12.923	0.000	67.733	0.145

pcc	(u5)	123.084	7.518	16.371	0.000	123.084	0.326
bkd	(u6)	174.414	9.939	17.549	0.000	174.414	0.382
par	(u7)	264.134	14.447	18.283	0.000	264.134	0.428
oas	(u8)	141.675	7.495	18.901	0.000	141.675	0.476

Group 4 [older]:

Latent Variables:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc	=~						
inf		1.000				19.333	0.893
com	(11)	0.883	0.025	34.966	0.000	17.079	0.851
sim	(12)	1.000	0.030	33.314	0.000	19.331	0.831
voc	(13)	1.103	0.026	42.728	0.000	21.320	0.933
gf	=~						
pcc		1.000				16.804	0.835
bkd	(14)	1.054	0.036	28.947	0.000	17.708	0.802
par	(15)	1.179	0.043	27.506	0.000	19.814	0.773
oas	(16)	0.783	0.030	25.959	0.000	13.155	0.742

Covariances:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc	~~						
gf		275.569	36.410	7.568	0.000	0.848	0.848

Intercepts:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc		-0.558	1.885	-0.296	0.767	-0.029	-0.029
gf		-22.387	1.688	-13.264	0.000	-1.332	-1.332
inf	(t1)	56.312	1.070	52.621	0.000	56.312	2.600
com	(t2)	56.768	0.967	58.711	0.000	56.768	2.829
sim	(t3)	57.339	1.107	51.785	0.000	57.339	2.465
voc	(t4)	56.566	1.157	48.892	0.000	56.566	2.475
pcc	(t5)	75.805	0.864	87.730	0.000	75.805	3.765
bkd	(t6)	57.953	0.935	62.000	0.000	57.953	2.623
par	(t7)	62.078	1.071	57.940	0.000	62.078	2.422
oas	(t8)	73.433	0.732	100.321	0.000	73.433	4.139

Variances:

		Estimate	Std.Err	Z-value	P(> z)	Std.lv	Std.all
gc		373.783	46.033	8.120	0.000	1.000	1.000
gf		282.381	37.756	7.479	0.000	1.000	1.000
inf	(u1)	95.207	5.780	16.472	0.000	95.207	0.203
com	(u2)	110.977	6.075	18.267	0.000	110.977	0.276
sim	(u3)	167.392	8.904	18.800	0.000	167.392	0.309
voc	(u4)	67.733	5.241	12.923	0.000	67.733	0.130
pcc	(u5)	123.084	7.518	16.371	0.000	123.084	0.304
bkd	(u6)	174.414	9.939	17.549	0.000	174.414	0.357
par	(u7)	264.134	14.447	18.283	0.000	264.134	0.402
oas	(u8)	141.675	7.495	18.901	0.000	141.675	0.450